



Cloud

finite set of points \Rightarrow Simplicial Complex

Def (Čech complex) $\check{C}_\epsilon(P) = X$ defined by
 $X^{(k)} = P$ $(k-1)$ -simplex whenever
 k ϵ -balls have nonempty common
homotopy intersection

Čech thm $\check{C}_\epsilon(P) \simeq \overline{B_{\frac{\epsilon}{2}}(P)}$ (of ambient space "nice")

Def (Rips complex) $R_\epsilon(P) = X$ defined by

$$\Delta \quad \cancel{R_\epsilon(P)} \quad X^{(k)} = (\check{C}_\epsilon(P))^{(k)}$$

$$(R_\epsilon(P))^{(k)}$$

$R_\epsilon(P)$ is the flag simplicial complex
with this 1-skeleton.

(whenever there is a simplex, there is)

Lemma $R_\epsilon(P) \hookrightarrow \check{C}_{\epsilon/2}(P) \hookrightarrow R_{\epsilon/2}(P)$

Cloud \rightarrow Filtration of simplicial complexes

$$\dots \rightarrow X_0 \rightarrow X_1 \rightarrow X_2 \rightarrow \dots$$

increasing more simplices

$$\dots \rightarrow H_2(X_0) \rightarrow H_2(X_1) \rightarrow H_2(X_2) \rightarrow \dots$$

$$\rightarrow C_2(X_0) \rightarrow C_2(X_1) \rightarrow C_2(X_2)$$

$$\rightarrow C_1(X_0) \rightarrow C_1(X_1) \rightarrow C_1(X_2)$$

$$\rightarrow C_0(X_0) \rightarrow C_0(X_1) \rightarrow C_0(X_2)$$

$$\dots \rightarrow H_1(X_0) \rightarrow H_1(X_1) \rightarrow H_1(X_2) \rightarrow \dots$$

$$\begin{aligned} \dots \rightarrow H_2(X_0) \rightarrow H_2(X_1) \rightarrow H_2(X_2) \rightarrow \dots \\ \dots \rightarrow H_1(X_0) \rightarrow H_1(X_1) \rightarrow H_1(X_2) \rightarrow \dots \end{aligned}$$

inclusion of vector space

take direct limit \Rightarrow persistence module

Point Cloud \rightarrow Filtration of Simplicial Complexes \rightarrow Persistent homology

(Carlsson Zomorodian) A persistence-module is a graded $k[X]$ -module

Thm: (Crawley-Boevey) M is a persistence module

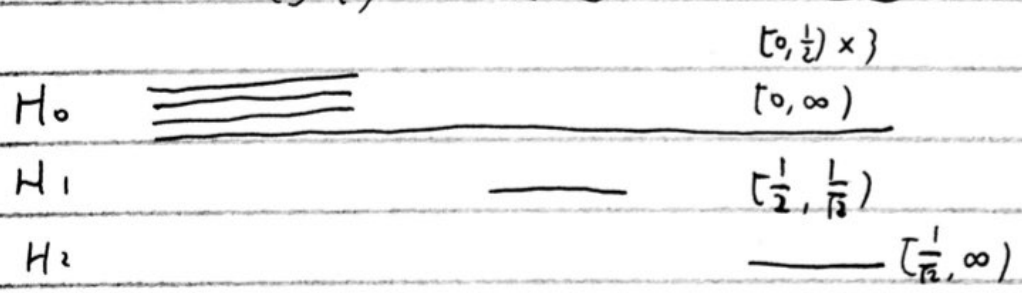
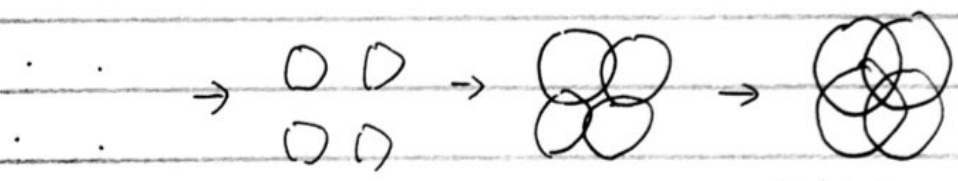
all v.s. f.d., then \exists multiset of intervals $\{I_1, \dots, I_t\}$

