Reference-Dependent Procedural Decision Making*

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June 22, 2006

Abstract

We derive, by using the revealed preference method, an individual decision making model that allows for an agent not only to exhibit status quo bias, but also to make use of her default option as a reference point. The resulting model contains elements of the classical rational choice model – indeed it reduces to that model in the case of choice problems without default options – but can also be viewed as arising from a basic choice procedure that involves multiple objectives. Another important feature of the model is that, while it permits status quo bias, it does not necessarily lead to the overvaluation of one's endowment, and hence it is duly consistent with the *absence* of a gap between one's willingness to sell and buy a given choice alternative.

JEL Classification: D11, D81.

Keywords: Revealed Preferences, Incomplete Preferences, Status Quo Bias, Endowment Effect, Overpricing.

*Various conversations with Jean-Pierre Benoît, Faruk Gul, Gil Riella, Ariel Rubinstein, and Neslihan Uler have helped to the development of this paper to a significant degree; we gratefuly acknowledge our intellectual debt to them. We are also grateful to the support of the C. V. Starr Center for Applied Economics at New York University.

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1 Introduction

A large number of empirical studies, both within economics and psychology, have established that decision makers settle various types of choice problems in a *reference-dependent* manner. In particular, there is now a widespread agreement among both behavioral economists and rational decision theorists that individuals behave in the same choice situation markedly differently, depending on what sort of a "reference" they are given in the form of an initial entitlement, endowment and/or default option. Indeed, in a plethora of experimental and field studies, the relative value of an alternative is found to be accentuated for agents who possess that alternative as current endowment. (Following Samuelson and Zeckhauser (1988), this effect is called the *status quo bias* phenomenon.) Moreover, it is well established that the status quo position of a decision maker affects the behavior of the agent even if the agent chooses to move away from her status quo (as in *reason-based* decision making, or what is called the *attraction effect*).[1]

However, the basic nature of reference-dependent decision making is not well understood. For instance, a good part of the experimental literature assumes that status quo bias phenomenon is tantamount to the so-called *endowment effect* (Thaler, 1980) which maintains that the minimum compensation demanded by an agent for a good that she owns is less than the maximum price she is ready to pay for the same good. In fact, as we demonstrate below, status quo bias (properly defined) does *not* entail the endowment effect. Moreover, it turns out that the endowment effect is suspect as a behavioral trait, at least in the case of experienced traders. Indeed, there is now good evidence that shows that this effect is likely to dissipate with market experience, and there need not be a gap between willingness to accept (WTA) and willingness to pay (WTP).[2] To be sure, this does not mean traders,

[1] The empirical/experimental literature on reference-dependent individual decision making is too large to be cited here. We refer the reader to Camerer (1995) and Sugden (1999) for insightful surveys on this matter. For discussions of the attraction effect and related phenomena, see Simonson (1993), Shafir, Simonson and Tversky (1993), Sen (1998), Malaviya and Sivakumar (2002), and references cited therein.

[2] See, for instance, Shogren *et al.* (1994) and List (2003, 2004). In a more recent study, Plott and Zeiler (2005, pp. 532 and 542) summarize their experimental findings in complete accordance with this position: "When an incentive-compatible mechanism is used to elicit valuations, and subjects are provided with (a) a detailed explanation of the mechanism and how to arrive at valuations; (b) paid practice using the mechanism; and (c) anonymity, we observe no WTA-WTP gap. ... The primary conclusion reported here is that (previously) observed WTA-WTP gaps do not reflect a fundamental feature of human preferences.

experienced or otherwise, do not use their initial entitlements as reference points. Instead, it shows that the nature of reference dependent choice is less straightforward than what one may initially surmise.

One major difficulty in this branch of literature is the lack of a canonical model within which a formal discussion can be conducted. Indeed, none of the few proposed models are general enough to serve for this purpose, for most of these models are couched within the particular setup of certain types of experiments, as opposed to being a general model of individual decision making. (This is, of course, in sharp contrast with the classical rational choice theory.) Consequently, these models either *necessitate* the presence of the endowment effect, or concentrate on only one aspect of the effect of one's endowment acting as a reference point.[3] In addition, more often than not, their foundational bases are suspect.

The main objective of the present paper is to develop an individual choice model which is free of the difficulties that surround the available models of reference-dependent choice. We follow an axiomatic approach that builds on classical revealed preference theory. First, we posit rationality axioms on choice behavior (for problems in which the agent may or may not have a status quo option) along the guidelines of standard choice theory. Second, we introduce a basic status quo bias axiom, and hence depart from the classical rational choice paradigm in a way that is consonant with related experimental findings. Finally, we advance two further properties that "control" the extent of reference dependence the sought model allows for. It is found that these behavioral (and hence directly testable) postulates lead to an individual decision making model that is based on a basic procedure.

Loosely speaking, the procedural structure of this model can be summarized as follows. If the choice problem of the agent has a status quo (default) option, then she makes her choices upon maximizing a utility function U. (See Figure 1.a.) If there is a status quo, say x, in the problem, she uses the following procedure:

Step 1. The agent employs a mental "constraint set" that depends on her status quo, say $\mathcal{Q}^1(x)$, and eliminates (in her mind) all feasible alternatives that do not belong to this constraint set.[4] If, other than the default option x, at least one feasible alternative passes

That is, endowment effect theory does not seem to explain observed gaps."

[3] In Section 2, we briefly consider the two main strands of models that are considered in the literature on reference-dependent behavior, and point to the fundamental deficiencies of these approaches.

[4] One may think of $\mathcal{Q}^1(x)$ as arising from a psychological phenomenon according to which the presence of the status quo point x, acting as a reference, leads the agent to concentrate *only* on those alternatives that

this test, then the agent chooses among such alternatives the ones that yield the highest satisfaction in terms of her utility function U. (See Figure 1.b.) Otherwise, she moves to the next stage of her procedure.

Step 2. At this stage, the agent reasons exactly as she did in the first stage, except now, perhaps realizing that she made an "overuse" of her default option x as a reference point, she relaxes her mental constraint set to a bigger set $\mathcal{Q}^2(x)$.[5] She settles her problem, then, upon searching for feasible alternatives in $\mathcal{Q}^2(x)$. If at least one of her options passes this test, among these, those with the highest U value constitute her set of choices. (See Figure 1.c.) If no feasible alternative passes this second test either, then the agent chooses not to move from her status quo point x. (See Figure 1.d.)

FIGURE 1 ABOUT HERE

There are several advantages of this model. It is a *boundedly* rational choice model, to be sure, but the departure from the rational choice model that it envisages emanates solely from reference dependence, as the model reduces to the standard choice model for problems without initial entitlements. It exhibits a definitive status quo bias — one of its defining axioms is, after all, a formalization of such a bias. But one can show that it allows for the absence of the endowment effect, so it is arguably suitable for modeling experienced as well as nonexperienced traders. Moreover, it is based on a well-defined procedure, and hence lies within the realms of both reference-dependent and procedural choice. (In fact, as we show below, one can also view this model as one of multicriteria choice.) Finally, this model is directly testable in that it is given here a foundational basis in terms of simple behavioral postulates.

The paper proceeds as follows. In Section 2 we provide a quick overview of the literature on reference-dependent choice, and point to the deficiencies of the models proposed in this body of work. We then sketch in Section 3 our axiomatic framework, delineating carefully

are better than x in some (subjectively) unambiguous sense. Alternatively, one may think of the elimination of those feasible alternatives outside $\mathcal{Q}^1(x)$ as a "simplification" the agent uses to settle her possibly complex choice problem.

