Math 116 — Practice for Exam 1

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Name: SOLUTIONS

Instructor: ______________________ Section Number: __________

1. This exam has 3 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2016	1	7		6	
Winter 2015	1	3		13	
Winter 2016	1	5		16	
Total				35	

Recommended time (based on points): 32 minutes

7. [6 points] Suppose that g is a continuous function, and define another function G by

$$G(x) = \int_0^x g(t)\,dt.$$

Given that $\int_0^7 g(x)\,dx = 5$, compute

$$\int_0^7 g(x)(G(x))^2\,dx.$$

Show each step of your computation.

Solution: Substitution gives

$$\int_0^7 g(x)(G(x))^2\,dx = \int_{G(0)}^{G(7)} u^2\,du = \frac{u^3}{3}\bigg|_0^5 = \frac{125}{3}.$$

Alternatively, integrate by parts to obtain

$$\int_0^7 g(x)(G(x))^2\,dx = (G(x))^3\big|_0^7 - 2\int_0^7 g(x)(G(x))^2\,dx,$$

which after rearranging gives

$$\int_0^7 g(x)(G(x))^2\,dx = \frac{1}{3}\left((G(x))^3\big|_0^7\right) = \frac{125}{3}.$$

3. [13 points] Use the table and the fact that

$$\int_0^{10} f(t)dt = 350$$

to evaluate the definite integrals below exactly (i.e., no decimal approximations). Assume $f'(t)$ is continuous and does not change sign between any consecutive t-values in the table.

t	0	10	20	30	40	50	60
$f(t)$	0	70	e^5	e^3	0	$\pi/2$	π

a. [4 points] $\displaystyle\int_0^{10} tf'(t)dt$

Solution:

$$\begin{aligned}\int_0^{10} tf'(t)dt &= tf(t)|_0^{10} - \int_0^{10} f(t)dt \\ &= 10f(10) - \int_0^{10} f(t)dt \\ &= 700 - 350 \\ &= 350.\end{aligned}$$

b. [4 points] $\displaystyle\int_{20}^{30} \frac{f'(t)}{f(t)}dt$

Solution:

$$\begin{aligned}\int_{20}^{30} \frac{f'(t)}{f(t)}dt &= \int_{f(20)}^{f(30)} \frac{1}{u}du \\ &= \ln|u||_{f(20)}^{f(30)} \\ &= \ln|f(30)| - \ln|f(20)| \\ &= 3 - 5 \\ &= -2.\end{aligned}$$

c. [5 points] $\displaystyle\int_{50}^{60} f(t)f'(t)\sin(f(t))dt$

Solution:

$$\begin{aligned}\int_{50}^{60} f(t)f'(t)\sin(f(t))dt &= \int_{f(50)}^{f(60)} w\sin(w)dw \\ &= -w\cos(w)|_{f(50)}^{f(60)} + \int_{f(50)}^{f(60)} \cos(w)dw \\ &= -\pi\cos(\pi) + \int_{\pi/2}^{\pi} \cos(w)dw \\ &= \pi - 1\end{aligned}$$

5. [16 points] Suppose that $f(x)$ is a function with the following properties:

- $\int_0^1 f(x)\,dx = -5.$
- $\int_0^3 f'(x)\,dx = 10.$
- The average value of $f(x)$ on $[1, 1.5]$ is -4.
- $\int_2^4 xf'(x)\,dx = 8.$

In addition, a table of values for $f(x)$ is given below.

x	0	1	2	3	4
$f(x)$	-7	-2	-2	m	0

Calculate (a)-(d) **exactly**. Show your work and do not write any decimal approximations.

a. [4 points] $m = 3$

Solution: Using the Fundamental Theorem in $\int_0^3 f'(x)\,dx = 10$ we get $f(3) - f(0) = 10$ which gives $m - (-7) = 10$ so $m = 3$.

b. [4 points] $\int_0^{1.5} f(x)\,dx = -7$

Solution:

$$\int_0^{1.5} f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^{1.5} f(x)\,dx = -5 + 0.5(-4) = -7$$

c. [4 points] $\int_2^4 f(x)\,dx = -4$

Solution: Using integration by parts in $\int_2^4 x f'(x)\,dx = 8$ we get $(4f(4) - 2f(2)) - \int_2^4 f(x)\,dx = 8$ which gives $\int_2^4 f(x)\,dx = 0 - 2(-2) - 8 = -4$.

d. [4 points] $\int_4^{16} f'(\sqrt{x})\,dx = 16$

Solution: Using the substitution $u = \sqrt{x}$ we get

$$\int_4^{16} f'(\sqrt{x})\,dx = \int_2^4 f'(u) \cdot 2u\,du = 2 \cdot 8 = 16$$