Many associated primes of powers of primes

Irena Swanson
(& Jesse Kim)

McCullough + Peeva: $\exists$ prime ideals in polynomial rings with regularity that is not bounded by any polynomial in #variables

"Computially bad" $\longleftrightarrow$ many associated primes

i.e. permanental ideals

Mayer-Meyer ideals

<u>Changed problem</u>: $P \in$ a polynomial ring

\# ass primes of P^2 is large

$$R[It] \qquad R[It, t^{-1}] \qquad R[It, t^2]$$

$$\nwarrow \qquad \uparrow \qquad \nearrow$$

$S = R[w_1, \dots w_{\#\text{generators of } I}, T]$

R polynomial ring

$Q =$ Kernel.

Q_0 = prime defining the extended Rees alg of I_0

$$\mathrm{Ass}(S/Q_0^2) = \{Q_0, (x, y, z, \underline{w}), (x, y, z, \underline{w}, T)\}$$

Def Let A be a ring. An A-splitting is an A-alg homo $\varphi: A[t] \to A[u_1, \cdots, u_n]$ such that $t \longmapsto u_1^{r_1} \cdots u_n^{r_n}$

Today $r_1 = r_2 = \cdots = 1$

Facts: 1. φ is faithfully flat

2. If I is prime (primary) in $A[t]$ and $t \notin \sqrt{I}$, then $\varphi(I)A[\underline{u}]$ is ~~primary (prime~~ prime (primary)

3. If $I = \mathfrak{q}_1 \cap \cdots \cap \mathfrak{q}_s$ p.d. with $t \in \sqrt{\mathfrak{q}_1}, \cdots \sqrt{\mathfrak{q}_r}$ & $t \notin \sqrt{\mathfrak{q}_{r+1}}, \cdots, \sqrt{\mathfrak{q}_s}$

then $\varphi(I) = \bigcap_{i=1}^{r} \left(\bigcap_{j=1}^{n} \varphi(\mathfrak{q}_i) : (u_1 \cdots \hat{u}_j \cdots u_n)^\infty \right) \cap \bigcap_{i=r+1}^{s} \varphi(\mathfrak{q}_i)$

is a p.d.

Ex: $\varphi: x \mapsto x_1 \cdots x_m$

$y \mapsto y_1 \cdots y_n$

$z \mapsto z_1 \cdots z_l$

$\varphi(Q_1)$ is prime

$\varphi(Q_1^2)$ has $m \cdot n \cdot l$ embedded primes

Ways of generating P s.t. # vars in an associated prime gets larger.

1. (attempt) $(Q_1+Q_2)^2 = (Q_1^{(2)} + Q_1Q_2 + Q_2^{(2)})$

$$\cap\ (\text{embed}(Q_1^2) + Q_2)$$

$$\cap\ (\text{embed}(Q_2^2) + Q_1)$$

2. ~~Spreading~~:

~~S~~ ~~Set~~-up:

2. $Q_1 + (z - z_1, \cdots, z - z_m)$

Embedded of Q_1^2 is $\underbrace{(X, Y, Z, Z_1 \cdot Z_m)}_{m+3 \text{ var}}$

splitting P in a poly of $n(m+3)$ var

embed (P^2) is n^{m+3}

Spreading: