Dim 0 Category

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Def A ring R is dim 0 if every f.g. R-module is finite length

$\mathbb{Q}S_4$ is a dim 0 ring.

(R commutative $\Rightarrow$ dim 0 = Artinian)

Generate S_4 rep

$\mathbb{Q}S_4 \cdot \{\{1,3\},\{4,2\}\}$

$$\frac{\langle 12, 34\rangle}{\{\langle 12,34\rangle - \langle 12,43\rangle,\ \langle 12,34\rangle - \langle 34,12\rangle\}}$$

$$\operatorname{Coker}\left((\mathbb{Q}S_4)^{\oplus 2} \xrightarrow{\left[\boxed{1234} - \boxed{2134} \,\middle|\, \boxed{1234} - \boxed{3412}\right]} (\mathbb{Q}S_4)^{\oplus 1} \right) = \text{Module}$$

$$\operatorname{Ker}\left[(1)-(1) \,\middle|\, (1)-(1)\right] = \ker (0\ 0)$$

so 1 copy of $\boxed{\ \ \ \ }$

0 copy of $\boxed{\begin{smallmatrix} \\ \\ \\ \end{smallmatrix}}$

$$\ker\left[\begin{pmatrix}1&0\\0&1\end{pmatrix} - \begin{pmatrix}1&0\\1&-1\end{pmatrix} \,\middle|\, \begin{pmatrix}1&0\\0&1\end{pmatrix} - \begin{pmatrix}1&0\\0&1\end{pmatrix}\right] = \ker\left(\begin{array}{cc|cc}0&0&0&0\\-1&2&0&0\end{array}\right) = 1\text{-dim}$$

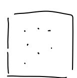
so 1 copy of $\boxed{+}$

Module $\simeq \boxed{\ \ \ \ } \oplus \boxed{+}$

dim $= 1+2 = 3$

Configuration Space

$\mathbb{R}^2$ 

Distinct Points in $\mathbb{R}^2$ 

$\mathrm{Conf}_n(\mathbb{R}^2)$

Thm $$H^2(\mathrm{Conf}_n(\mathbb{R}^2)) = \frac{\langle \omega_{ij} \otimes \omega_{kl} \rangle}{\langle \omega_{ij}\otimes\omega_{kl} - \omega_{ji}\otimes\omega_{kl},\ \omega_{ii}\otimes_{jk},\ \omega_{ij}\otimes\omega_{kl} + \omega_{kl}\otimes\omega_{ij},\ \omega_{ij}\otimes\omega_{jk} + \omega_{jk}\otimes\omega_{ki} + \omega_{ki}\otimes\omega_{ij} \rangle}$$

$i, j, k, l \in \{1, \dots, n\}$

$$\downarrow$$

$$\frac{\langle \boxed{1234} \rangle}{\langle \boxed{1234} - \boxed{2134},\ \boxed{1123},\ \boxed{1234} + \boxed{3412},\ \boxed{1223} + \boxed{2331} + \boxed{3112} \rangle}$$

Thm Fin is dim 0

The simples are in bijection with partitions

Example $\dim C_{\boxed{+}} = 2 \cdot \binom{n}{3}$

$\dim C_\lambda = \dim(\lambda) \cdot \binom{n}{|\lambda|}$

unless $\lambda = \boxed{\begin{smallmatrix} \\ \\ \\ \end{smallmatrix}}$, $\dim C_{\boxed{\begin{smallmatrix} \\ \\ \end{smallmatrix}}\}k} = \binom{n-1}{k-1}$

$$H^2(\mathrm{Conf}(\mathbb{R}^2)) = C_{\boxed{+}} \oplus C_{\boxed{+\ }}$$

$$\dim = 2 \cdot \binom{n}{3} + 3 \cdot \binom{n}{4}$$