Plane Algebraic Curves 10/06/2015

Greeks - Rulers & Compass

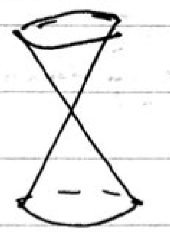
Two examples: ① Duplicate a Cube

② Trisect an angle

Thm (Wentzel, 1837) Both are impossible

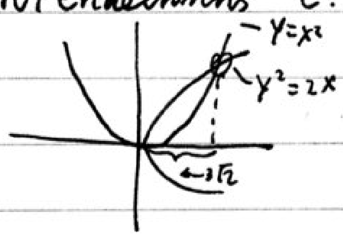
Pf: Galois Theory #

Conic Sections



$y^2 = px + qx^2$

Menaechmus C. 350BC

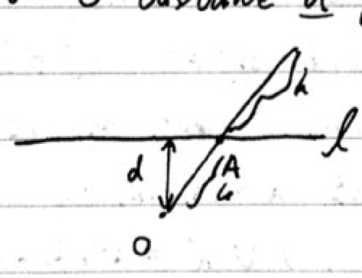


Conchoids of Nicomedes (C. 180B.C)

Quartic $(x^2+y^2)(d-y)^2 - k^2y^2 = 0$

line $\underline{l}$ point O distance $\underline{d}$ from $\underline{l}$

Length $\underline{k}$



Analytic Geometry

Femart & Desecrtes Use algebraic equations

1629 1619

Classify Curves

degree = d $\quad f(x,y)=0$

$d=1$ Lines ....

$d=2$ Conics ...

$d=3$ Cubics ...

for Cubics, Netwon (1710)

- Look at asymptotic behavior as $x, y \to \infty$

$\hat{f}(x,y) = ax^3 + bx^2y + cxy^2 + dy^3$

Cases: I : $xy^2 + ey =$ ~~$ax^3+bx^2y+cxy^2+dy^3$~~ ...

II : $xy = ax^3+bx^2+cx+d$

III $y^2 = ax^3+bx^2+cx+d$

IV $y = ax^3+bx^2+cx+d$

Netwon found 72 different cases (missed 6)

Observation (Newton, 1710)

Project a cubic from a pt P onto other hyperplane you get a cubic

Projective Geometry

Poncelet 1822

Plücker 1834

Def: $\mathbb{RP}^2 = \mathbb{R}^3 - \{0\} / \sim$

"$\mathbb{R}^2 \cup \{$a line at $\infty\}$"

Thm (Desargues, 1648)

Let ABC, abc two $\triangle$, let $\overline{Aa}, \overline{Bb}, \overline{Cc}$ pass through P

$\overline{AB} \cap \overline{ab}$, $\overline{AC} \cap \overline{ac}$, $\overline{BC} \cap \overline{bc}$ collinear other nice things about $\mathbb{P}^2$

Thm (Bézout, 1779)

If C, C' two curves in $\mathbb{CP}^2$ that do not share a common component, $C = V(f)$, $C' = V(g)$
(f deg m, g deg n)

