Math 116 — Second Midterm — March 21, 2022

Write your 8-digit UMID number very clearly in the box to the right.

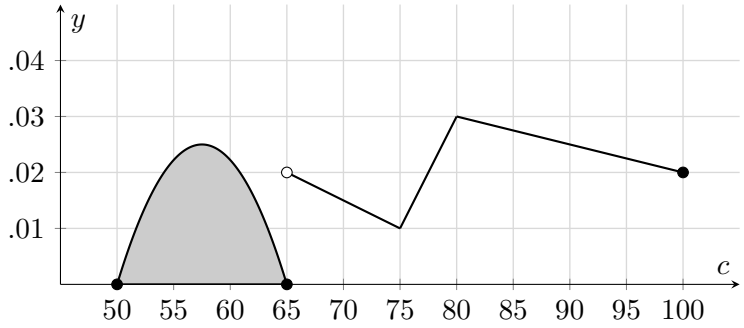
Your Initials Only: _____ Instructor Name: _____ Section #: _____

1. This exam has 10 pages including this cover.
2. There are 10 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but how you obtained it.
5. You are allowed notes written on two sides of a $3'' \times 5''$ note card.
6. You are NOT allowed other resources, including, but not limited to, notes, calculators or other devices.
7. For any graph or table that you use to find an answer, be sure to sketch the graph or write out the entries of the table. In either case, include an explanation of how you used the graph or table to find the answer.
8. Include units in your answer where that is appropriate.
9. Problems may ask for answers in *exact form*. Recall that $x = \sqrt{2}$ is a solution in exact form to the equation $x^2 = 2$, but $x = 1.41421356237$ is <u>not</u>.
10. You must use the methods learned in this course to solve all problems.

Problem	Points	Score
1	6	
2	7	
3	12	
4	15	
5	12	
6	12	

Problem	Points	Score
7	12	
8	8	
9	8	
10	8	
Total	100	

1. [6 points] Brad and Joan have developed a new strategy to analyze baseball players, except now instead of focusing on home run distance, they need to know the probability a pitcher throws a ball at a given speed. Shown below is a graph of the function $f(c)$, a probability density function (pdf) describing the probability a certain pitcher throws the ball at a speed of c miles per hour (mph). Assume that $f(c) = 0$ when $c \leq 50$ and $c > 100$.



a. [3 points] What is the probability this pitcher throws a pitch between 50 and 65 mph?

b. [3 points] What is the median speed of this player's pitches, in mph?

2. [7 points] Brad and Joan are examining another pitcher's probability density function (pdf) when Brad spills coffee on the paper and smudges some of the ink. After drying off the paper, they are left with the incomplete probability density function, $g(v)$ given below, where v is in **hundreds of miles per hour.**

$$g(v) = \begin{cases} r + qv & 0 < v \leq 1 \\ 0 & otherwise \end{cases}$$

Brad and Joan know that this player has a mean pitch speed of $\frac{2}{3}$ hundreds of miles per hour. Find the values of r and q which make this function a probability density function.

$r =$ _____ $q =$ _____

3. [12 points]

a. [6 points] Suppose $F(x)$ is a cumulative distribution function for the height x, in meters, of the students on the University of Michigan campus. For each of the following, circle MUST BE, COULD BE, or CANNOT BE if the statement must be true, could be true, or cannot be true.

- $F(2) < F(1)$.

  MUST BE COULD BE CANNOT BE

- $\lim_{x\to\infty} F(x) = 1$.

  MUST BE COULD BE CANNOT BE

- $F(1.8) = 0.6$.

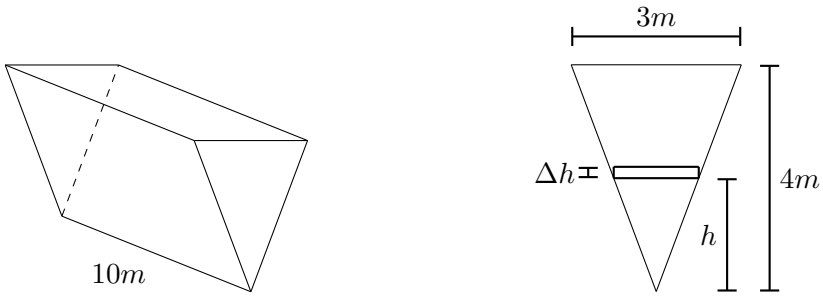
  MUST BE COULD BE CANNOT BE

b. [6 points] Which of the following series converge? Circle all that apply. If none converge, circle NONE.

$$\sum_{n=1}^{\infty} \frac{e^n}{n} \qquad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^{0.1}} \qquad \sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$$

$$\sum_{n=1}^{\infty} e^{-1/n} \qquad \text{NONE}$$

4. [15 points] A gas station needs to pump gas out of a subterranean tank. The tank is 10 meters in length, and has cross-sections shaped like isosceles triangles, with base 3 meters and height 4 meters. **The top of the tank is 15 meters below the surface of the earth**. Recall that $g = 9.8m/s^2$ is the gravitational constant.



