Math 116 — Practice for Exam 1

Generated January 18, 2023

Name: SOLUTIONS

Instructor: ______________________ Section Number: __________

1. This exam has 2 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2021	1	6		13	
Winter 2022	1	3		12	
Total				25	

Recommended time (based on points): 23 minutes

6. [13 points]

a. [6 points] Split the function $\frac{4-9x}{(x-2)^2(x+5)}$ into partial fractions with 2 or more terms. **Do not integrate these terms**. Please show all work to obtain your partial fractions.

Solution: Let

$$\frac{4-9x}{(x-2)^2(x+5)} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2} + \frac{C}{(x+5)}.$$

Then,

$$A(x-2)(x+5) + B(x+5) + C(x-2)^2 = 4 - 9x.$$

Comparing coefficients of x^2

$$A + C = 0 \text{ , so } A = -C.$$

Comparing coefficients of x,

$$3A + B - 4C = -9 \text{ , so } B = -9 - 7A.$$

Comparing the coefficients of the constant term,

$$4C + 5B - 10A = 4 \text{ , so } A = -1.$$

Substituting back, we see that $B = -2$ and $C = 1$, so

$$\frac{4-9x}{(x-2)^2(x+5)} = \frac{-1}{(x-2)} + \frac{-2}{(x-2)^2} + \frac{1}{(x+5)}.$$

b. [7 points] Use the fact that $\frac{5x}{(x^2+1)(x-2)} = \frac{2}{x-2} + \frac{-2x+1}{(x^2+1)}$ to solve the indefinite integral

$$\int \frac{5x}{(x^2+1)(x-2)} dx.$$

Solution: We first split the integral into 3 terms and use substitution $w = x^2 + 1$ on the second term:

$$\begin{aligned} \frac{5x}{(x^2+1)(x-2)dx} &= \int \frac{2}{x-2} dx + \int \frac{-2x}{x^2+1} dx + \int \frac{1}{x^2+1} dx \\ &= \int \frac{2}{x-2} dx - \int \frac{1}{w} dw + \int \frac{1}{x^2+1} dx \\ &= 2\ln|x-2| - \ln|w| + \arctan x + C \\ &= 2\ln|x-2| - \ln(x^2+1) + \arctan x + C. \end{aligned}$$

Note that we have substituted back in for x.

3. [12 points]

a. [6 points] Split the following function into partial fractions. Do not integrate the result. Be sure to show all your work.

$$\frac{10x+2}{(x-1)^2(x+2)}$$

Solution: Start by splitting:

$$\frac{10x+2}{(x-1)^2(x+2)} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$$

By giving terms a common denominator, we get:

$$10x+2 = A(x-1)(x+2) + B(x+2) + C(x-1)^2. \qquad (1)$$

Method 1: (Comparing Coefficients) If we distribute terms, we get

$$0x^2 + 10x + 2 = (A+C)x^2 + (A+B-2C)x + (-2A+2B+C).$$

This gives the system of equations:

$$A + C = 0 \qquad A + B - 2C = 10 \qquad -2A + 2B + C = 2$$

which we can solve for the values: $A = 2$, $B = 4$, $C = -2$.

Method 2: (Pluggin In Values) If we plug $x = 1$ into (1) we get

$$10(1) + 2 = A(1-1)(1+2) + B(1+2) + C(1-1)^2$$

This simplifies to give $12 = 3B$ and therefore $B = 4$.
If we plug in $x = -2$

$$10(-2) + 2 = A(-2-1)(-2+2) + B(-2+2) + C(-2-1)^2$$

This gives $-18 = 9C$ and therefore $C = -2$.
Then, plugging in B, C and $x = -1$, we get:

$$\begin{aligned} 10(-1) + 2 &= A(-1-1)(-1+2) + 4(-1+2) + (-2)(-1-1)^2 \\ &\Rightarrow -8 = -2A + 4 - 8 \\ &\Rightarrow -4 = -2A \Rightarrow A = 2. \end{aligned}$$

So we have $A = 2$, $B = 4$, $C = -2$.

b. [6 points] Given the partial fraction decomposition

$$\frac{-x-10}{(x-3)(x^2+4)} = \frac{-1}{(x-3)} + \frac{x+2}{x^2+4},$$

evaluate the following indefinite integral, show all your steps:

$$\int \frac{-x-10}{(x-3)(x^2+4)}dx$$

Solution: Start by substituting and splitting up the integral:

$$\int \frac{-x-10}{(x-3)(x^2+4)}dx = \int \frac{-1}{x-3}dx + \int \frac{x+2}{x^2+4}dx$$

Then we split up the second integral to get:

$$\int \frac{-x-10}{(x-3)(x^2+4)}dx = \int \frac{-1}{x-3}dx + \int \frac{x}{x^2+4}dx + \int \frac{2}{x^2+4}dx$$

For the first integral we have:

$$\int \frac{-1}{x-3}dx = -\ln|x-3| + C$$

For the second integral we use u-substitution with $u = x^2+4$ and $du = 2xdx$ to get

$$\int \frac{x}{x^2+4}dx = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln(x^2+4) + C.$$

For the final integral, we first rewrite $\frac{2}{x^2+4} = \frac{2}{4((\frac{x}{2})^2+1)} = \left(\frac{1}{2}\right)\frac{1}{(\frac{x}{2})^2+1}$. Then we have:

$$\int \frac{2}{x^2+4}dx = \frac{1}{2}\int \frac{1}{(\frac{x}{2})^2+1}dx$$

Using substitution with $u = \frac{x}{2}$, so $du = \frac{1}{2}dx$, this becomes:

$$\int \frac{2}{x^2+4}dx = \frac{1}{2}\int \frac{1}{(\frac{x}{2})^2+1}dx = \int \frac{1}{u^2+1}du = \arctan(u) + C = \arctan\left(\frac{x}{2}\right) + C$$

Putting this all together, we get

$$\int \frac{-x-10}{(x-3)(x^2+4)}dx = -\ln|x-3| + \frac{1}{2}\ln(x^2+4) + \arctan\left(\frac{x}{2}\right) + C.$$