Math 116 — Practice for Exam 1

Generated September 28, 2017

Name: SOLUTIONS

Instructor: ______________________________ Section Number: __________

1. This exam has 5 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2017	1	4		12	
Fall 2013	3	9	olive oil	13	
Winter 2016	1	9	chocolate prism	9	
Winter 2014	1	7	drag queens	8	
Fall 2010	1	1		12	
Total				54	

Recommended time (based on points): 53 minutes

4. [12 points] For each of the questions below, circle <u>all</u> of the available correct answers. Circle "NONE OF THESE" if none of the available choices are correct.

a. [3 points] Which of the following are antiderivatives of the function $2\sin(x)\cos(x)$?

i. $\frac{1}{2}\cos^2(x) + \frac{1}{2}\sin^2(x)$

ii. $\boxed{\sin^2(3) - \cos^2(x)}$

iii. $\displaystyle\int_0^\pi 2\sin(x)\cos(x)\,dx$

iv. $\boxed{\sin^2(x)}$

v. NONE OF THESE

b. [3 points] Which of the following integrals give the arc length of the curve $y = e^{2x}$ from $x = 0$ to $x = 2$?

i. $\displaystyle\int_0^2 \sqrt{1+4e^{2x}}\,dx$

ii. $\displaystyle\int_0^2 \sqrt{1+e^{4x}}\,dx$

iii. $\displaystyle\frac{1}{2}\int_0^1 \sqrt{1+4e^{2s}}\,ds$

iv. $\boxed{\displaystyle\int_0^2 \sqrt{1+4e^{4u}}\,du}$

v. NONE OF THESE

c. [3 points] Which of the following are antiderivatives of the function $\dfrac{1}{\ln x}$?

i. $\ln(\ln(x)) + 4$

ii. $\displaystyle\int_2^e \frac{1}{\ln t}\,dt$

iii. $\boxed{\displaystyle\int_1^{\ln x} \frac{e^t}{t}\,dt}$

iv. $\boxed{\displaystyle\int_2^x \frac{1}{\ln t}\,dt}$

v. NONE OF THESE

d. [3 points] An object with variable mass is lifted up 30 meters at a constant rate. This process takes 10 seconds. Suppose that $m(t)$ is the mass of the object, in kilograms, t seconds after the lifting begins. Let g be the acceleration due to gravity in m/s^2. (So $g \approx 9.8$.) Which of the following expressions give the work, in joules, required to raise the object?

i. $\boxed{\displaystyle 3\int_0^{10} g\cdot m(t)\,dt}$

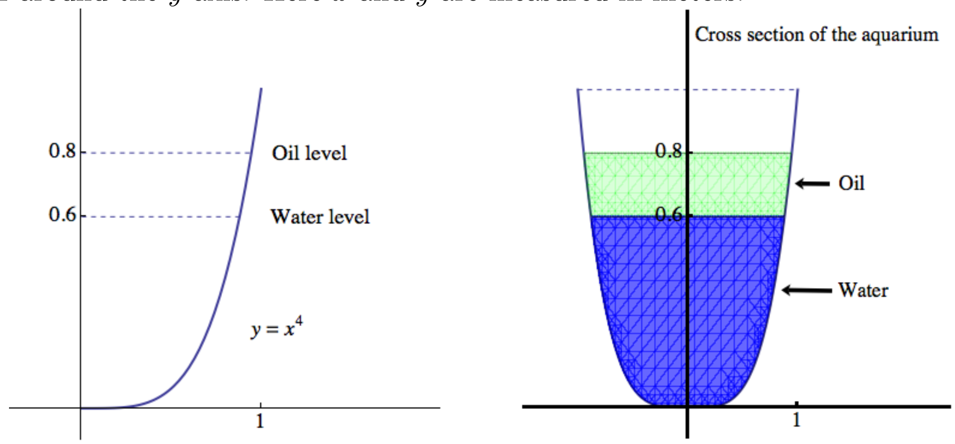
ii. $\boxed{\displaystyle\int_0^{30} g\cdot m\left(\frac{x}{3}\right)\,dx}$

iii. $\displaystyle\frac{1}{3}\int_0^{30} g\cdot m(x)\,dx$

iv. $\displaystyle\int_0^{10} g\cdot 3t\cdot m(t)\,dt$

v. NONE OF THESE

9. [13 points] Olive oil have been poured into the Math Department's starfish aquarium! The shape of the aquarium is a solid of revolution, obtained by rotating the graph of $y = x^4$ for $0 \le x \le 1$ around the y-axis. Here x and y are measured in meters.



The aquarium contains water up to a level of $y = 0.6$ meters. There is a layer of oil of thickness 0.2 meters floating on top of the water. The water and olive oil have densities 1000 and 800 kg per m^3, respectively. Use the value of $g = 9.8$ m per s^2 for the acceleration due to gravity.