$$m \cdot n = \#\{\text{intersection points w/ multiplicity}\}$$

Pf: Think the coh ring

$$H^*(\mathbb{CP}^2) = \mathbb{Z}[x]/x^2$$

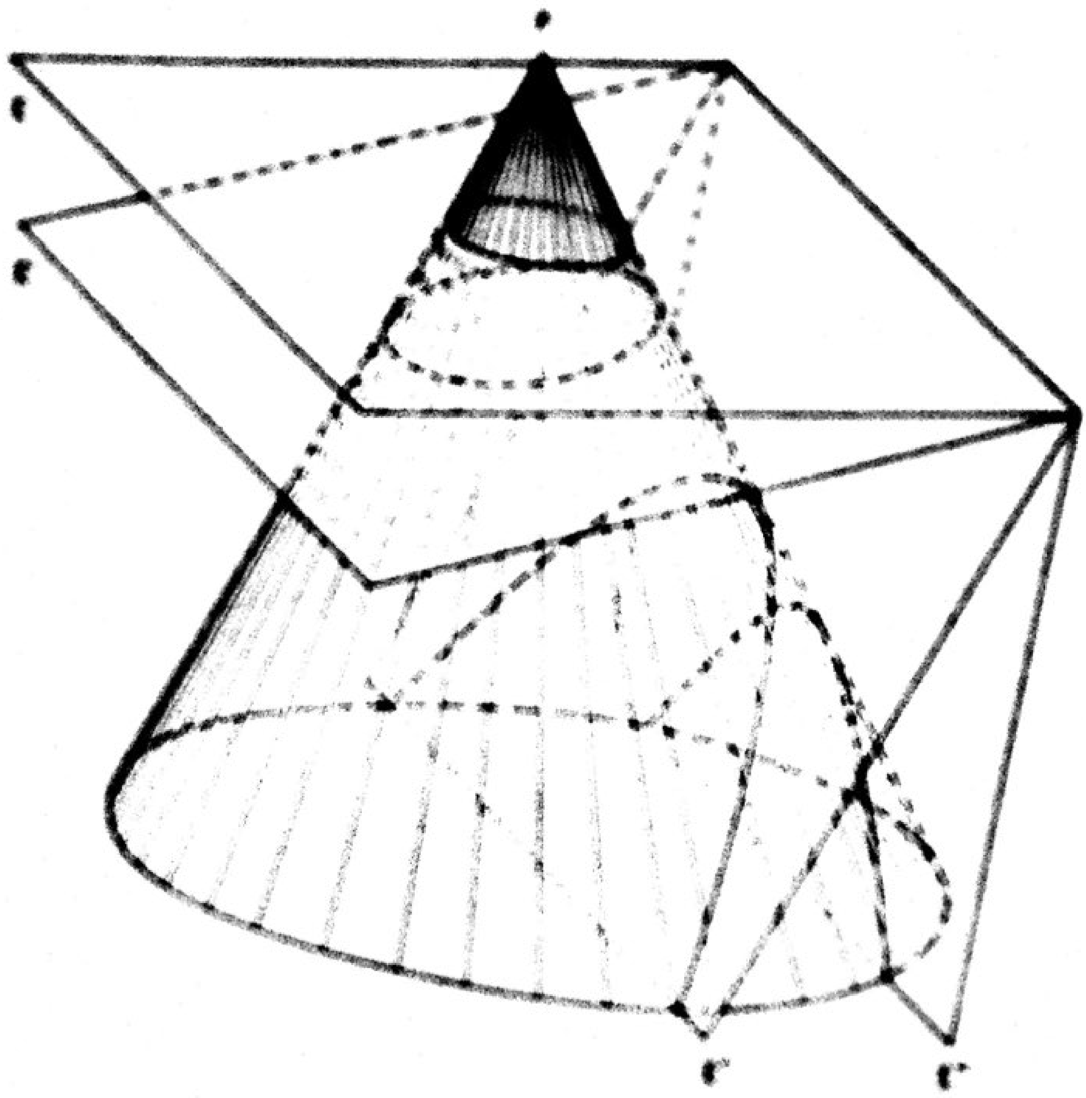
(x: class generated by a line L)

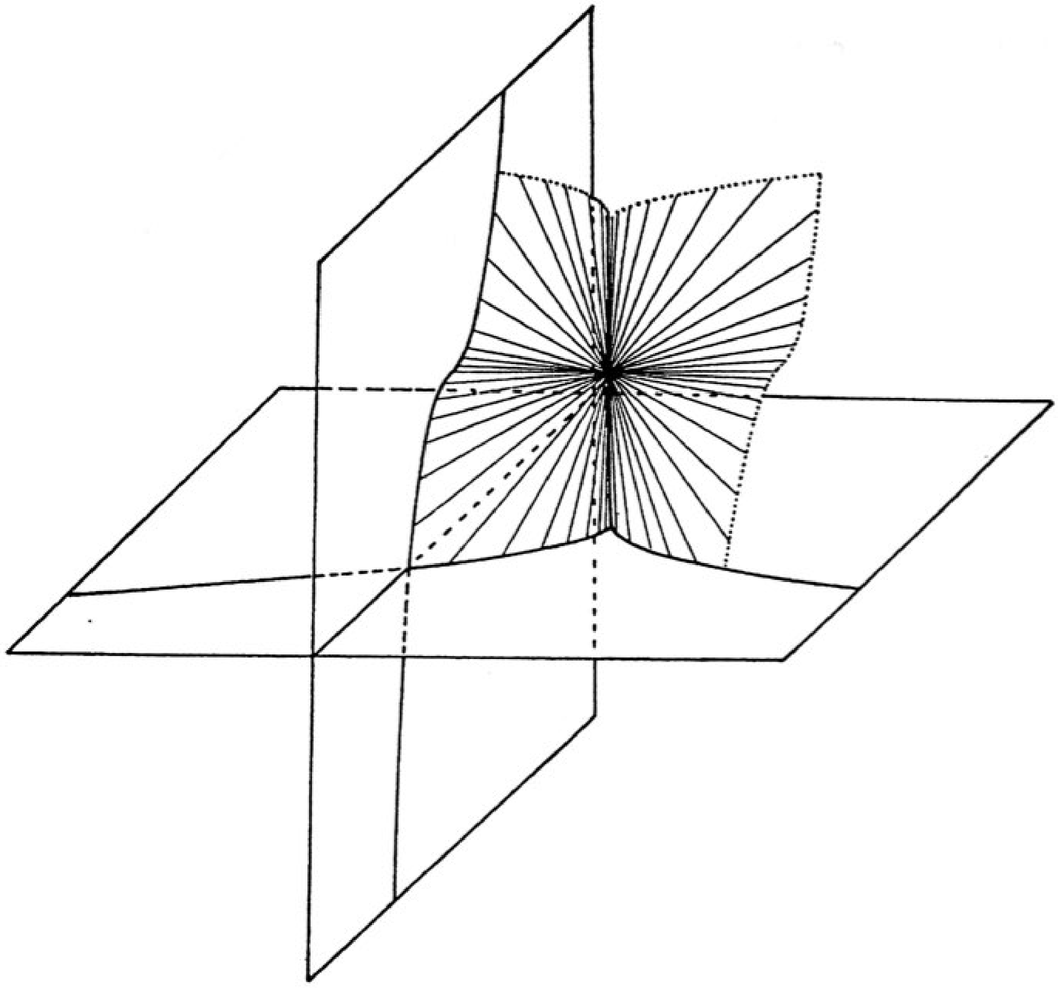
Thm (Pascal 1640) "Hexagrammum Mysticum"

