From Cluster Algebra to Polytopes and beyond

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~~Bg~~ Physics: Solve equations $\nabla^2 V = 0$

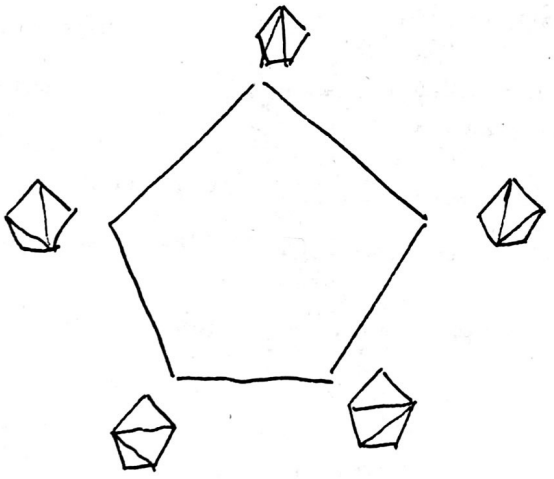
on some ~~boundary~~ area

A cluster alg is (X, X_0, B)

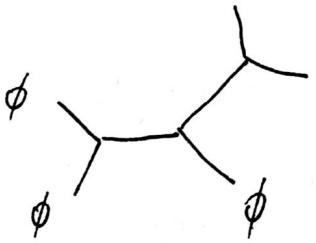
↑ cluster vars ↑ frozen vars ↑ mutation matrix

Both comes down to how much info. we need to determine the whole system.

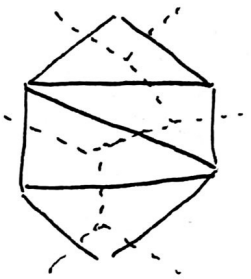
Polytopes:



Physics:



(diagram perturbation)



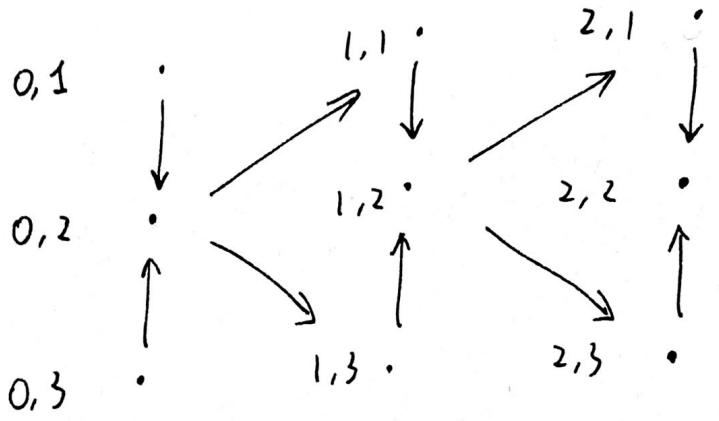
Given a polytope P, $\bar{P}$ is the dual polytope

then the volume form of $\bar{P}$ is the

canonical form of P up to the scalar

It turns out that the canonical form is exactly the observables

Ex: A_3 : ① → ② ← ③



$(i,k) \in Q$

$\rightsquigarrow (i,k) \to (i+1, j) \in \tilde{Q}$

(mega cluster)

each vertex corresponds to the set of all cluster variables

For polytopes, each facet has an associated facet variable W_i.

then we can calculate facet vectors using the mega cluster.

This gives a much faster way to calculate observables.