[5] By way of interpretation, we can think of the agent as allowing for more trade-offs at Step 2, and hence relaxing her reference-dependence somewhat. Another interpretation is that she concludes from the outcome of Step 1 that using $\mathcal{Q}^1(x)$ to eliminate alternatives may have "oversimplified" her problem, thereby adopting a less restrictive rule of elimination.

the behavioral basis of our choice postulates. Section 4 presents our characterization of the procedural choice model discussed informally above, and Section 5 relates this model to the theory of multicriteria decision making. Finally, in Section 6, we examine the standing of our model relative to the endowment effect, and show that it is consistent with both the absence and presence of a gap between one's WTA and WTP. Section 7 concludes, and Section 8 outlines the proofs of the main results.

2 Review of Reference-Dependent Choice Models

In this section we briefly discuss the two basic approaches towards modeling individual choice behavior in the presence of status quo or reference alternatives. We intend to show the need for further theoretical work in this area, thereby clarifying the contribution of the present investigation (or its lack thereof).[6]

2.1 The Loss Aversion Model(s)

The prototypical example of loss aversion models is the one introduced by Tversky and Kahneman (1991) in the context of riskless choice.[7] Indeed, this model is viewed within the behavioral economics literature as the "standard" model of reference-dependent decision making. It is couched in a framework where the objects of choice have multiple, say $n \geq 2$, dimensions, and all of these dimensions are observable. It is thus particularly suitable to study decisions of individuals over all possible bundles of n goods — indeed, this is primarily how the model is used in practice.

The basic premise of the Tversky-Kahneman model is that an agent, whose initial entitlement is some $x \in \mathbb{R}^n_+$, chooses those alternatives from a given feasible subset of $\mathbb{R}^n_+$ upon maximizing a utility function $U_x : \mathbb{R}^n_+ \to \mathbb{R}$ of the following form:

$$U_x(y) := \sum_{i=1}^{n} u_i(y_i - x_i). \tag{1}$$

[6]We concentrate exclusively on works in which one's reference is modeled exogenously as her (observable) initial holdings and/or default option. Equally interesting is, of course, the case where one's reference is not observable, such as one's aspirations, as in the work of Köszegi and Rabin (2006). We have little to say about that type of reference dependence in the present paper, however.

[7]See Sugden (2003) for a version of this model within the context of choice under uncertainty.

Here, for each i, $u_i : \mathbb{R} \to \mathbb{R}$ is a continuous and strictly increasing function that satisfies the following two properties:

(1) $u_i(0) = 0$ and $u_i(t) < -u_i(-t)$ for each $t \neq 0$;

(2) $u_i|_{\mathbb{R}_+}$ is concave and $u_i|_{\mathbb{R}_-}$ is convex.

Property (1) captures the phenomenon of *loss aversion* which says that, in terms of the ith commodity, a gain is less important to the agent than a loss of equal size. In turn, Property (2) says that the marginal value of both gains and losses are decreasing, and hence corresponds to the so-called *diminishing sensitivity* effect.

The implications of this model is found to be consistent with most experiments in which the presence of an "initial entitlement" of an agent affects her choice behavior. Moreover, it has a particularly simple mathematical structure that makes it amenable to applications, which is surely an appealing property.

Notwithstanding its incipient popularity, however, there are several difficulties that surround this model. First of all, it is not a "basic" choice model in that it does not apply in a straightforward manner to individual choice problems in which the objects of choice are not consumption bundles. Indeed, it is difficult to see how to make use of the model, say, in the context of voting over political candidates, choosing between insurance or retirement policies, comparing job offers, etc..[8]

The Tversky-Kahneman model is thus best situated within the context of consumption decisions alone. Considering, in addition, the fact that this model is based on an "additive aggregation" hypothesis, it becomes transparent that it cannot serve as a canonical model of reference-dependent choice.

Another difficulty with the Tversky-Kahneman model stems from the fact that, even in

[8] A somewhat typical response to this, say, in the context of job offers, is that every "job contract" is a multidimensional object — the dimensions being, for instance, work location, salary, job quality, etc. — and hence, once the dimensions are specified, the Tversky-Kahneman model becomes applicable to the context of job search. The difficulty here is that these dimensions are not observable, simply because which dimensions are rendered relevant to the problem is known only to the decision maker. Furthermore, even when they are prespecified, these dimensions need not be quantifiable, so one has to view x in this context as the "utility profile" that the agent derives from her current job. In turn, $y_i - x_i$ corresponds to the gain/loss of utility (of switching from x to y) with respect to the ith dimension. This leads one to subscribe to unacceptably strong cardinality-of-utility assumptions just to be able to view $y_i - x_i$ as a meaningful expression, and surely makes the quantity $u_i(y_i - x_i)$ rather difficult to interpret. (Other loss aversion models proposed in the literature, such as that of Munro and Sugden (2003), suffer as well from the same shortcoming.)

the context of choosing among consumption bundles, not all of its implications are in concert with empirical facts. Indeed, this model deviates from the standard rational choice paradigm to the extent that certain reasonable implications of the latter are lost. In particular, and as nicely demonstrated by Munro and Sugden (2003), the Tversky-Kahneman model permits choice cycles in the sense that, according to this model, an agent may strictly prefer y to x when endowed with x, and strictly prefer z to y when endowed with y, and finally, x to z when endowed with z). Needless to say, it would be unreasonable to expect a consumer to depict such cyclical patterns in daily discourse, and indeed, there is no known market evidence to this effect.[9]

Finally, we note that the Tversky-Kahneman model is not consistent with the absence of a gap between willingness to accept (WTA) and willingness to pay (WTP).[10] At first, this may seem like a strong point of the theory in that there are many "endowment effect" experiments that show that the minimum compensation demanded by an agent for a good that she owns is often less than the maximum price she is ready to pay. However, as we have noted in Section 1, careful empirical studies have shown that it may be a mistake to consider the WTA-WTP gap as a fundamental feature of individual preferences, and one that goes away with sufficient market experience. Of course, this does not mean that the preferences of (experienced) traders are not reference-based or status quo biased. Instead, it shows that a model that necessarily implies a gap between WTA and WTP may well leave something to be desired.[11]

[9] Such cycles are permitted by the Tversky-Kahneman theory because the diminishing sensitivity effect (for losses) may act counter to the loss aversion effect. If the former is eliminated, that is, if we consider the *constant loss aversion* version of the model (in which each $u_i|_{\mathbb{R}_+}$ and $u_i|_{\mathbb{R}_-}$ are assumed to be linear functions), then no choice cycles may occur. But, unfortunately, the explanatory power of the constant loss aversion model is nowhere near that of the Tversky-Kahneman model (cf. Masatlioglu and Uler, 2006).

[10] This claim is formalized in Section 6.2.

[11] To give an example, List (2004, pp. 616 and 624) summarizes the findings of his field experiment within the sportscard market as follows: "An interesting finding is that individual trading rates of inexperienced consumers are consonant with predictions from prospect theory. The endowment effect anomaly is not universal, however: consumers that have significant market experience do not exhibit behavior consistent with prospect theory: rather, their behavior is in line with neoclassical predictions. ... The overall data pattern observed uncovers important successes and failures of the theoretical literature, and provides challenges for both neoclassical and reference-dependent theorists."

