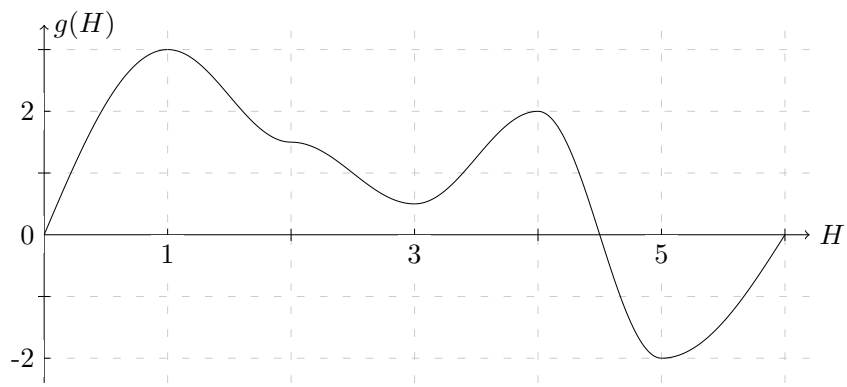
6. [10 points] After receiving a termination notice, The Intern has begun to read up on the global job market. A dubious pop-economics book he is reading claims that the rate at which interns are hired or terminated in a large company is purely a function of the number of interns at the company. Specifically, it states that

$$\frac{dH}{dt} = g'(H),$$

where $H(t)$ gives the number of interns at a company, in thousands, after t days, and $g(H)$ is a differentiable function. A graph of $g(H)$ (**not** $g'(H)$) is given in the book:



a. [2 points] What are the units of $g'(H)$?

Solution: The units are thousands of interns per day.

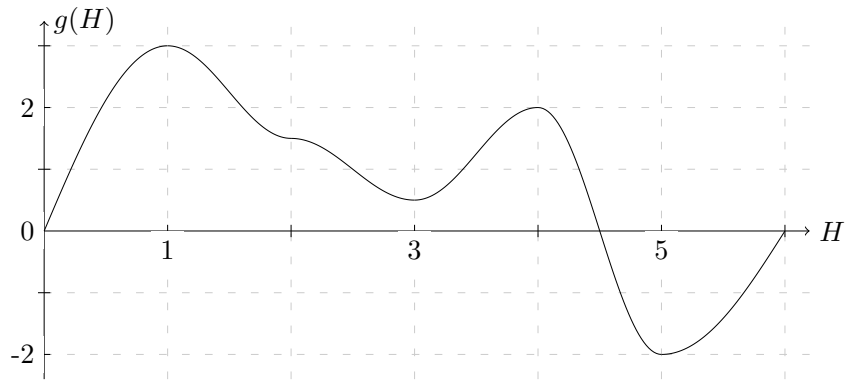
b. [3 points] Are there any **stable** equilibrium solutions of the differential equation? If so, what are they?

Solution: Yes; the stable equilibrium solutions are $H = 1$ and $H = 4$.

6. (continued). Recall that the number of interns in thousands $H(t)$ satisfies

$$\frac{dH}{dt} = g'(H),$$

where a graph of $g(H)$ (**not** $g'(H)$) is given below:



c. [2 points] If a company starts with 3,500 interns, what will happen to the number of interns in the long run?

Solution: The number of interns will approach 4,000 asymptotically from below.

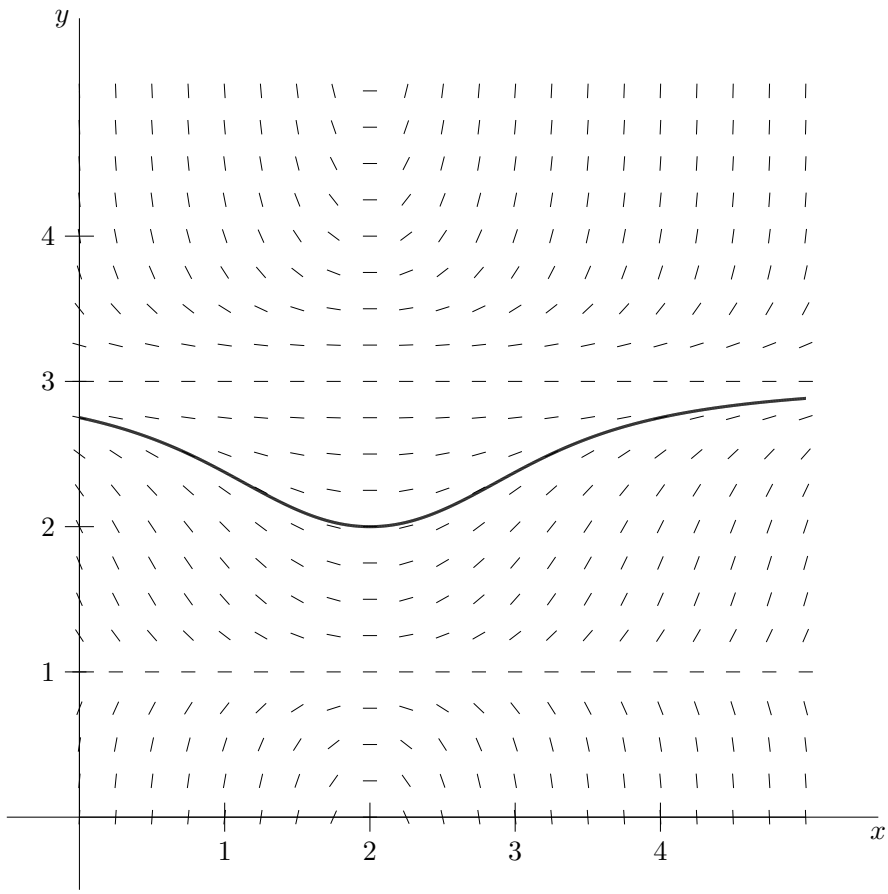
d. [1 point] Estimate the number of interns at which the number of interns is decreasing the fastest.

Solution: The number of interns is decreasing the fastest when there are $\approx 4,500$ interns.

e. [2 points] Suppose that a company begins with 5,500 interns. If you used Euler's method to estimate how many interns there will be 5 days from now, would you expect an underestimate or an overestimate? Justify your answer briefly.

Solution: The corresponding solution of the differential equation is concave up, so we expect Euler's method to yield an underestimate.

7. [9 points] The graph of a slope field corresponding to a differential equation is shown below.



a. [3 points] On the slope field, carefully sketch a solution curve passing through the point $(2, 2)$ with domain $0 \le x \le 5$.

b. [4 points] The slope field pictured above is the slope field for one of the following differential equations. Which one? Circle your answer. You do not need to show your work.

$\boxed{\dfrac{dy}{dx} = (x-2)(y-1)(y-3)^2}$ $\qquad$ $\dfrac{dy}{dx} = (x+2)(y+1)(y+3)^2$

$\dfrac{dy}{dx} = (x-2)(y-1)^2(y-3)^2$ $\qquad$ $\dfrac{dy}{dx} = (x-2)(y-1)^2(y-3)$

c. [2 points] If we use Euler's method starting at the point $(2, 2)$ and use $\Delta x = 0.1$, would we get an overestimate or an underestimate for the value of $y(2.5)$? Circle your answer. You do not need to show your work.

overestimate $\qquad$ $\boxed{\text{underestimate}}$