

7. [12 points] Solve each of the equations below. *Show your work step-by-step and write the solutions in exact form in the answer blanks provided.*

a. [3 points] $5(1.7)^{2y} = 2.4$

Solution: We first divide both sides of the equation by 5 and then use logarithms to find y .

$$5(1.7)^{2y} = 2.4$$

$$(1.7)^{2y} = \frac{2.4}{5} = 0.48$$

$$\ln(1.7^{2y}) = \ln(0.48)$$

$$2y \ln(1.7) = \ln(0.48)$$

$$y = \frac{\ln(0.48)}{2 \ln(1.7)}$$

Answer: $y = \frac{\ln(0.48)}{2 \ln(1.7)}$

b. [3 points] $3t - 1 = \log(2(10)^{4.6t})$

Solution: Using properties of logarithms, we see that $\log(2(10)^{4.6t}) = \log(2) + \log(10^{4.6t}) = \log(2) + 4.6t$, so it remains to solve the equation $3t - 1 = \log(2) + 4.6t$. Then we find $-1.6t = \log(2) + 1$ so $t = \frac{\log(2) + 1}{-1.6}$

Answer: $t = \frac{\log(2) + 1}{-1.6}$

c. [3 points] $e^{\ln(w-4)} = \ln(3.2) - \ln(4)$

Solution: Applying basic properties of the natural logarithm, we see that $e^{\ln(w-4)} = w - 4$ and $\ln(3.2) - \ln(4) = \ln\left(\frac{3.2}{4}\right) = \ln(0.8)$. Thus $w - 4 = \ln(0.8)$ so $w = 4 + \ln(0.8)$. However, note that we cannot plug this value of w into the original equation (since this would involve $\ln(\ln(0.8))$, which is undefined because $\ln(0.8) < 0$). So, if there were a solution, to the equation, it would be $w = 4 + \ln(0.8)$, but there is actually no solution.

Answer: $w = \text{No solution}$

d. [3 points] $\log(2p + 1) - \log(p - 3) = 3$

Solution: We apply a basic property of logarithms and then use the definition of the logarithm (or exponentiate).

$$\log(2p + 1) - \log(p - 3) = 3$$

$$\log\left(\frac{2p + 1}{p - 3}\right) = 3$$

$$\frac{2p + 1}{p - 3} = 10^3 = 1000$$

$$2p + 1 = 1000(p - 3) = 1000p - 3000$$

$$3001 = 998p$$

$$\frac{3001}{998} = p$$

Answer: $p = \frac{3001}{998}$