2.2 The Multicriteria Choice Model(s)

Of more recent origin than the loss aversion models are choice models in which reference-dependence of an individual is captured by means of multiple rationales. To give an example, and for drawing comparisons later, we consider here the model that was introduced in Masatlioglu and Ok (2005).[12] That model posits that a decision maker evaluates all choice alternatives in some grand choice space X according to multiple, say $m \geq 2$ many, criteria. Suppose the preferences of the agent over alternatives in X with respect to criterion i is represented by some utility function $u_i : X \to \mathbb{R}$. Now, if the agent does not have a default (status quo) option, then she chooses from a given feasible set those alternatives that maximize a utility function $U : X \to \mathbb{R}$. Here U is obtained through some aggregation of the objectives $u_1, ..., u_m$ (that is, $U(\omega) = f(u_1(\omega), ..., u_m(\omega))$ for each $\omega \in X$, where $f : \mathbb{R}^m \to \mathbb{R}$ is a strictly increasing (aggregation) function.) If, on the other hand, the agent has a status quo choice in the problem, say $x \in X$, then she proceeds to settle that problem by comparing every feasible alternative with x *with respect to all criteria*. If none of the feasible alternatives y dominates x with respect to all objectives — that is, for any feasible y, there is a rationale i such that $u_i(x) > u_i(y)$ — then the agent chooses to stay with her status quo. If, however, at least one feasible alternative "beats" x with respect to all objectives, then the agent concentrates (only) on such alternatives, and chooses among them the one(s) that yield the highest satisfaction in terms of the reference-*in*dependent utility function U.

This choice model relates closely to those suggested by Simon (1955) and Bewley (1986), it reduces to the standard model of rational choice for problems *without* status quo, and is fully consistent with the status quo bias phenomenon.[13] Moreover, unlike the loss aversion model(s), it is universal (in the sense that it applies to any choice situation), it forbids cyclical choice patterns, and it is consistent with both the presence and absence of a gap between WTA and WTP. Yet, it would surely be a mistake to deem this model superior to the Tversky-Kahneman model. After all, the said Masatlioglu-Ok model fails to capture the notion of reference-dependence beyond what is entailed by the status quo phenomenon. The following example illustrates the nature of the difficulty.

Example 1. Consider an agent whose problem is to choose among feasible consumption

[12] See Houy (2006) and Sagi (2006) for related choice models.

[13] The results obtained in the experiments of Masatlioglu and Uler (2006) show that this model fits the data significantly better than the *constant* loss aversion model.

bundles (with two distinct commodities). Consider Figure 2, and suppose the initial endowment of the agent is x, and that this agent chooses y from the set $\{x, x', y, z\}$. Now consider the case where the initial endowment of the agent is x'. The model outlined above necessitates that the agent choose in this case y as well. (For, this model maintains that "worsening" of one's *unchosen* endowment cannot affect the final choice(s) of the agent.) A moment's reflection, however, shows that it is not at all unreasonable for the decision maker to choose instead z in this situation. After all, it is quite conceivable that z looks much better when one looks at the situation "from x'," so if the utility of y is only slightly higher than that of z, reference-dependence of the agent may easily lead her to choose z from $\{x, x', y, z\}$ where her reference point is x'.[14] □

FIGURE 2 ABOUT HERE

This example illustrates that, despite its appealing features, the model of Masatlioglu and Ok (2005), among other multicriteria choice models, falls short of being a satisfactory model of "reference-dependent" decision making.

2.3 Synopsis

While it is far from being a comprehensive survey, the discussion above nevertheless highlights the fact that the literature on reference-dependent choice is far from mature. In particular, it lacks a rational choice model which (1) allows for the status quo bias phenomenon; (2) reduces to the standard rational choice model in the case of problems with no explicitly given reference points; (3) is free from cyclical choice patterns; (4) is consistent with both the presence and absence of a gap between WTA and WTP; and (5) permits reference-dependent decision making in ways that go beyond the status quo bias phenomenon.

Last, but not least, the said literature does not contain any model that could properly be thought of as "procedural," despite the obvious plausibility of such choice behavior (especially in contexts where reference alternatives are explicitly designated). In the rest of the paper, we conduct a decision-theoretic analysis towards "deriving" a procedural choice model that

[14] Indeed, the experiments of Masatlioglu and Uler (2006) show that such sort of reference-dependence is quite common. Of course, given the strong empirical support for "reason-based choice," this is only to be expected.

satisfies all of the criteria outlined above.

3 Rational Choice with Status Quo Bias

3.1 The Basic Framework

We designate an arbitrary compact metric space X to act as the universal set of all mutually exclusive alternatives. The set X is thus viewed as the grand alternative space, and is kept fixed throughout the exposition. The members of X are denoted as x, y, z, etc.. For reasons that will become clear shortly, we designate the symbol $\Diamond$ to denote an object that does not belong to X. We shall use the symbol τ to denote a generic member of $X \cup \{\Diamond\}$.

We let Ω_X denote the set of all nonempty closed subsets of X. By a **choice problem**, we mean a list (S, τ) where $S \in \Omega_X$ and either $\tau \in S$ or $\tau = \Diamond$. The set of all choice problems is denoted as $\mathcal{C}(X)$.

Given any $x \in X$ and $S \in \Omega_X$ with $x \in S$, the choice problem (S, x) is called a **choice problem with a status quo**. The interpretation is that the individual is confronted with the problem of choosing an alternative from the feasible set S while either she is currently endowed with the alternative x or her default option is x. Viewed this way, choosing an alternative $y \in S \backslash \{x\}$ means that the subject individual gives up her status quo x and switches to y. We denote by $\mathcal{C}_{\text{sq}}(X)$ the set of all choice problems with a status quo.[15]

On the other hand, many real-life choice situations do not have a natural status quo alternative. Within the formalism of this paper, the choice problems of the form $(S, \Diamond)$ model such situations. Formally, then, we define a **choice problem without a status quo** as the list $(S, \Diamond)$ for any set S in Ω_X. (While the use of the symbol $\Diamond$ is clearly redundant here, it will prove quite convenient in the foregoing analysis.)

By a **choice correspondence** on $\mathcal{C}(X)$ in the present setup, we mean a map $\mathbf{c} : \mathcal{C}(X) \rightarrow$

[15] The restriction $x \in S$ in the definition of the problem (S, x) is mildly restrictive. It disallows, for instance, choice problems in which the agent must move away from her status quo point. (To wit, consider job-search of an agent who is just fired of her current job.) The only related study that (we know and) allows for x not to belong to S in the definition of a choice problem with a reference point is Rubinstein and Zhou (1997). That study however, works within an entirely different context than ours, and hence utilizes a structure that is unsuitable for a theory of choice with status quo and/or reference effects.

Ω_X such that

$$\mathbf{c}(S,\tau) \subseteq S \quad \text{for all } (S,\tau) \in \mathcal{C}(X).$$

(Notice that a choice correspondence on $\mathcal{C}(X)$ must be nonempty-valued by definition.)

3.2 Rational Choice

We now recall the basic rationality properties imposed on choice correspondences in the classical theory of revealed preference. These properties allow one to identify when a "choice" can be viewed as an outcome of a (well-behaved) utility maximization exercise. The most well-known of such properties is the famous *weak axiom of revealed preference* (also known as *Arrow's choice axiom* (Arrow (1959)). As is standard in the related literature, we present this axiom here as partitioned into two parts.

