MATH 116 — PRACTICE FOR EXAM 1

Generated September 20, 2017

NAME: SOLUTIONS

INSTRUCTOR: ______________________ SECTION NUMBER: __________

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1. This exam has 3 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	**Exam**	**Problem**	**Name**	**Points**	**Score**
Winter 2010	1	5		10	
Winter 2016	1	1	catapult	16	
Fall 2011	1	7	AC current	13	
Total				39	

Recommended time (based on points): 35 minutes

5. [10 points] For each statement below, circle TRUE if the statement is *always* true; otherwise, circle FALSE. There is no partial credit on this page.

a. [2 points] The function $\frac{\sin x}{x}$ has an anti-derivative.

[✓ True] False

b. [2 points] $\frac{d}{dx}\int_x^{x^2} e^{t^2}\,dt = 4x^3e^{x^4} - 2xe^{x^2}$.

True [✓ False]

c. [2 points] The average of the function $f(x) = \frac{1}{x}$ from $x = 1$ to $x = 3$ is $\ln(\sqrt{3})$.

[✓ True] False

d. [2 points] $\int_a^b f(x)dx$ is greater than or equal to at least one of LEFT(n), RIGHT(n), TRAP(n), or MID(n) regardless of what $f(x)$ or n is.

True [✓ False]

e. [2 points] If $\int_a^b f(x)dx > 0$ then $f(b) > f(a)$.

True [✓ False]

1. [16 points] At a time t seconds after a catapult throws a rock, the rock has horizontal velocity $v(t)$ m/s. Assume $v(t)$ is monotonic between the values given in the table and does not change concavity.

t	0	1	2	3	4	5	6	7	8
$v(t)$	47	34	24	16	10	6	3	1	0

a. [4 points] Estimate the average horizontal velocity of the rock between $t = 2$ and $t = 5$ using the trapezoid rule with 3 subdivisions. Write all the terms in your sum. Include **units**.

Solution:

$$\frac{\int_2^5 v(t)\,dt}{5-2} = \frac{Left(3) + Right(3)}{2 \cdot 3} = \frac{(v(2) + v(3) + v(4)) + (v(3) + v(4) + v(5))}{6} =$$

$$= \frac{24 + 16 + 10 + 16 + 10 + 6}{6} = \frac{82}{6} = \frac{41}{3}$$

The average horizontal velocity of the rock is 41/3 m/s.

b. [4 points] Estimate the total horizontal distance the rock traveled using a left Riemann sum with 8 subdivisions. Write all the terms in your sum. Include **units**.

Solution:

$$\int_0^8 v(t)\,dt = Left(8) = v(0) + v(1) + v(2) + v(3) + v(4) + v(5) + v(6) + v(7) =$$

$$= 47 + 34 + 24 + 16 + 10 + 6 + 3 + 1 = 141$$

The total horizontal distance the rock traveled is approximately 141 meters.

c. [4 points] Estimate the total horizontal distance the rock traveled using the midpoint rule with 4 subdivisions. Write all the terms in your sum. Include **units**.

Solution:

$$\int_0^8 v(t)\,dt = Mid(4) = 2(v(1) + v(3) + v(5) + v(7)) =$$

$$= 2(34 + 16 + 6 + 1) = 114$$

The total horizontal distance the rock traveled is approximately 114 meters.

d. [4 points] A second rock thrown by the catapult traveled horizontally 125 meters. Determine whether the first rock or the second rock traveled farther, or if there is not enough information to decide. Circle your answer. Justify your answer.

Solution:

the first rock [✓ the second rock] not enough information

The function $v(t)$ is concave up since for example

$$-10 = \frac{v(2) - v(1)}{2 - 1} > \frac{v(1) - v(0)}{1 - 0} = -13$$

The trapezoid rule gives $Trap(4) = 121$ (or $Trap(8) = 117.5$). Since $v(t)$ is concave up, this is an overestimate which means that the first rock traveled at most 121 meters, that is less than the second.

7. [13 points] Household electricity in the United States is supplied in the form of an alternating current that varies sinusoidally with a frequency of 60 cycles per second (Hz). The voltage is given by the equation

$$E(t) = 170\sin(120\pi t),$$

where t is given in seconds and E is in volts.

a. [7 points] Using integration by parts, find $\int \sin^2\theta d\theta$. Show all work to receive full credit. (Hint: $\sin^2\theta + \cos^2\theta = 1$.)

Solution: We first note that $\int \sin^2\theta d\theta = \int \sin\theta(\sin\theta)d\theta$, so that we may take $u = \sin\theta, dv = \sin\theta d\theta$ (and $du = \cos\theta d\theta, v = -\cos\theta$). Then integration by parts gives

$$\begin{aligned}\int \sin^2\theta d\theta &= \sin\theta(-\cos\theta) - \int -\cos\theta(\cos\theta)d\theta\\ &= -\sin\theta\cos\theta + \int \cos^2\theta d\theta.\end{aligned}$$

Using the trig. identity given in the hint, we obtain

$$\int \sin^2\theta d\theta = -\sin\theta\cos\theta + \int\left(1-\sin^2\theta\right)d\theta = -\sin\theta\cos\theta + \int d\theta - \int \sin^2\theta.$$

The integral on the far right also appears on the left, so combining like terms, we get

$$\begin{aligned}2\int \sin^2\theta d\theta &= -\sin\theta\cos\theta + \int d\theta\\ \int \sin^2\theta d\theta &= \frac{1}{2}\left(-\sin\theta\cos\theta + \int d\theta\right)\\ &= -\frac{1}{2}\sin\theta\cos\theta + \frac{\theta}{2} + C.\end{aligned}$$

b. [6 points] Voltmeters read the root-mean-square (RMS) voltage, which is defined to be the square root of the average value of $[E(t)]^2$ over one cycle. Find the exact RMS voltage of household current.

Solution: Since the frequency of the current is 60 cycles per second, one cycle is completed every $\frac{1}{60}$ seconds. Thus

$$\begin{aligned}\text{RMS voltage} &= \sqrt{\frac{1}{\frac{1}{60}-0}\int_0^{\frac{1}{60}} E(t)^2 dt}\\ &= \sqrt{60\int_0^{\frac{1}{60}} 170^2\sin^2(120\pi t)\,dt}.\end{aligned}$$

Substituting $w = 120\pi t, dw = 120\pi dt$, we get

$$\begin{aligned}\text{RMS voltage} &= \sqrt{60\int_{w(0)}^{w(\frac{1}{60})} 170^2\sin^2(w)\cdot\frac{1}{120\pi}dw}\\ &= \sqrt{\frac{170^2}{2\pi}\int_0^{2\pi}\sin^2(w)\,dw}.\end{aligned}$$

Using the antiderivative we found in part (a) with $C = 0$, the Fundamental Theorem of Calculus gives

$$\begin{aligned}\text{RMS voltage} &= \sqrt{\frac{170^2}{2\pi}\left(-\frac{1}{2}\sin\theta\cos\theta+\frac{\theta}{2}\right)\Big|_0^{2\pi}}\\ &= \sqrt{\frac{170^2}{2\pi}\left((-\frac{1}{2}\sin(2\pi)\cos(2\pi)+\frac{2\pi}{2})-(-\frac{1}{2}\sin(0)\cos(0)+\frac{0}{2})\right)} = \frac{170}{\sqrt{2}}\text{ Volts}.\end{aligned}$$

Note that due to the periodicity of the sine function, the average value over one cycle could also have been computed over $0 \le t \le 1$ (or any other number of periods).