Math 115 — Practice for Exam 2

Generated February 20, 2017

Name: _____

Instructor: _____ Section Number: _____

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1. This exam has 6 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2010	2	5	hyperbola	13	
Fall 2010	2	5		12	
Fall 2011	2	8	rose curve	12	
Winter 2012	2	3		12	
Fall 2012	2	1		12	
Winter 2013	2	7		10	
Total				71	

Recommended time (based on points): 64 minutes

5. [13 points] The equation below implicitly defines a hyperbola.

$$x^2 - y^2 = 2x + xy + y + 2.$$

a. [5 points] Find $\frac{dy}{dx}$.

b. [4 points] Consider the two points $(4, 2)$ and $(2, -1)$. Show that one of these points lies on the hyperbola defined above, and one does not.

c. [4 points] For the point in part (b) which lies on the hyperbola, find the equation of the line which is tangent to the hyperbola at this point.

5. [12 points] Suppose a curve in the plane is given by the equation

$$\sin(\pi xy) = y - 1.$$

a. [3 points] Verify that the point $(x, y) = (1, 1)$ is on the curve.

b. [5 points] Calculate $\dfrac{dy}{dx}$.

c. [4 points] Find the equation for the tangent line to the curve at the point $(1, 1)$.

8. [12 points] The equation $(x^2 + y^2)^2 = 4x^2y$ describes a two-petaled rose curve.

a. [2 points] Verify that the point $(x, y) = (1, 1)$ is on the curve.

b. [7 points] Calculate dy/dx at $(x, y) = (1, 1)$.

c. [3 points] Find the equation of the tangent line to the rose curve at the point $(x, y) = (1, 1)$.

3. [12 points] The following questions relate to the implicit function

$$y^2 + 4x = 4xy^2.$$

a. [4 points] Compute $\dfrac{dy}{dx}$.

b. [4 points] Find the equation for the tangent line to this curve at the point $(\frac{1}{3}, 2)$.

c. [4 points] Find the x- and y-coordinates of all points at which the tangent line to this curve is vertical.

1. [12 points] The following questions relate to the implicit curve $2x^2 + 4x - x^2y^2 + 3y^4 = -1$.

a. [6 points] Calculate $\frac{dy}{dx}$.

b. [2 points] Q is the only point on the curve that has a y-coordinate of 1. Find the x-coordinate of Q.

c. [4 points] Find the equation of the tangent line to the curve at Q.

7. [10 points] For each real number k, there is a curve in the plane given by the equation

$$e^{y^2} = x^3 + k.$$

a. [4 points] Find $\frac{dy}{dx}$.

b. [3 points] Suppose that $k = 9$. There are two points on the curve where the tangent line is horizontal. Find the x and y coordinates of each one.

c. [3 points] Now suppose that $k = \frac{1}{2}$. How many points are there where the curve has a horizontal tangent line?