α**-Axiom.** *For any* $(S,\tau),(T,\tau) \in \mathcal{C}(X)$ *with* $T \subseteq S$,

$$y \in \mathbf{c}(S,\tau) \text{ and } y \in T \quad \textit{imply} \quad y \in \mathbf{c}(T,\tau).$$

β**-Axiom.** *For any* $(S,\tau),(T,\tau) \in \mathcal{C}(X)$ *and* $z \in T$,

$$y,z \in \mathbf{c}(S,\tau) \text{ and } y \in \mathbf{c}(T,\tau) \quad \textit{imply} \quad z \in \mathbf{c}(T,\tau).$$

Either of these properties condition the behavior of a decision maker across two choice problems with *identical* status quo structure. In this sense, they are merely reflections of the classical weak axiom of revealed preference.

The next rationality property that we consider here obtains as a simple modification of the β-Axiom.

β^***-Axiom.** *For any* $(S,x) \in \mathcal{C}_{\text{sq}}(X)$, $T \in \Omega_X$, *and* $z \in T$,

$$y,z \in \mathbf{c}(S,x) \text{ and } y \in \mathbf{c}(T,\Diamond) \quad \textit{imply} \quad z \in \mathbf{c}(T,\Diamond).$$

In words, this property says the following: If two alternatives are declared "equally good" in a given choice problem with a status quo, this must be because these alternatives are equally good in an objective sense, so they should also be deemed "equally good" in choice problems without a status quo. Needless to say, this property is a necessary condition for behavior which is couched in terms of a utility maximization exercise: If two alternatives y and z are both deemed "optimal" in a given choice problem, then the agent must be *indifferent* between y and z.

The final property we state here is a reflection of the standard continuity property for choice correspondences. It is a condition that ensures that the choices of an individual for "similar" choice problems are "similar." (Of course, it is trivially satisfied when X is finite.)

Upper Hemicontinuity (UHC). (a) *For any* $S, S_m \in \Omega_X$, $m = 1, 2, ...,$ *if* $y_m \in \mathbf{c}(S_m, \diamondsuit)$ *for each* m, $y_m \to y$ *for some* $y \in X$, *and* $S_m \to S$ (*with respect to the Hausdorff metric*), *then* $y \in \mathbf{c}(S, \diamondsuit)$.

(b) *For any* $(S, x) \in \mathcal{C}_{\text{sq}}(X)$, *if* (y_m) *is a sequence in* $X \backslash \{x\}$ *such that* $y_m \in \mathbf{c}(S \cup \{y_m\}, x)$ *for each* m, *and* $y_m \to y$ *for some* $y \in X$, *then* $y \in \mathbf{c}(S \cup \{y\}, x)$.

All four of the properties considered above are "normative" postulates, anyone of which has a counterpart in the classical theory of rational choice. Mainly for expositional reasons, we thus call a choice correspondence that satisfy these properties as "rational" in this paper.

Definition. *A choice correspondence* $\mathbf{c}$ *on* $\mathcal{C}(X)$ *is said to be* ***rational***, *if it satisfies UHC and the* α-, β- *and* β^*-*Axioms.*

Due to the richness of the present choice domain (which we owe to the presence of default options), the class of rational choice correspondences is quite large, and in fact, include choice models that can be viewed as *boundedly rational* at best. Uncovering such choice models that deviate from the classical paradigm in order to conform with certain experimental regularities is the main objective of the present investigation.

3.3 Status Quo Bias

The primary "descriptive" axiom that we are concerned with here derives from the phenomenon that "an alternative is more likely to be chosen when it is the default option of a decision maker." The following postulate is an ordinal formulation of this notion.

Weak Status Quo Bias (WSQB). *For any* $x, y \in X$, *if* $y \in \mathbf{c}(\{x, y\}, x)$ *or* $y \in \mathbf{c}(\{x, y\}, \diamondsuit)$, *then* $\{y\} = \mathbf{c}(\{x, y\}, y)$.

The idea is that if the decision maker reveals the superiority of y over x even when x is the status quo, then, when y is itself the default option of the agent, its position can only be stronger relative to x. This postulate, which is referred to as "conservatism" by Munro and Sugden (2003), posits that in this case y must be the *only* choice from the alternative set $\{x, y\}$, thereby preconditioning a choice correspondence to exhibit a bias towards the status quo alternatives.[16]

WSQB property seems almost unexceptionable for a rational choice theory which aims at modeling the status quo bias phenomenon. Indeed, versions of this property is recently adopted in a few other studies (cf. Sugden (2003), Munro and Sugden (2003), Apesteguia and Ballester (2004), Masatlioglu and Ok (2005) and Sagi (2006)).[17] Furthermore, the experimental literature on individual choice provides direct verifications of WSQB.[18]

3.4 Referential Nature of Status Quo Alternatives

Needless to say, creating a status quo bias is not the only way the presence of a default option may affect the choice behavior of an economic agent. Such an option may act also as a "reference" relative to which the desirability of the feasible alternatives are assessed. We now turn to discuss how to discern this effect from the choice behavior of a decision maker. We need to introduce two auxiliary concepts for this purpose.

Definition. *Let* $\mathbf{c}$ *be a choice correspondence on* $\mathcal{C}(X)$*. For any* $x, y \in X$*, we say that* $\mathbf{c}$ ***renders*** y ***superior to*** x*, or simply* y *is* $\mathbf{c}$***-superior to*** x*, if there exists a choice problem* $(T, x) \in \mathcal{C}_{\text{sq}}(X)$ *such that* $\{y\} = \mathbf{c}(T, x)$*. A subset* S *of* X *with* $x \in S$ *is said to be* $\mathbf{c}$***-superior to*** x*, if* y *is* $\mathbf{c}$*-superior to* x *for every* $y \in S$*.*

[16] The following stronger property can be considered at this junction: *For any* $(S, \tau) \in \mathcal{C}(X)$*, if* $y \in \mathbf{c}(S, \tau)$*, then* $\{y\} = \mathbf{c}(S, y)$. (Masatlioglu and Ok (2005) refer to this property as "Status Quo Bias.") To the best of our knowledge, there is no direct test of this stronger property in the literature. Moreover, there is a variety of reasons for viewing this axiom as too demanding. (More on this shortly.)

[17] Sugden calls his version of SQB the "strict exchange aversion," and Sagi the "no-regret" condition.

[18] A classic experiment in this regard is that of Knetsch (1989). In this experiment, subjects were partitioned (at random) into two groups, with each member of one group being endowed with a coffee mug and each member of the other with a chocolate bar. A few minutes later, when each subject were presented with the opportunity of exchanging her endowment with the other one, a pronounced preference for default options was observed.

In words, the $\mathbf{c}$-superiority of an alternative y over x means simply that if $\mathbf{c}$ reveals y to be superior to x (even) when x is designated as the default option for the agent.

The following definition is key to the present investigation.

Definition. *Let* $\mathbf{c}$ *be a choice correspondence on* $\mathcal{C}(X)$. *For any* $x, y \in X$, *we say that* $\mathbf{c}$ ***renders*** y ***essential in reference to*** x, *or simply* y *is* $\mathbf{c}$***-essential in reference to*** x, *if there exists an alternative* $z \in X$ *such that* z *is* $\mathbf{c}$*-superior to* x, *and for some* $S, T \in \Omega_X$ *with* $y, z \in T \cap S$,

$$z \in \mathbf{c}(S, \Diamond) \quad \textit{while} \quad \{y\} = \mathbf{c}(T, x).$$

If y *is not* $\mathbf{c}$*-essential in reference to* x, *we say that it is* $\mathbf{c}$***-inessential in reference to*** x.