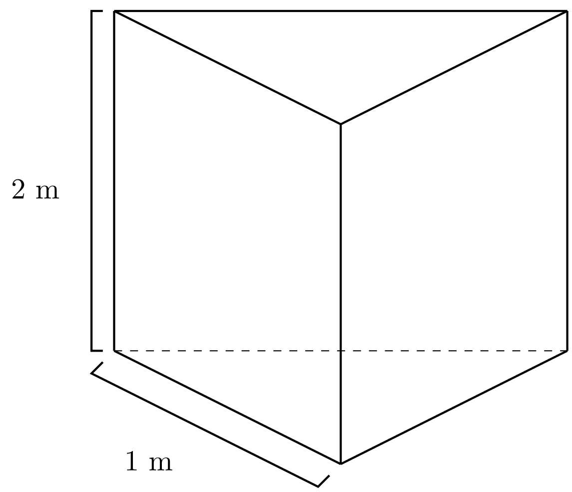
a. [6 points] Give an expression involving definite integrals that computes the total mass of the water in the aquarium.

Solution: $\text{Mass}_{water} = \int_0^{0.6} \pi(\sqrt[4]{y})^2(1000)dy = \int_0^{0.6} \pi\sqrt{y}(1000)dy$

b. [7 points] Give an expression involving definite integrals that computes the work necessary to pump all the olive oil to the top of the aquarium.

Solution: $\text{Work}_{oil} = \int_{0.6}^{0.8} \pi(\sqrt[4]{y})^2(800)(9.8)(1-y)dy = \int_{0.6}^{0.8} \pi\sqrt{y}(800)(9.8)(1-y)dy$

9. [9 points] The tank pictured below has height 2 meters, and the top and bottom are equilateral triangles with sides of length 1 meter. It is filled **halfway** with hot chocolate. The hot chocolate has uniform density 1325 kg/m^3. The acceleration due to gravity is 9.8 m/s^2. Calculate the work needed to pump all the chocolate to the top of the tank. Show all your work. Give an **exact** answer. Include **units**.



Solution: We take a horizontal slice at height y meters from the bottom of the tank. It has mass $1325 \cdot \frac{\sqrt{3}}{4} 1^2 \Delta y$. We need to move it $2 - y$ meters up. Thus, the work needed to pump all the chocolate to the top is

$$\int_0^1 1325 \cdot \frac{\sqrt{3}}{4} 1^2 \cdot 9.8 \cdot (2-y)\, dy = \frac{1325 \cdot 9.8\sqrt{3}}{4}\left[2y - \frac{y^2}{2}\right]_{y=0}^{y=1} = \frac{1325 \cdot 9.8\sqrt{3}}{4} \cdot \frac{3}{2} \text{ Joules}$$

7. [8 points] Alyssa Edwards wants to play a prank on Coco Montrese by spilling a bucket of orange cheese powder on her. To do this Alyssa lifts the bucket at a constant speed from the ground to a height of 10 meters. Unfortunately the bucket has a small hole and the cheese begins leaking out at a constant rate as soon as the bucket leaves the ground. The bucket initially weighs 10kg and when it reaches a height of 10 meters it only weighs 5kg. Recall the gravitational constant is $g = 9.8\text{m/s}^2$.

a. [3 points] Write an expression giving the mass of the bucket $m(h)$ when the bucket is h meters above the ground.

Solution: The bucket is being lifted and is leaking at a constant rate. Therefore the mass of the bucket at height h will be a linear function. $m(h) = 5 + (\frac{10-h}{2}) = 10 - \frac{h}{2}$

b. [5 points] How much work is required to lift the bucket from the ground to a height of 10 meters? Include units.

Solution: The force on the bucket at height h is $gm(h)$. Therefore the work is $\int_0^{10} gm(h)dh = \int_0^{10} g(10 - \frac{h}{2})dh = 75g = 735$ joules.

1. [12 points] Indicate if each of the following is true or false by circling the correct answer (Justify your answer):

a. [3 points] If $F(t)$ and $G(t)$ are antiderivatives of the function $f(t)$ with $F(0) = 1$ and $G(0) = 3$ then $F(2) - G(2) = 1$.

True **[False]**

Solution: $F(2) - G(2) = F(0) - G(0) = 1 - 3 = -2$

b. [3 points] If $h(t) > 0$ for $0 \le t \le 1$, then the function $H(x) = \int_0^x h(t)dt$ is concave up for $0 \le x \le 1$.

True **[False]**

Solution: $H'(x) = h(x) > 0$, hence $H(x)$ is increasing in $0 \le x \le 1$, but not necessarily concave up. You need $H''(x) = h'(x) > 0$ for $H(x)$ to be concave up.

c. [3 points] If $\int_0^2 g(t)dt = 6$ then $\int_2^3 3g(2t-4)dt = 9$.

[True] False

Solution: Using $u = 2t - 4$ then $\int_2^3 3g(2t-4)dt = \frac{3}{2}\int_0^2 g(u)du = 9$

d. [3 points] $\frac{d}{dx}\left(\int_{-x^2}^{\sin x} e^{t^3}dt\right) = \cos x\ e^{x^3} + 2xe^{x^3}$.

True **[False]**

Solution: $\frac{d}{dx}\left(\int_{-x^2}^{\sin x} e^{t^3}dt\right) = \cos x\ e^{\sin^3 x} + 2xe^{-x^6}$.