Local Cohomology & Hartshorne's Conj

Takumi

[Lyubeznik] "A survey of problems and results on the number of defining equations" (1989)

k = field R = comm unital Noetherian ring

§1 Introduction

<u>Ex</u> (Twisted Cubic)

$$I = (x^2 - yw,\ xz - y^2,\ xy - zw) \subseteq k[x, y, z, w]$$

(1) I is not a complete intersection

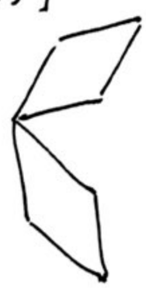
Recall: $I \subseteq R$ an ideal is a c.i if I is generated by ht I elements

(2) I is a set-theoretic c.i.

Recall: $I \subseteq R$ is a set-c.i. if $\sqrt{I} = \sqrt{(f_1, \cdots, f_{\mathrm{ht}\, I})}$

i.e. $\sqrt{I} = \sqrt{(x^2 - yw,\ z(xy - zw) + y(xz - y^2))}$

Ex [Hartshorne's]

Y  $\subseteq \mathbb{A}^4_k$ $I(Y) = (x, y) \cap (u, v) \subseteq k[x, y, u, v]$

$\dim Y = 2$ $\sqrt{I(Y)} = \sqrt{(xu,\ yv,\ xv + yu)}$

but we know $H^3_I(R) \neq 0 \Rightarrow Y$ is not set-c.i

<u>Conj</u> [Hartshorne] Every homogeneous <u>prime</u> ideal of height 2 in $k[x,y,u,v]$ is a set-c.i.

§2 More Exs

<u>Ex</u> [Macaulay] $I = \ker\left(k[x,y,z,w] \to k[s^4, s^3t, st^3, t^4] \subseteq k[s,t]\right)$

$\uparrow$ prime

$ht\, I = 2$

(1) complete intersection

$I_2 = 10 - 9 = 1$

$\downarrow$

$I_3 = 20 - 13 - 4 = 3$

<u>NOT</u> a c.i.

(2) set-c.i.? $\forall M$

$H^3_I(M) = 0$, so LC can't detect whether or not it's a set c.i

(3) <u>Is</u> a set-ci in char $p > 0$

[Hartshorne]

(4) Who knows in char 0?