If y is $\mathbf{c}$-essential in reference to x, then we understand that, when the agent evaluates the feasible alternatives by using x as a reference point, the appeal of the alternative y seems accentuated. Indeed, in this case, the agent (whose choice behavior is modeled by $\mathbf{c}$) would choose y over an alternative z, even though (i) z is $\mathbf{c}$-superior to the reference point x; and (ii) the agent is known to like z at least as much as y in an objective sense (i.e. in the absence of a status quo). In this sense, the referential nature of the status quo point x leads the agent to favor y more than she would have in the absence of a status quo.

The notions of $\mathbf{c}$-superiority and $\mathbf{c}$-essentialness are defined through a choice correspondence, and hence, they are, at least in principle, observable concepts. We now use these notions to introduce the final two properties for a choice correspondence. This will complete our axiomatic model.

First, we introduce a property that identifies those situations in which $\mathbf{c}$ disregards the presence of a status quo alternative.

Status-quo Independence (SQI). *For any* $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ *such that* S *is* $\mathbf{c}$*-superior to* x, *if every* $y \in S \backslash \{x\}$ *is* $\mathbf{c}$*-inessential in reference to* x, *then* $\mathbf{c}(S, x) = \mathbf{c}(S, \Diamond)$.

Consider a feasible set S of alternatives and a status quo point $x \in S$ such that x is not only "undesirable" in S (in the sense that all alternatives in S are deemed superior to x, even when x acts as the default option), but it also does not distinguish between alternatives in S by acting as a reference point (in the sense that it deems none of the alternatives in $S \backslash \{x\}$ to be $\mathbf{c}$-essential). It seems in this case that the "best" choice in S is independent of whether or not x is given to the agent as a default option. Because x is an undesirable alternative that does not play any sort of a referential role in the choice problem, it would be entirely reasonable – from the normative as well as a descriptive angle – to expect the agent to view the presence of x as a status quo as irrelevant for her final choice, and thus settle her problem by comparing the alternatives in S objectively (that is, as if there is no status quo in the problem). This is the gist of SQI.[19]

Dominance (D). *For any* $S \in \Omega_X$ *and* $(T, x) \in \mathcal{C}_{\rm sq}(X)$,

$$y \in \mathbf{c}(S, \Diamond) \quad \textit{and} \quad \{y\} = \mathbf{c}(T, x) \quad \textit{implies} \quad y \in \mathbf{c}(S, x),$$

provided that $T \backslash \{x\}$ *contains at least one alternative that is* **c***-essential in reference to* x.

To see the crux of this postulate, consider a feasible set S of alternatives, and suppose that y is revealed to be a "best" alternative in S, absent any status quo considerations. Now consider the alternative scenario in which the agent faces the choice problem S with some $x \in S$ being her default option. Under which circumstances would it be reasonable to expect y to be deemed "choosable" by the agent in this situation as well?

There are at least two reasons for y not to be chosen from the problem (S, x) even though it is chosen from the problem $(S, \Diamond)$. First of all, due to "status quo bias" x may look more desirable to the agent in the problem (S, x), and hence she may opt for x in that problem instead of y. But what if $\{y\} = \mathbf{c}(T, x)$ for some choice problem (T, x)? Then, apparently, this is not the case —after all, this observation tells us that y is, in fact, **c**-superior to x— the agent thinks y is a better choice than x even when x is her dafault option. Still, this may not be enough to view y "choosable" in the problem (S, x). For, perhaps, utilizing x as a reference point takes away from the relative desirability of y in the mind of the agent, leading to the choice of some other (presumably **c**-essential) alternative in the problem (S, x). Now if we also knew that this is not the case, that is, if y is deemed "choosable" even in the presence of a **c**-essential alternative in reference to x (when, of course, x is the status quo) – as it would be the case if $T \backslash \{x\}$ contained some **c**-essential alternatives in reference to x – then this objection dissipates as well. In this case, speaking both descriptively and

[19] To wit, consider the following alternatives: $z :=$ "watching a new episode of '24' tonight," $y :=$ "watching a new episode of 'Lost' tonight," and $x :=$ "getting killed tonight by painful torture." It is hardly disputable that, in this case, it is indeed irrelevant that x is given as a default option to the agent – we are bound to declare y and z **c**-inessential in reference to x, and have $\mathbf{c}(\{x, y, z\}, x) = \mathbf{c}(\{x, y, z\}, \Diamond)$, as posited by SQI.

normatively, it seems quite reasonable to expect y to be a "choice" in the problem (S,x) – this is precisely what the property D envisages.

4 Procedural Choice with Status Quo Bias

4.1 The Main Result

We need to introduce a final bit of terminology before we state the main result of this paper.

Definition. *Let A be any set. By a **self-correspondence** on A, we mean a map Γ from A into $2^A\backslash\{\emptyset\}$. We say that Γ is **injective** if $a \neq b$ implies $\Gamma(a) \neq \Gamma(b)$.*

Notation. *The lower contour set of an alternative $x \in X$ with respect to the (utility) function $U : X \to \mathbb{R}$ is denoted as $\mathcal{L}_U(x)$, that is,*

$$\mathcal{L}_U(x) := \{y \in X : U(x) \geq U(y)\}.$$

The following theorem provides a complete characterization of those rational choice correspondences that satisfy the three "descriptive/normative" properties considered above. We find that any such choice correspondence can, in fact, be rationalized by means of a simple choice "procedure."

Theorem A. *Let $\mathbf{c} : \mathcal{C}(X) \to 2^X$ be any map. Then, $\mathbf{c}$ is a rational choice correspondence on $\mathcal{C}(X)$ that satisfies WSQB, SQI and D if, and only if, there exist a continuous map $U : X \to \mathbb{R}$ and two closed-valued injective self-correspondences $\mathcal{Q}^1$ and $\mathcal{Q}^2$ on X such that, for every $x \in X$,*

$$\mathcal{L}_U(x) \cap \mathcal{Q}^1(x) = \{x\} = \mathcal{L}_U(x) \cap \mathcal{Q}^2(x) \ \text{ and } \ \mathcal{Q}^1(x) \subseteq \mathcal{Q}^2(x), \tag{2}$$

while, for every $S \in \Omega_X$,

$$\mathbf{c}(S, \Diamond) = \underset{\omega \in S}{\arg\max}\, U(\omega), \tag{3}$$

and, for every $(S,x) \in \mathcal{C}_{\mathrm{sq}}(X)$,

$$\mathbf{c}(S,x) = \begin{cases} \underset{\omega \in S \cap \mathcal{Q}^1(x)}{\arg\max}\, U(\omega), & \text{if } S \cap \mathcal{Q}^1(x) \neq \{x\} \\ \\ \underset{\omega \in S \cap \mathcal{Q}^2(x)}{\arg\max}\, U(\omega), & \text{otherwise.} \end{cases} \tag{4}$$

To understand the nature of the choice model obtained through Theorem A, consider a decision maker whose choice correspondence **c** is rational and satisfies WSQB, SQI and D. When dealing with a choice problem *without* a status quo, this agent makes her choices upon maximizing a utility function U — this is, of course, in complete accordance with the weak axiom of revealed preference.

