So many uses of simplicial objects

① Models for spaces

② Modes for ∞-categories

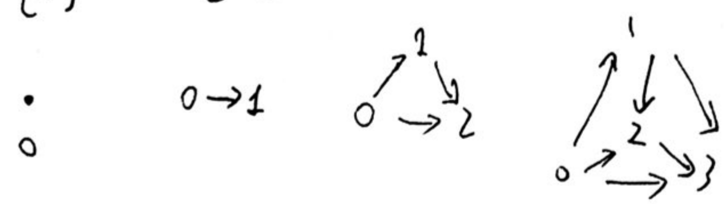
③ Resolutions

④ Higher alg structure

---

① $\Delta = \{ [n] = \{0, 1, \cdots, n\}$ non-decreasing maps $\}$

[0] [1] [2] [3]



Simplicial Set: $X : \Delta^{op} \to Set$

Maps: natural transformation

$$X_0 \rightleftarrows X_1 \;\substack{\longrightarrow\\ \longleftarrow \\ \longrightarrow \\ \longleftarrow \\ \longrightarrow}\; X_2 \;\cdots$$

$d_n^i : [n-1] \to [n]$ $\quad$ $s_n^i : [n+1] \to [n]$

E.g. ① $\Delta^n = Hom(-, [n]) : \Delta^{op} \to Set$

Δ^3? 1-simplicies: $[1] \to [3]$ $\quad 0 \to 1 \;\; \to$ 

② C category

$(NC)_n = \{ a_0 \to a_1 \cdots \to a_n$ length n chains $\}$

faces : compose

degenerations : insert identity

<u>Realization</u> $|\cdot| : sSet \to Top$

$|\Delta^n| = \Delta^n_{geom}$

$|X| = \coprod_{n \geq 0} (X_n \times \Delta^n_{geom}) / \sim$

if $\theta : [m] \to [n]$

$(x, \theta_* p) \sim (\theta^* x, p)$

Singular simplicial Sets

$S : Top \to sSet$

$X \in Top \quad S(X)_n = \{ \Delta^n_{geom} \to X \}$

Homotopy theory in sSet

$$X \rightrightarrows Y \quad \Leftrightarrow \quad \begin{tikzcd} X \times \Delta^1 \arrow[r] & Y \\ X \times \Delta^0 \arrow[u, shift left] \arrow[u, shift right] \end{tikzcd}$$

Have homotopy equivalence: weak homotopy equivalence.

$$\rightsquigarrow \quad sSet[w.h.e^{-1}] \underset{\longleftarrow}{\overset{\sim}{\longrightarrow}} Top[w.h.e^{-1}]$$

---

② $0 \le k \le n$

Λ^n_k

Λ^2_1
$$\begin{tikzcd} & 2 \\ 0 \arrow[r] & 1 \arrow[u] \end{tikzcd}$$

Λ^2_0
$$\begin{tikzcd} & 2 \\ 0 \arrow[r] \arrow[ur] & 1 \end{tikzcd}$$

Λ^3_3

$$\Lambda^n_k \subseteq \Delta^n$$

$$(\Lambda^n_k)_p = \{ f : [p] \to [n] \mid Im(f) \not\supseteq \{0, 1, \cdots, \hat{k}, n\} \}$$

∞-category

$$C : \Delta^{op} \to Set$$

s.t.
$$\begin{tikzcd} \Lambda^n_k \arrow[r] \arrow[d, hook] & C \\ \Delta^n \arrow[ur, dashed, "\exists \text{ not necessarily unique}"'] & \end{tikzcd} \qquad 0<k<n$$

---

a category : $\underset{obj}{C_0} \leftrightarrows \underset{arrows}{C_1}$ + composition data

∞-cat $C_0 \leftrightarrows C_1 \leftrightarrows C_2 \cdots$ + extension condition

$f, g \in C_1 \quad f : x \to y \quad g : y \to z$

$$\Lambda^2_1 \quad \begin{tikzcd} & 1 \arrow[dr] & \\ 0 \arrow[ur] & & 2 \end{tikzcd}$$

$$\rightsquigarrow \quad \begin{tikzcd} \Lambda^2_1 \arrow[r] \arrow[d, hook] & C \\ \Delta^2 \arrow[ur] & \end{tikzcd}$$

$$\begin{tikzcd} & 1 \arrow[dr] & \\ 0 \arrow[ur] & & 2 \end{tikzcd} \longmapsto \begin{tikzcd} & y \arrow[dr, "g"] & \\ x \arrow[ur, "f"] & & z \end{tikzcd}$$

$$\begin{tikzcd} & y \arrow[dr, "g"] & \\ x \arrow[ur, "f"] \arrow[rr, "g\circ f"'] & & z \end{tikzcd}$$