Math 116 — Practice for Exam 3

Generated November 19, 2017

Name: ____

Instructor: ______________________ Section Number: __________

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1. This exam has 4 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2012	3	9		10	
Fall 2013	3	8		12	
Winter 2014	3	3		10	
Winter 2015	3	4	Bessel	9	
Total				41	

Recommended time (based on points): 49 minutes

9. [10 points] A second way to approximate the function

$$F(x) = \int_0^x \sqrt{1+9t^4}dt.$$

is by using its Taylor polynomials.

a. [2 points] Find the first three nonzero terms in the Taylor series for the function $\sqrt{1+u}$ about $u = 0$.

b. [2 points] Find the first three nonzero terms in the Taylor series for $\sqrt{1+9t^4}$ about $t = 0$.

c. [2 points] Find the first three nonzero terms in the Taylor series for $F(x)$ about $x = 0$.

d. [2 points] For which values of x do you expect the Taylor series for $F(x)$ about $x = 0$ to converge? Justify your answer.

e. [2 points] Use the fifth degree Taylor polynomial for $F(x)$ about $x = 0$ to approximate the value of $F(\frac{1}{2})$.

8. [12 points]

a. [4 points] Let a be a positive constant. Determine the first three nonzero terms of the Taylor series for

$$f(x) = \frac{1}{(1+ax^2)^4}$$

centered at $x = 0$. Show all your work. Your answer may contain a.

b. [2 points] What is the radius of convergence of the Taylor series for $f(x)$? Your answer may contain a.

c. [3 points] Determine the first three nonzero terms of the Taylor series for

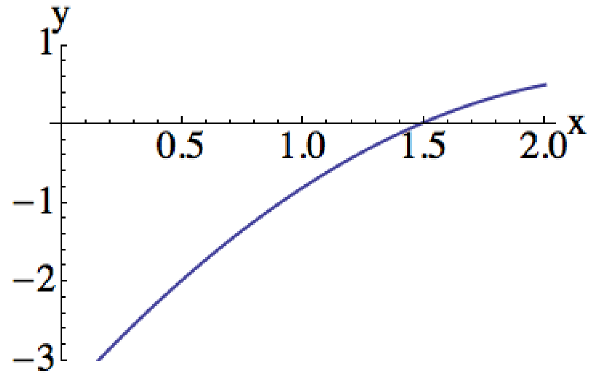
$$g(t) = \int_0^t \frac{1}{(1+ax^2)^4} dx,$$

centered at $t = 0$. Show all your work. Your answer may contain a.

d. [3 points] The degree-2 Taylor polynomial of the function $h(x)$, centered at $x = 1$, is

$$P_2(x) = a + b(x-1) + c(x-1)^2.$$

The following is a graph of $h(x)$:



What can you say about the values of a, b, c? You may assume a, b, c are nonzero. Circle your answers. No justification is needed.

a is: Positive Negative Not enough information

b is: Positive Negative Not enough information

c is: Positive Negative Not enough information

3. [10 points] For each of the following questions circle the correct answer.

a. [2 points] What is the value of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n}}{n!}$?

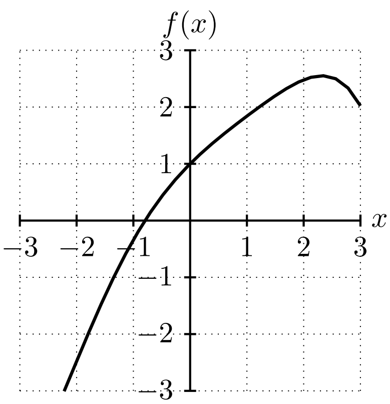
$\cos(2)$ $\qquad e^{-2}$ $\qquad \cos(4)$ $\qquad e^{-4}$

b. [2 points] What is the value of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^{2n}(-1)^n}{(2n+1)!}$?

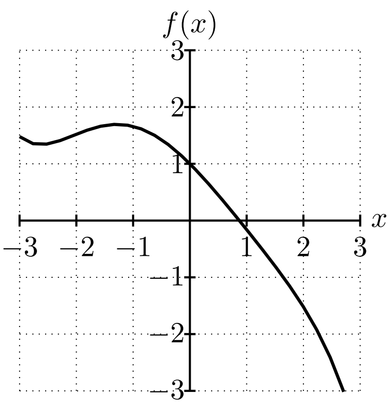
$\frac{1}{2}\sin(2)$ $\qquad \sin(2) - 2$ $\qquad \sin(2)$ $\qquad \frac{1}{2}(\sin(2) - 2)$

c. [2 points] Suppose that $1 + x - \frac{1}{4}x^2 + \frac{1}{10}x^3$ is the 3rd degree Taylor polynomial for a function $f(x)$. Which of the following pictures could be a graph of $f(x)$?

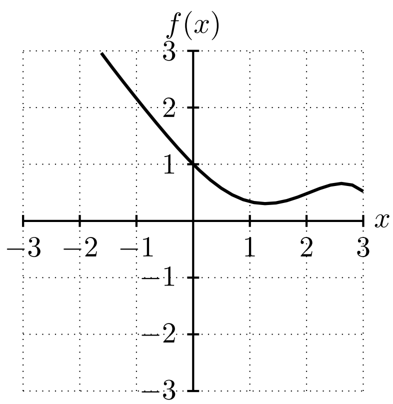
I



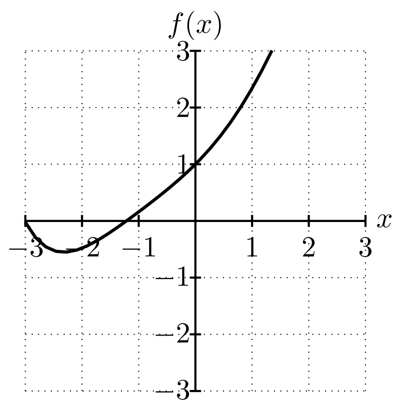
II



III



IV



d. [2 points] What is the Taylor series of $2xe^{x^2}$ centered at $x = 0$?

$\displaystyle\sum_{n=0}^{\infty} \frac{2x^{2n+1}}{n!}$ $\qquad \displaystyle\sum_{n=1}^{\infty} \frac{2x^{2n-1}}{n!}$ $\qquad \displaystyle\sum_{n=1}^{\infty} \frac{2x^{2n+1}}{(n-1)!}$ $\qquad \displaystyle\sum_{n=0}^{\infty} \frac{2x^{2n-1}}{n!}$

e. [2 points] The radius of convergence of the Taylor series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+5)^n 5^{-n}}{n+5}$ is $R = 5$. What is the interval of convergence of the series?

$[-10, 0)$ $\qquad (-10, 0)$ $\qquad (0, 10]$ $\qquad [-10, 0]$ $\qquad [0, 10)$

4. [9 points] We can define the Bessel function of order one by its Taylor series about $x = 0$,
$$J_1(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}.$$

a. [3 points] Compute $J_1^{(2015)}(0)$. Write your answer in exact form and do not try evaluate using a calculator.

$J_1^{(2015)}(0) =$ ______________

b. [4 points] Find $P_5(x)$, the Taylor polynomial of degree 5 that approximates $J_1(x)$ near $x = 0$.

$P_5(x) =$ ______________

c. [2 points] Use the Taylor polynomial from the previous part to compute
$$\lim_{x\to 0} \frac{J_1(x) - \frac{1}{2}x}{x^3}.$$

$\lim_{x\to 0} \frac{J_1(x) - \frac{1}{2}x}{x^3} =$ ______________