Rep of Quivers

Thursday, February 4, 2016 5:10 PM

Quiver (Q_0, Q_1, h, t)

Q_0: vertices $\quad h, t: Q_0 \to Q_1$

Q_1: edges

- A quiver representation V consists of familia of f.d. vector spaces $V(x)$ and linear maps: $V(\alpha) : V(t\alpha) \to V(h\alpha)$

- A subrep of a quiver rep $(V(x), V(\alpha))$ is a representation $(W(x), W(\alpha))$ s.t. $W(x) \subseteq V(x)$ $\forall x \in Q_0$

  $W(\alpha) = V(\alpha)|_{W(t\alpha)}$

- An irreducible rep is a quiver rep whose only subrep are 0 & itself

Comparable to kG : kQ the path alg

kQ: vector space spanned by paths

$p, q \in Q$ $\quad p \cdot q = \begin{cases} pq & \text{if } h(q) = t(p) \\ 0 & \text{otherwise} \end{cases}$

<u>Example</u>: $Q: \overset{x}{\cdot} \xrightarrow{\alpha} \overset{y}{\cdot}$

$kQ = k\alpha \oplus ke_x \oplus ke_y$

$e_x^2 = e_x \qquad e_y\alpha = \alpha e_x = \alpha$

$e_y^2 = e_y \qquad e_x\alpha = \alpha e_y = 0$

$\alpha^2 = 0 \qquad e_x e_y = e_y e_x = 0$

<u>Prop</u> $\operatorname{Rep}(Q)$ is an equivalent category to kQ-mod

V rep of Q

$\bar{V} = \bigoplus_{x \in Q_0} V(x)$

A is a f.d. ass alg $/ k$

A is <u>Morita equivalent</u> to kQ/I for some Quiver Q

A is Morita equivalent to kQ/I for some Quiver Q & ideal I

the equivalence between Rep Category

The irreducibles are very nice in kQ/I

$\exists$ very simple (short) projective resolutions of kQ/I - modules.

R (Artinian) ring

a short exact seq of R-modules

$$0 \to L \to M \to N \to 0$$ is

called almost split if

① if the seq is not split

② N is indecomposable and $\varphi : A \to N$ is not an isom where A is indecomposable then φ factor through M

③ $\varphi : L \to A$ where L, A indecomposable and φ not iso, then φ factor through M

Example: $R = k[X]/(X^n)$ $\quad k[X]/(X^m)$ $\quad 1 \le m < n$

$$0 \to k[X]/(X^m) \to k[X]/(X^{m+1}) \oplus k[X]/(X^{m-1}) \longrightarrow k[X]/(X^m) \to 0$$

$$a \longmapsto (Xa, a)$$

$$(b, c) \longmapsto b - Xc$$

Given a finite-dim alg want a mode at ... indecomposabl...

Given a finite-dim alg, put a node at each indecomposable module,
put arrows between nodes for each irreducible maps.

$f: V \to W$ is irred if neither

$$0 \to V \xrightarrow{f} W \to \operatorname{coker} f \to 0$$

$$0 \to \ker f \to V \xrightarrow{f} W \to 0$$

splits.

<u>Toward's Gabriel's Thm</u>

Q is of finite type if there are finitely many isomorphism classes of indecomposables

<u>Thm</u> (Gabriel) A connected quivers Q of finite type has underlying graph (that is, forgetting the orientation on edges that is one of A_n, D_n, E_6, E_7, E_8)