A computational Introduction to Rep Stability

Monday, February 15, 2016 4:11 PM

Thm (Hib - Serre)

$R = \mathbb{Q}[a,b,c,d,e]$ Suppose V is f.g.

$V = \bigoplus_{i=0}^{\infty} V_i$

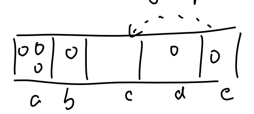
$n \mapsto \dim V_n$

eventually coincide with a polynomail

$X \rightsquigarrow V$

Toy Problem # 1

jump $\longrightarrow$ relations $(ab - bc,\ bc - cd,\ cd - ce) = M$



$= a^3bde \longleftrightarrow a^3bcd$

$\frac{0}{1}$ $\frac{1}{5}$ $\frac{2}{12}$ $\frac{3}{20}$

$\operatorname{coker}\left(R \oplus R \oplus R \xrightarrow{M} R\right) =$ graded module

$\mathbb{Q} G$ $\mathbb{Q}[a,b,c,d,e] \longleftrightarrow \mathbb{Q}\mathbb{N}^5$ $\mathbb{N}^5 \longrightarrow End(V)$

$\circlearrowright$
V

R:

object $\mathbb{N}$

Morphism $Hom(m,n) = \{$monomial of degree $n-m\}$

Defn: $\mathcal{D}$ a cat, a repn of $\mathcal{D}$ is a functor $F : \mathcal{D} \to Vect$

a rep of R is the same as an R-mod

	2	1	4	5 $\leftarrow$ obj in Cat
0				$\leftarrow \mathbb{Q}\,Hom(0,5)$
3				
4				

Defn $M \in \mathrm{Mat}(\oplus d_i, \oplus d_j')$ $\quad d \longmapsto \dfrac{\mathrm{Mat}(\oplus d_i, d)}{M\,\mathrm{Mat}(\oplus d_j', d)}$

$\quad\; 2 \qquad\; 2 \qquad\; 2$

$0\,(ab-bc,\ bc-cd,\ cd-de)$ $\qquad \dfrac{\mathrm{Mat}(0, d)}{M \cdot \mathrm{Mat}(2 \oplus 2 \oplus 2, d)}$ = coker of degree d

---

Rep Stability:

$X_0 \rightleftarrows X_1 \rightleftarrows X_2 \quad \cdots\cdots$

$HX_0 \rightleftarrows HX_1 \rightleftarrows HX_2 \quad \cdots$

want this to be predictable