Underground Tank

a. [5 points] Write an expression for the volume (in cubic meters) of a horizontal rectangular slice of gasoline at height h above the bottom of the tank, with thickness Δh. Your answer should not involve an integral.

b. [3 points] Gasoline has a density of 800 kg/m^3. Write an expression for the weight (in newtons) of the slice of gasoline mentioned in part (a). Your answer should not involve an integral.

c. [4 points] Write an expression for the work (in joules) needed to pump the slice of gasoline mentioned above to the surface of the earth. Your answer should not involve an integral.

d. [3 points] Write an integral for the total work (in joules) needed pump all of the gasoline to the surface of the earth.

5. [12 points] A tech startup is growing quickly, and the company needs to understand its customers data-storage needs to properly scale its infrastructure. Over the course of each month, the users each store 5 gigabytes of new data. Additionally, because users are conscious of their digital footprint, at the beginning of each month, each user deletes 20% of all data they had stored in previous months.

a. [4 points] Let D_n be the amount of data stored per user at the end of the n^{th} month. If $D_1 = 5$, write expressions for D_2 and D_3. The letter D should not appear in your final answers.

$D_2 =$ _____

$D_3 =$ _____

b. [4 points] Find a closed form expression for D_n. This means your answer should be a function of n, should not contain Σ, and should not be recursive.

$D_n =$ _____

c. [4 points] What is the long-term expected data storage of a user in gigabytes?

Answer = _____

6. [12 points] Answer the following questions relating the the sequences shown here:

$$a_n = -\cos\left(\frac{\pi}{n}\right) \qquad b_n = \frac{(-1)^n(n+1)}{n} \qquad c_n = \left(\frac{4}{3}\right)^n \qquad d_n = \sum_{k=1}^{n}\left(-\frac{3}{4}\right)^k$$

Assume all sequences start at the index $n = 1$.

a. [3 points] Which of the sequences are bounded?

a_n b_n c_n d_n *none*

b. [3 points] Which of the sequences shown above are monotone increasing?

a_n b_n c_n d_n *none*

c. [3 points] Which of the sequences shown above are monotone decreasing?

a_n b_n c_n d_n *none*

d. [3 points] Which of the sequences shown above converge?

a_n b_n c_n d_n *none*

7. [12 points] The parts of this problem are unrelated to each other.

a. [7 points] **Compute** the value of the following improper integral if it converges. If it does not converge, use a **direct computation** of the integral to show its divergence. Be sure to show your full computation, and be sure to use **proper notation.**

$$\int_1^2 \frac{1}{\sqrt{t-1}} dt$$

b. [5 points] Compute the following limit. Fully justify your answer including using proper notation.

$$\lim_{x \to 0} \quad \frac{1-\cos(x)}{x^2}$$

8. [8 points] Determine whether the following improper integral converges or diverges. **Circle your final answer choice**. Fully justify your answer including using proper notation and showing mechanics of any tests you use.

$$\int_1^\infty \frac{t^2 + \ln(t)}{t^3 - \cos(t) + 2} dt$$

Circle one: **Converges** **Diverges**

9. [8 points] Determine whether the following series converges or diverges. If it converges, determine if it is absolute or conditional convergence **Circle your final answer choice**. Fully justify your answer including using proper notation and showing mechanics of any tests you use.

$$\sum_{n=1}^{\infty} \frac{n(-2)^n}{3^n}$$

Circle one: **Absolutely Converges** **Conditionally Converges** **Diverges**

10. [8 points] Determine whether the following series converges or diverges. If it converges, determine if it is absolute or conditional convergence **Circle your final answer choice**. Fully justify your answer including using proper notation and showing mechanics of any tests you use.

$$\sum_{n=1}^{\infty} \frac{(-1)^n (n^{1/2}+4)}{3n^{3/2}+2}$$

Circle one: **Absolutely Converges** **Conditionally Converges** **Diverges**