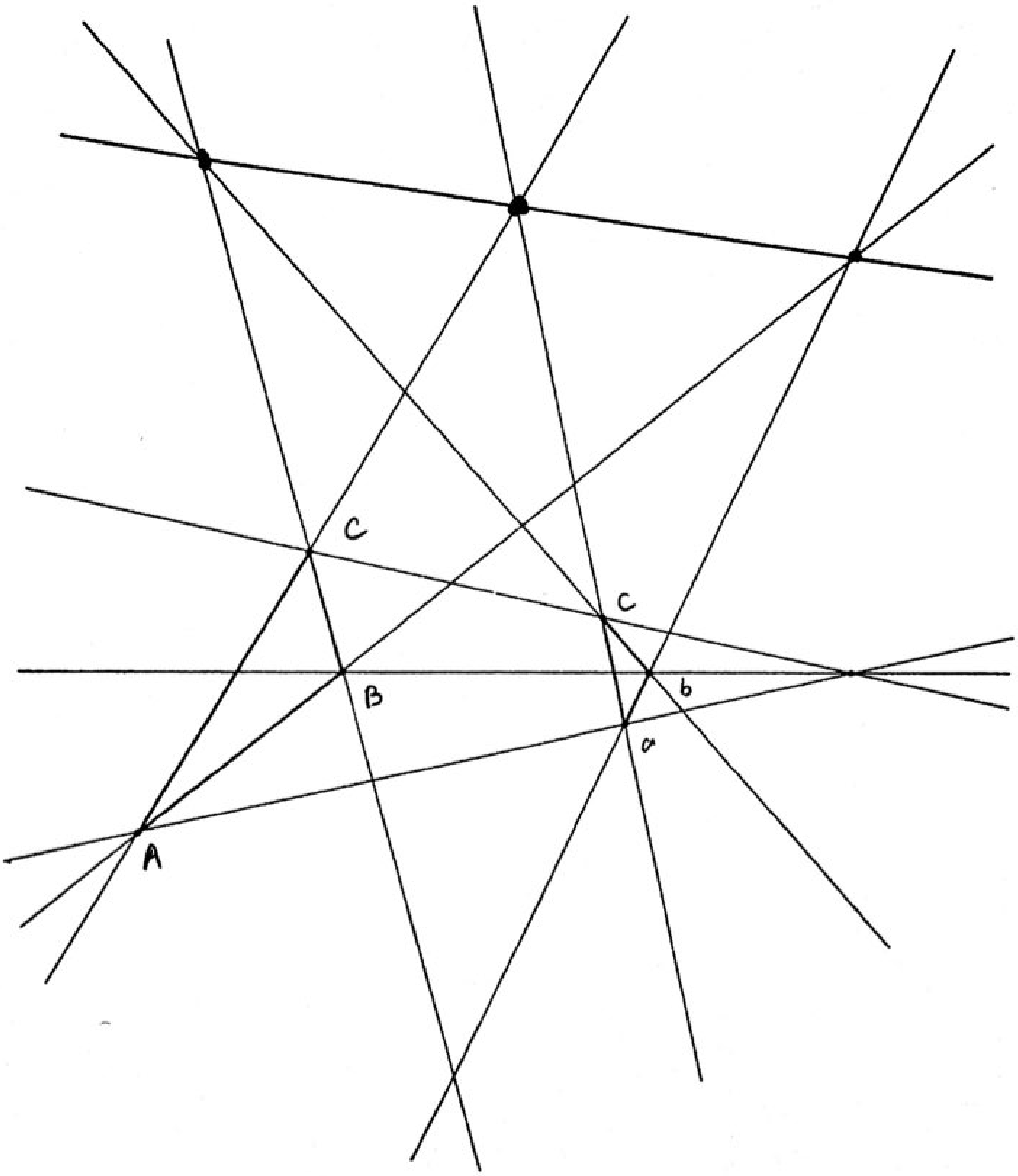
$C \subseteq \mathbb{P}^2$ conic, $\{P_i\}_{i=1}^6 \subseteq C$ distinct

L_{ij} = lines $\overline{P_iP_j}$

$L_{12} \cap L_{45}$, $L_{23} \cap L_{56}$, $L_{34} \cap L_{16}$ are collinear






C
C
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