$$\text{st. } M = \bigoplus_{j=1}^t D^{I_j}$$

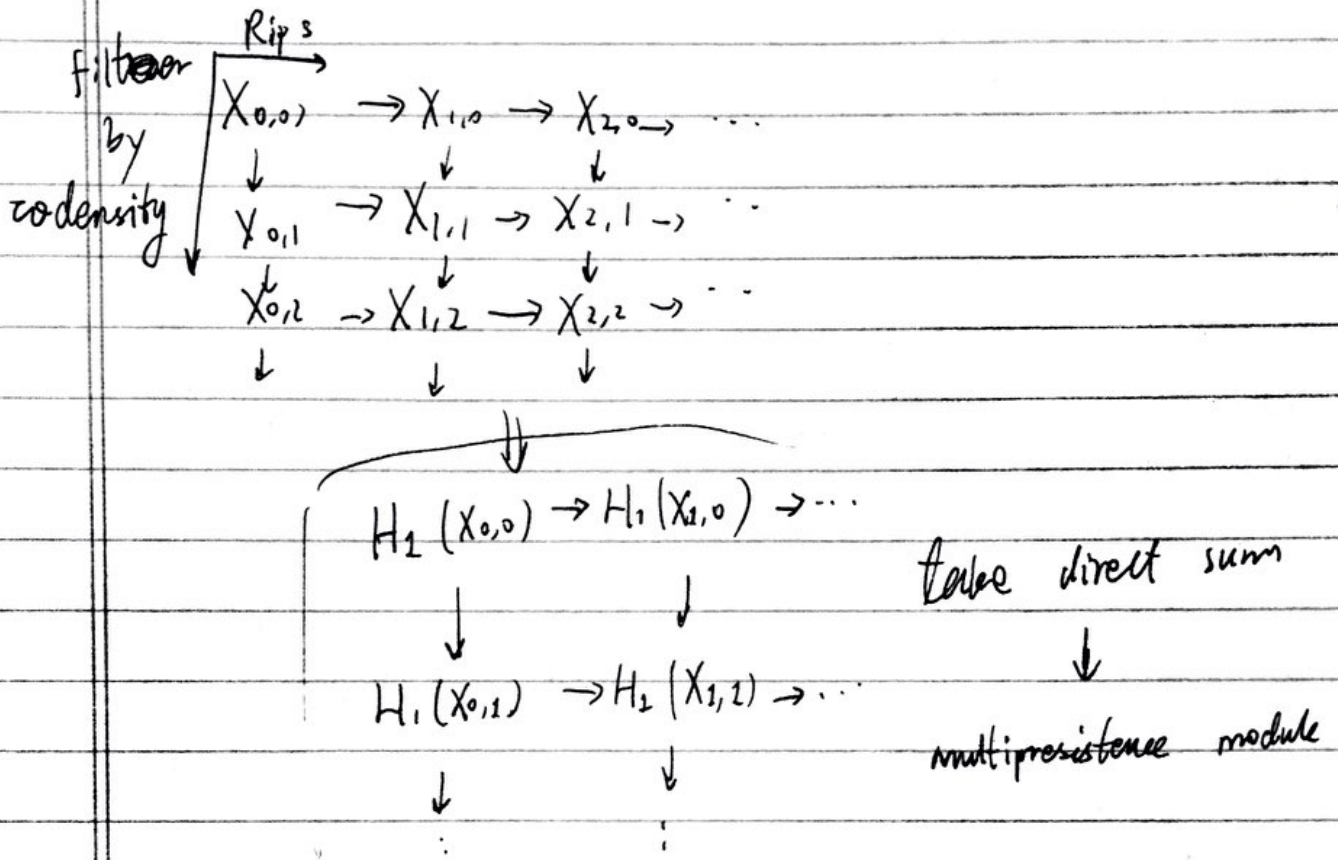
$B''(M)$

$$\text{where } (D^{I_j})_a = \begin{cases} k & a \in I_j \\ 0 & \text{else} \end{cases}$$

\rightarrow Barcodes \hookrightarrow Persistence diagram



Cloud \rightarrow ~~the~~ Bi (Multi) filtration of complexes



\Rightarrow multipersistence
homology

How to describe:

- Barcodes along 1-D slices
- Betti numbers $\sum_{0,2}^k \quad \sum_{1,1}^k$
- Rank invariant

\Rightarrow Vineyards