Math 116 — Practice for Exam 3

Generated October 26, 2017

Name: SOLUTIONS

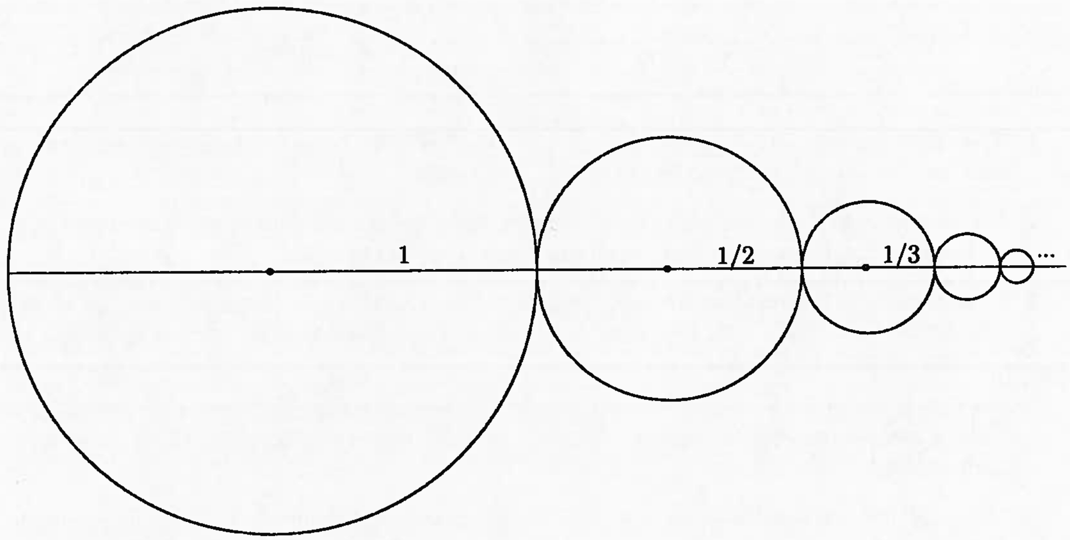
Instructor: ______________________ Section Number: __________

1. This exam has 3 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2016	3	5	pizzas	6	
Winter 2014	3	4		10	
Fall 2015	3	13		10	
Total				26	

Recommended time (based on points): 32 minutes

5. [6 points] O-guk is eating pizzas! All is well now, so he got hungry. He has put them next to each other, as depicted below, so that he can devour them one after another. There are infinitely many pizzas, and they have radii $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$
The following figure shows the first five pizzas.



a. [4 points] Write infinite series for the total area and the total perimeter of the pizzas. You must write your series in sigma notation.

Total area: $\displaystyle\sum_{n=1}^{\infty} \frac{\pi}{n^2}$

Total perimeter: $\displaystyle\sum_{n=1}^{\infty} \frac{2\pi}{n}$

b. [2 points] In the next two questions **circle** the correct answer.

Is the total area a finite number?

$\boxed{\text{YES}}$ **NO**

Is the total perimeter a finite number?

YES **$\boxed{\text{NO}}$**

4. [10 points] Determine whether the following series converge or diverge. Show all of your work and justify your answer.

a. [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{8^n + 10^n}{9^n}$

Solution: $\lim_{n\to\infty} \frac{8^n+10^n}{9^n} = \infty$ therefore by the n^{th} term test the series diverges.

b. [5 points] $\displaystyle\sum_{n=4}^{\infty} \frac{1}{n^3 + n^2\cos(n)}$

Solution: $\displaystyle\sum_{n=4}^{\infty} \frac{1}{n^3 + n^2\cos(n)} \leq \sum_{n=4}^{\infty} \frac{1}{n^2(n-1)} \leq \sum_{n=4}^{\infty} \frac{1}{n^2}$. The final series is a convergent p series since $p = 2 > 1$. Therefore the original series converges by comparison.

OR, once you know the Limit Comparison Test:

Since $\displaystyle\lim_{n\to\infty} \frac{\frac{1}{n^3+n^2\cos n}}{\frac{1}{n^3}} = 1$

and $\displaystyle\sum_{n=4}^{\infty} \frac{1}{n^3}$ is a convergent p-series with $p=3$,

$\therefore \displaystyle\sum_{n=4}^{\infty} \frac{1}{n^3+n^2\cos n}$ converges by the Limit Comparison Test.

13. [10 points] Suppose a_n and b_n are sequences with the following properties.

- $\sum_{n=1}^{\infty} a_n$ converges.
- $n \le b_n \le e^n$.

For each of the following statements, decide whether the statement is always true, sometimes true, or never true. Circle your answer. No justification is necessary. **You only need to answer 5 of the 7 questions.** Only answer the 5 questions you want graded. If it is unclear which 5 questions are being answered, the first 5 questions you answer will be graded.

a. [2 points] The sequence $\frac{1}{b_n}$ diverges.

ALWAYS SOMETIMES $\boxed{\textbf{NEVER}}$

Since $b_n \geq n$
$\Rightarrow 0 \le \frac{1}{b_n} \le \frac{1}{n}$
$\Rightarrow \lim_{n\to\infty} b_n = 0$

b. [2 points] The sequence a_n is bounded.

$\boxed{\textbf{ALWAYS}}$ SOMETIMES NEVER

Since $\sum_{n=1}^{\infty} a_n$ converges
$\Rightarrow \lim_{n\to\infty} a_n = 0$
$\Rightarrow a_n$ bounded

c. [2 points] The series $\sum_{n=1}^{\infty} \frac{1}{b_n}$ diverges.

ALWAYS $\boxed{\textbf{SOMETIMES}}$ NEVER

For instance, since if $b_n = n$ then it diverges but if $b_n = e^n$ then it converges

d. [2 points] The series $\sum_{n=1}^{\infty} e^{-a_n}$ converges.

ALWAYS SOMETIMES $\boxed{\textbf{NEVER}}$

Since $\sum_{n=1}^{\infty} a_n$ converges
$\Rightarrow \lim_{n\to\infty} a_n = 0$
$\Rightarrow \lim_{n\to\infty} e^{-a_n} = e^{-0} = 1$
Now use n^{th} term test

Assume $a_n \geq 0$.

e. [2 points] The series $\sum_{n=1}^{\infty} a_n^2$ diverges.

ALWAYS ~~SOMETIMES~~ NEVER

Since $\sum_{n=1}^{\infty} a_n$ converges
$\Rightarrow \lim_{n\to\infty} a_n = 0$
so for big n we have $0 \le a_n < 1$
$\therefore 0 \le a_n^2 < a_n$
so $\sum_{n=1}^{\infty} a_n^2$ converges by comparison test

f. [2 points] The series $\sum_{n=1}^{\infty} a_n b_n$ converges.

ALWAYS $\boxed{\textbf{SOMETIMES}}$ NEVER

For instance, it converges if $a_n = 0$ (for all n) but it diverges if $a_n = \frac{1}{n^2}$ and $b_n = e^n$

g. [2 points] The series $\sum_{n=1}^{\infty} \frac{b_n}{n!}$ converges.

$\boxed{\textbf{ALWAYS}}$ SOMETIMES NEVER

Since $0 < \frac{b_n}{n!} \le \frac{e^n}{n!}$ which is less than $\frac{1}{n^2}$ when n is big, and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.
(OR use the Ratio Test once we cover it in class)