looks as though we have treated dy/dx as a fraction, cross-multiplied, and then integrated. Although that's not exactly what we have done, you may find this a helpful way of remembering the method. In fact, the dy/dx notation was introduced by Leibniz to allow shortcuts like this (more specifically, to make the chain rule look like cancellation).

Exercises and Problems for Section 11.4

EXERCISES

1. Determine which of the following differential equations are separable. Do not solve the equations.

(a) $y' = y$	**(b)** $y' = x + y$
(c) $y' = xy$	**(d)** $y' = \sin(x + y)$
(e) $y' - xy = 0$	**(f)** $y' = y/x$
(g) $y' = \ln(xy)$	**(h)** $y' = (\sin x)(\cos y)$
(i) $y' = (\sin x)(\cos xy)$	**(j)** $y' = x/y$
(k) $y' = 2x$	**(l)** $y' = (x + y)/(x + 2y)$

■ For Exercises 2–5, determine if the differential equation is separable, and if so, write it in the form $h(y)\,dy = g(x)\,dx$.

2. $y' = xe^y$

3. $y' = xe^y - x$

4. $y' = xe^y - 1$

5. $y' - xy' = y$

■ In Exercises 6–32, use separation of variables to find the solution to the differential equation subject to the initial condition.

6. $\dfrac{dP}{dt} = -2P, \quad P(0) = 1$

7. $\dfrac{dP}{dt} = 0.02P, \quad P(0) = 20$

8. $\dfrac{dL}{dp} = \dfrac{L}{2}, \quad L(0) = 100$

9. $\dfrac{dQ}{dt} = \dfrac{Q}{5}, \quad Q = 50$ when $t = 0$

10. $P\dfrac{dP}{dt} = 1, \quad P(0) = 1$

11. $\dfrac{dm}{dt} = 3m, \quad m = 5$ when $t = 1$

12. $\dfrac{dI}{dx} = 0.2I, \quad I = 6$ where $x = -1$

13. $\dfrac{1}{z}\dfrac{dz}{dt} = 5, \quad z(1) = 5$

14. $\dfrac{dm}{ds} = m, \quad m(1) = 2$

15. $2\dfrac{du}{dt} = u^2, \quad u(0) = 1$

16. $\dfrac{dz}{dy} = zy, \quad z = 1$ when $y = 0$

17. $\dfrac{dy}{dx} + \dfrac{y}{3} = 0, \quad y(0) = 10$

18. $\dfrac{dy}{dt} = 0.5(y - 200), \quad y = 50$ when $t = 0$

19. $\dfrac{dP}{dt} = P + 4, \quad P = 100$ when $t = 0$

20. $\dfrac{dy}{dx} = 2y - 4,$ through $(2, 5)$

21. $\dfrac{dQ}{dt} = 0.3Q - 120, \quad Q = 50$ when $t = 0$

22. $\dfrac{dm}{dt} = 0.1m + 200, \quad m(0) = 1000$

23. $\dfrac{dR}{dy} + R = 1, \quad R(1) = 0.1$

24. $\dfrac{dB}{dt} + 2B = 50, \quad B(1) = 100$

25. $\dfrac{dy}{dt} = \dfrac{y}{3 + t}, \quad y(0) = 1$

26. $\dfrac{dz}{dt} = te^z,$ through the origin

27. $\dfrac{dy}{dx} = \dfrac{5y}{x}, \quad y = 3$ where $x = 1$

28. $\dfrac{dy}{dt} = y^2(1 + t), \quad y = 2$ when $t = 1$

29. $\dfrac{dz}{dt} = z + zt^2, \quad z = 5$ when $t = 0$

30. $\dfrac{dw}{d\theta} = \theta w^2 \sin\theta^2, \quad w(0) = 1$

31. $\dfrac{dw}{d\psi} = -w^2 \tan\psi, \quad w(0) = 2$

32. $x(x + 1)\dfrac{du}{dx} = u^2, \quad u(1) = 1$

PROBLEMS

33. **(a)** Solve the differential equation

$$\frac{dy}{dx} = \frac{4x}{y^2}.$$

Write the solution y as an explicit function of x.

(b) Find the particular solution for each initial condition below and graph the three solutions on the same coordinate plane.

$$y(0) = 1, \qquad y(0) = 2, \qquad y(0) = 3.$$