In turn, this agent deals with any given choice problem with a status quo point, say with (S, x), by means of a simple two-stage procedure. In the first stage, she uses a mental "constraint set" $\mathcal{Q}^1(x)$ and eliminates all feasible alternatives that does not belong to this constraint set. If, other than the default option x, exactly one feasible alternative passes this test, then that alternative is the unique choice of the agent in the problem. If more than one alternative in $S\backslash\{x\}$ pass this test, then the agent chooses among these alternatives the ones that yield the highest satisfaction in terms of her (objective) utility function U. If, on the other hand, every alternative in S, other than x, fails this test, that is, $S \cap \mathcal{Q}^1(x) = \{x\}$, then the agent moves to the second stage of her procedure.

At this stage, the agent reasons exactly as she did in the first stage, except now, perhaps realizing that she made an "overuse" of her default option x as a reference point, she relaxes her mental constraint set to a bigger set $\mathcal{Q}^2(x)$. The decision maker settles her problem, then, upon searching for alternatives in $S \cap \mathcal{Q}^2(x)$. If this search yields exactly one alternative in $S\backslash\{x\}$, then that's her choice. If more than one alternative pass this test, then the ones with the highest U value constitute the set of all choices of the agent. If no feasible alternative other than x passes this second test either, that is, $S \cap \mathcal{Q}^2(x) = \{x\}$, then the agent chooses not to move from her status quo point x.[20]

A few remarks on the interpretation of this choice model are in order. There are indeed multiple ways of thinking about the set $\mathcal{Q}^1(x)$. Descriptively speaking, one may think of this set as arising from a psychological phenomenon. Simply put, the presence of the status quo point x, acting as a reference, leads the agent to concentrate *only* on those alternatives that are better than x in some unambiguous sense for her. Normatively speaking, on the other

[20] There is a natural generalization of this choice procedure. Indeed, one can think of the agent as relaxing her mental constraint set further at this point to some $\mathcal{Q}^3(x)$ (with $\mathcal{L}_U(x) \cap \mathcal{Q}^3(x) = \{x\}$) and then evaluating the alternatives in $S \cap \mathcal{Q}^3(x)$, instead of simply opting for the default option x. If this does not work, that is $S \cap \mathcal{Q}^3(x) = \{x\}$, she might consider relaxing her constraint set further, and so on. This sort of a choice model is, of course, not without interest. Nonetheless, its axiomatic basis, and hence the determination of a complete (behavioral) test for it, seems like a notoriously difficult task.

hand, one may think of the elimination of those alternatives in $S \backslash \mathcal{Q}^1(x)$ (distinct from x) from consideration as a "simplification" the agent uses to settle her possibly complex choice problem. Either of these interpretations sits well with the "boundedly rational" choice model envisaged by Theorem A.

The interpretation of the set $\mathcal{Q}^2(x)$ follows a similar line of reasoning. From a descriptive angle, one can think of the agent as allowing for more trade-offs in her evaluation, and hence relaxing somewhat her reference-dependence, upon observing that using $\mathcal{Q}^1(x)$ to eliminate alternatives from selection contention leaves no room for choice (other than the default option). This is perhaps because she realizes that an alternative y, while not unambiguously better than x for her, may still have a much higher utility value than x, and hence it may be a "mistake" to eliminate y from consideration. Similarly, from a normative angle, the fact that $S \cap \mathcal{Q}^1(x) = \{x\}$ may make the agent realize that using $\mathcal{Q}^1(x)$ to eliminate alternatives may have "oversimplified" the problem. This realization leads her to adopt a less restrictive rule of elimination, focusing this time on the alternatives $S \cap \mathcal{Q}^2(x)$.

4.2 Rationality, Status Quo Bias and Reference Dependence

The model given by Theorem A carries elements of rationality as well as the phenomena of status quo bias and reference dependence. First, observe that an agent whose choice behavior abides by this model is indistinguishable from a standard utility maximizer in the context of choice problems without a status quo. Moreover, even in a choice problem with a given initial entitlement x, among the alternatives that pass the tests imposed by the presence of x, the final choice is determined upon maximizing the reference-independent utility function. Thus, while the standard choice model is a "*utility maximization model*," the choice model we derived here is a "*constrained utility maximization model*," where the constraint is induced by one's default option.

Second, the present model exhibits the phenomenon of status quo bias — it is, after all, built on the property SQB. We see this from the representation obtained in Theorem A due to the fact that $\mathcal{L}_U(x) \cap \mathcal{Q}^i(x) = \{x\}$, $i = 1, 2$. This condition ensures that if y is chosen from the problem (S, x), then we must have $U(y) > U(x)$ — this is only to be expected — but even if all alternatives in $S \backslash \{x\}$ has a strictly higher utility than x, one may still have $\{x\} = \mathbf{c}(S, x)$. In particular, if $U(y) > U(x)$, but $y \notin \mathcal{Q}^2(x)$, then $\{x\} = \mathbf{c}(\{x, y\}, x)$.

Having said this, we should note that the status quo bias allowed in the model is somewhat limited. In particular, if $y \in \mathbf{c}(S, x)$, then we do not have to have $y \in \mathbf{c}(S, y)$ as well. (Notice that SQB requires this only for the case $S = \{x, y\}$.) The following example illustrates.

Example 2. Let $X := \{x, y, z\}$, and consider a choice correspondence $\mathbf{c}$ of the form given by Theorem A, where $z \in \mathcal{Q}^2(x) \backslash \mathcal{Q}^1(x)$, $y \in \mathcal{Q}^1(x)$, and $z \in \mathcal{Q}^2(y)$. (See Figure 3.) In this case, we have $\{y\} = \mathbf{c}(\{x, y, z\}, x)$ whereas $\{z\} = \mathbf{c}(\{x, y, z\}, y)$.

FIGURE 3 ABOUT HERE

It follows that the present model allows for a curious *menu-dependence*. Consider, for instance, an individual who views the insurance policies z and y better than her current policy x. Suppose, because y is clearly superior to x in all relevant dimensions — while z is not so, even though it yields better utility than x in the aggregate — this agent chooses to buy policy y. It is quite conceivable that, when asked again, this agent then opts for z. After all, she has in fact chosen y due to x being the reference point, and now, y is instead the reference point, and this may well overturn her earlier decision.[21]

Finally, we note that the choice model of Theorem A uses a status quo point not only as an alternative whose "value" is somewhat accentuated for the agent, but also, it allows for it to be used as a *reference*. The next example illustrates.

Example 1. [Continued] We revisit the issue pointed out in Example 1. Let $X := \{x, x', y, z\}$, and consider a choice correspondence $\mathbf{c}$ of the form given by Theorem A, where $\mathbf{c}(\{y, z\}, \Diamond) = \{y\} = \mathbf{c}(\{x, y, z\}, x)$. Can this agent choose z instead of y in the problem $(\{x', y, z\}, x')$, where x' is an unambiguously worse alternative than x. In a model where one's initial endowment lacks referential status, the answer would be no, as worsening of a status quo alternative would then not alter the decisions of the agent. On the other hand, as shown in Figure 4, the present model duly allows for $\{z\} = \mathbf{c}(\{x', y, z\}, x')$, signifying how it incorporates the notion of "reference-dependent" decision making.[22]

[21] Needless to say, no "cycles" may occur through this reasoning, however. z remains to be the unique choice in this example (because $z \in \mathcal{Q}^2(x) \cap \mathcal{Q}^2(y)$ implies $U(z) > \max\{U(x), U(y)\}$).

[22] Incidentally, the considerations outlined above show in exactly what way the present work improves upon the earlier related work of Masatlioglu and Ok (2005). While this is not entirely obvious, one can show that the model obtained in the latter paper is a special case of that of Theorem A in which $\mathcal{Q}^1 = \mathcal{Q}^2$. It is for this reason that the latter model gives no "reference status" to the initial entitlement of an agent, and thus, it has a substantially less explanatory power than the present model (Section 2.2).

