

1. Write these using  $\sum$ :

(a)  $\frac{1}{2}x + \frac{1 \cdot 3}{2^2 \cdot 2!}x^2 + \frac{1 \cdot 3 \cdot 5}{2^3 \cdot 3!}x^3 + \dots$

(b)  $(x - 1)^3 - \frac{(x-1)^5}{2!} + \frac{(x-1)^7}{4!} - \frac{(x-1)^9}{6!} + \dots$

2. For which values of  $x$  do these power series converge? Diverge?

(a)  $\sum_{n=0}^{\infty} n^3 x^n$

(b)  $\sum_{n=1}^{\infty} \frac{2^n (x-1)^n}{n}$

(c)  $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$

(d)  $\sum_{n=0}^{\infty} \frac{n^2 x^{2n}}{2^{2n}}$

3. Find the interval of convergence. This INCLUDES checking the endpoints!!

(a)  $\sum_{n=1}^{\infty} \frac{x^{2n+1}}{n!}$

(b)  $\sum_{n=1}^{\infty} \frac{(5x)^n}{\sqrt{n}}$

4. For all  $t$ -values for which it converges, the function  $f$  is defined by the series

$$f(t) = \sum_{n=0}^{\infty} \frac{(t - 7)^n}{5^n}.$$

(a) Find  $f(4)$ .

(b) Find the interval of convergence of  $f(t)$ .

5. The series  $\sum C_n x^n$  converges when  $x = -4$  and diverges when  $x = 7$ . Decide whether each of the following is true or false, or whether this cannot be determined.

(a) The power series converges when  $x = 10$ .

- (b) The power series converges when  $x = 3$ .
  - (c) The power series diverges when  $x = 1$ .
  - (d) The power series diverges when  $x = 6$ .
6. If  $\sum C_n(x - 3)^n$  converges at  $x = 7$  and diverges at  $x = 10$ , what can you say about the convergence at  $x = 11$ ? At  $x = 5$ ? At  $x = 0$ ?
7. For all  $x$ -values for which it converges, the function  $f$  is defined by the series

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

- (a) What is  $f(0)$ ?
- (b) What is the domain of  $f$ ?
- (c) Assuming that  $f'$  can be calculated by differentiating the series term-by-term, find the series for  $f'(x)$ . What do you notice?
- (d) Guess what well-known function  $f$  is.