Integral Closure of An Ideal

Zhan Jiang

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1 Definition

Definition 1.1. *Let $I \subseteq R$ be an ideal and $u \in R$, the **integral closure** $\overline{I}$ of I is the set of elements u such that for some n, it satisfies a monic polynomial*

$$x^n + r_1 x^{n-1} + r_2 x^{n-2} + \cdots + r_n = 0$$

where $r_j \in I^j$ for $1 \leq j \leq n$.

We have an alternative description: One forms the *Rees ring*

$$R[It] = R + It + I^2t^2 + \cdots + I^n t^n + \cdots \subseteq R[t]$$

where t is an indeterminate, the integral closure of $R[It]$ in $R[t]$ has the form

$$R + J_1 t + J_2 t^2 + \cdots + J_n t^n + \cdots$$

where $J_n \subseteq R$ is an ideal. Then $J_k = \overline{I^k}$ for any k. In particular, J_1 is the integral closure of I.