FIGURE 4 ABOUT HERE

5 Multicriteria Decision Making with Status Quo Bias

The choice model derived in Theorem A is a bit too general in that it does not put any restrictions about how the mental constraints captured by the correspondences $\mathcal{Q}^1$ and $\mathcal{Q}^2$ change across alternatives. For instance, consider a choice correspondence $\mathbf{c}$ that is represented as in Theorem A, and suppose $y \in \mathcal{Q}^2(x)$ and $z \in \mathcal{Q}^2(y)$. Thus, a decision maker with this choice correspondence deems y better than x even when x is her default option, and z better than y even when y is her default choice. So, informally speaking, there is a sense in which y is "much better than" x and z is "much better than" y for this agent. It is thus only reasonable to expect z to be deemed "much better than" x, that is, to posit that $z \in \mathcal{Q}^2(x)$. The model at hand, however, does not guarantee this to be the case.

As similar remarks apply also to the correspondence $\mathcal{Q}^1$, it is natural to inquire in what way we can ensure that $\mathcal{Q}^1$ and $\mathcal{Q}^2$ in Theorem A behave *transitively.* The following property expresses this behavioral requirement in terms of the primitive choice correspondence $\mathbf{c}$.

Transitivity of the Reference Effect (TRE). *Given any* $(S, x), (T, y) \in \mathcal{C}_{\text{sq}}(X)$, *let* $\{y\} = \mathbf{c}(S, x)$ *and* $\{z\} = \mathbf{c}(T, y)$. *Then* $z \in \mathbf{c}(\{x, z\}, x)$. *Moreover, if* $S \backslash \{x\}$ *contains a* $\mathbf{c}$*-essential alternative in reference to* x, *and* $T \backslash \{y\}$ *contains a* $\mathbf{c}$*-essential alternative in reference to* y, *then* $z \in \mathbf{c}(S \cup \{z\}, x)$.

Adding this "'transitive evaluation" requirement to our basic axiomatic system warrants the properties $\mathcal{Q}^1 \circ \mathcal{Q}^1 \subseteq \mathcal{Q}^1$ and $\mathcal{Q}^2 \circ \mathcal{Q}^2 \subseteq \mathcal{Q}^2$. In turn, perhaps somewhat unexpectedly, this allows one to view the resulting model as one of multicriteria decision making, at least when the grand alternative space X is finite.

To formalize this latter point, we need to introduce the following notation.

Notation. *For any positive integer* d *and vector-valued function* $\mathbf{w} : X \to \mathbb{R}^d$, *the strict upper contour set of an alternative* $x \in X$ *with respect to* $\mathbf{w}$ *is denoted as* $\mathcal{U}_{\mathbf{w}}(x)$, *that is,*

$$\mathcal{U}_{\mathbf{w}}(x) := \{y \in X : \mathbf{w}(y) > \mathbf{w}(x)\}.$$

The next result describes how Theorem A modifies in the presence of the transitive evaluation postulate TRE, when X is finite.

Theorem B. *Let X be a nonempty finite set, and take any $\mathbf{c} : \mathcal{C}(X) \to 2^X$. Then, $\mathbf{c}$ is a choice correspondence on $\mathcal{C}(X)$ that satisfies WSQB, D, SQI, and TRE if, and only if, there exist positive integers m and n, two injections $\mathbf{u} : X \to \mathbb{R}^m$ and $\mathbf{v} : X \to \mathbb{R}^n$, and a map $f : \mathbb{R}^{m+n} \to \mathbb{R}$ such that*

$$\mathbf{c}(S, \diamondsuit) = \arg\max_{\omega \in S} U(\omega) \quad \textit{for all } S \in \Omega_X,$$

where $U : X \to \mathbb{R}$ is defined by $U(\omega) := f(\mathbf{u}(\omega), \mathbf{v}(\omega))$, and, for every $(S, x) \in \mathcal{C}_{\text{sq}}(X)$,

$$\mathbf{c}(S,x) = \begin{cases} \arg\max\limits_{\omega \in S \cap \mathcal{U}_{(\mathbf{u},\mathbf{v})}(x)} U(\omega), & \textit{if } S \cap \mathcal{U}_{(\mathbf{u},\mathbf{v})}(x) \neq \emptyset \\ \arg\max\limits_{\omega \in S \cap \mathcal{U}_{\mathbf{u}}(x)} U(\omega), & \textit{if } S \cap \mathcal{U}_{(\mathbf{u},\mathbf{v})}(x) = \emptyset \textit{ and } S \cap \mathcal{U}_{\mathbf{u}}(x) \neq \emptyset \\ \{x\}, & \textit{otherwise.} \end{cases}$$

One can thus view the present choice model as one of "*constrained multiobjective maximization*," at least when X is a finite set.[23] Let us demonstrate the "as if" interpretation of this model by means of a concrete example. Consider a professor at an economics department – let us call her Prof. σ – who needs to evaluate the files of applicants for the research assistantship position that she is able to offer for the coming year. Prof. σ has identified, say, five criteria as relevant for her evaluation (e.g. the GPA, overall computer skills, strength of recommendation letters, interest in the project, and previous research experience). She aggregates these criteria in some manner, and identifies those candidates whose aggregate score is the largest. (Thus, in the absence of a status quo choice, Prof. σ solves her problem as a utility maximizer, in full conformity with the model exhibited in Theorem B.)

For purposes of this illustration, suppose that the maximization exercise of Prof. σ results in a single choice, say, the candidate γ. Yet, just when she gets ready to call this candidate, a new pile of files finds its way to her desk. As a consequence, she gets on solving her new choice problem, but this time in the presence of a default option, namely, the candidate γ.

[23] For the reason why one needs the finiteness assumption here, see Masatlioglu and Ok (2005), Remark 2.

If her choice behavior abides with that of Theorem B, then, Prof. σ would get on comparing every new file with that of the candidate γ with respect to all five of her choice criteria. Suppose she finds none of the new files superior to that of γ in all dimensions. Then, she moves to the second stage of her decision rule, perhaps realizing that her status quo bias may have resulted in the elimination of certain candidates who perform great in all but one criterion (with respect to which γ's file looks better). At this stage, she uses only a subset of her original criteria (e.g. disregards the strength of recommendation letters, and interest in the project), and examines again if any of the new files dominates that of γ with respect to the remaining three rationales. If the answer is yes, then she will identify all the files that dominate the applicant γ with respect to the three criteria and choose the one(s) that perform best in terms of all five of her objectives according to her aggregation rule.

This example illustrates in what way we can think of the choice model of Theorem B as a procedural model of multicriteria decision making. It also shows that the basic makeup of this model is hardly superficial, it surely has elements of the actual decision procedures familiar from daily discourse. It is a *boundedly rational choice* model, to be sure. Yet, it is one that embodies a good deal of rationality. Indeed, it satisfies all of the rationality axioms considered in Section 3.2 as well as the transitive evaluation requirement TRE, and moreover, it never leads to a cyclical choice pattern.

In passing, we note that Theorem B makes it clear the exact manner in which the present work improves upon the model of Masatlioglu and Ok (2005) discussed in Section 2.2. In particular, if the behavior of Prof. σ above was modeled according to that model (with the same set of criteria), then her evaluation process would have stopped at the stage where she did not find any of the new files superior to her default choice. The said model maintains that her final choice be surely the applicant γ, thereby exhibiting a perhaps unacceptably high amount of a status quo bias.

6 Overvaluation

6.1 Overvaluation of Reference Alternatives

The presence of an "objective" utility function U in the choice model under consideration allows one to pose a number of economic questions in a formally meaningful manner. Indeed, this function tells us the reference-independent evaluation of alternatives, and hence acts as a natural yardstick for assessing the value of an alternative, say, when this alternative serves as a reference for the decision maker. In particular, we may define the **smallest valuation** for an alternative x **as a reference** – denoted by $U^{\mathbf{c}}(x)$ – as the smallest "reference-independent value" for which the agent needs to be offered to be willing to depart from x, *when* x *is designated as her endowment.* That is:

$$U^{\mathbf{c}}(x) := \inf\{U(y) : x \neq y \in \mathbf{c}(\{x, y\}, x)\}, \quad x \in X.$$

When $U^{\mathbf{c}}(x) > U(x)$, we say that x is "overvalued" when it is an endowment.

The following result clarifies the position of the choice model characterized in Theorem A with respect to the relation between the maps U and $U^{\mathbf{c}}$. It turns out that this model is consistent with both the absence and presence of overvaluation of one's endowment, and there is an easy way of detecting which is the case.

Proposition C. *Let* $\mathbf{c}$ *be a choice correspondence on* $\mathcal{C}(X)$ *for which there exists a continuous (utility) function* U *on* X*, and two closed-valued injective self-correspondences* $\mathcal{Q}^1$ *and* $\mathcal{Q}^2$ *that satisfy* (2), (3) *and* (4). *Then, for any* $x \in X$ *with* $U^{\mathbf{c}}(x) < \infty$*, we have* $U^{\mathbf{c}}(x) = U(x)$ *if, and only if,* x *is an accumulation point of* $\mathcal{Q}^2(x)$.[24]

We can also use the reference-independent utility function U to define the **largest valuation of** x **as a non-reference** – denoted $U_{\mathbf{c}}(x)$ – as the largest "value" of endowment y at which the agent would be willing to exchange y with the alternative x. That is,

$$U_{\mathbf{c}}(x) := \sup\{U(y) : y \neq x \in \mathbf{c}(\{x, y\}, y)\}, \quad x \in X.$$

Studying the relation between $U_{\mathbf{c}}(x)$ and $U(x)$ turns out to be a delicate matter. While, as in Proposition C, it is possible to give a (topological) characterization of when the equation $U_{\mathbf{c}}(x) = U(x)$ would be true, this sort of a result is not likely to be useful in applications. Fortunately, there is an easy sufficiency condition for $U_{\mathbf{c}}(x) = U(x)$ to hold, at least in the case where X is also endowed with a linear structure (as in a Euclidean space). We report this result next.

[24] For any nonempty set A in a metric space X, we say that a point $x \in A$ is an *accumulation point* of A if there is a sequence in $A \backslash \{x\}$ that converges to x.

Proposition D. *Let X be a compact and convex subset of a normed linear space. Let* $\mathbf{c}$ *be a choice correspondence on* $\mathcal{C}(X)$ *for which there exists a continuous (utility) function* U *on* X, *and two closed-valued injective self-correspondences* $\mathcal{Q}^1$ *and* $\mathcal{Q}^2$ *that satisfy* (2), (3) *and* (4). *Then, for any* $x \in X$ *with* $U_{\mathbf{c}}(x) > -\infty$, *we have* $U_{\mathbf{c}}(x) = U(x)$, *provided that* $\mathcal{Q}^2$ *is a convex correspondence.*[25]

Our main interest in Propositions C and D derive from the implications of these results about the potential gap between WTP and WTA. We shall invoke them below to clarify the standing of our choice model with regard to this matter.

6.2 Willingness to Pay (WTP) vs. Willingness to Accept (WTA)

As we have discussed in Sections 1 and 2, a major channel through which the endowment effect was discovered in experimental environments is the discrepancy found between buying and selling prices of commodities by the individuals.[26] As we discussed earlier, however, the new evidence indicates that the gap between WTP and WTA need not at all exist, especially in the case of *experienced* traders. Of course, this does not mean that an experienced trader would necessarily behave in a reference-independent manner. It just means that the nature of the reference-dependence of such an agent is devoid of the endowment effect. It follows that the descriptive power of a reference-dependent choice model would be higher, if this model is able cope with the absence of the endowment effect, and yet allows for other types of reference dependence (such as status quo bias, etc.).

Our immediate goal is to formalize the comparison of WTP and WTA in terms of choice correspondences. (It is to our surprise that no such formalization is carried out in the literature, despite the substantial amount work on this matter.) By way of a consistency check, we will then prove that the Tversky-Kahneman model necessarily entails a gap between WTP and WTA. Finally, we will show that the present choice model may or may not allow for such a gap.

Consider an environment with two commodities, say, mugs and money. To conform with the previous experimental literature, we outline the arguments in terms of a single unit of a

[25] This condition simply means that the graph of $\mathcal{Q}^2$ is a convex subset of $X \times X$.

[26] See Kahneman, Knetsch and Thaler (1991) and Camerer (1995, pp. 665-670) for detailed surveys on this matter.

mug. Consequently, we designate

$$X := I \times [0,1]$$

as the grand alternative space of this environment, where I is an interval of the form $[0, \alpha]$ with $\alpha > 0$ being a sufficiently large positive number. (Here a pair like $(a, 1) \in X$ is interpreted as the agent possessing a dollars and "the" mug, while $(b, 0) \in X$ corresponds to the bundle that contains b dollars and no mug.)

Now consider a decision maker whose initial monetary endowment is $w_o \in I$ dollars.[27] We define the **willingness to accept** of this agent (for "the" mug) as

$$\mathit{wta}(\mathbf{c}) := \inf\{a \geq 0 : (w_o + a, 0) \in \mathbf{c}(\{(w_o + a, 0), (w_o, 1)\}, (w_o, 1))\},$$

where $\mathbf{c}$ corresponds her choice correspondence on $\mathcal{C}(X)$. That is, $\mathit{wta}(\mathbf{c})$ is the smallest amount of money that this agent, *if endowed with "the" mug*, would charge to sell the mug. The formulation of WTP is less straightforward. We follow in our demonstration the basic contention of Tversky and Kahneman (1991) and postulate that the act of giving up money to buy goods is viewed by the agent not as a loss, but as a foregone gain of money. This leads one to define the **willingness to pay** of this agent (for "the" mug) as

$$\mathit{wtp}(\mathbf{c}) := \sup\{w_o \geq a \geq 0 : (w_o, 1) \in \mathbf{c}(\{(w_o, 1), (w_o + a, 0)\}, (w_o + a, 0))\}.$$

That is, $\mathit{wtp}(\mathbf{c})$ is the largest amount of money increment the agent would be willing to give up to be able obtain "the" mug. To simplify the discussion, we assume that $w_o > \mathit{wtp}(\mathbf{c})$, that is, the agent is not willing to give up all of her monetary wealth just to be able to receive "the" mug.

Let us first consider how $\mathit{wta}(\mathbf{c})$ and $\mathit{wtp}(\mathbf{c})$ would compare if we modeled $\mathbf{c}$ according to the Tversky-Kahneman (loss aversion) model reviewed in Section 2.1. We adopt the notation introduced in that section, but here we label u_1 and u_2 as u and v, respectively. Clearly, the Tversky-Kahneman model maintains that

$$\mathit{wta}(\mathbf{c}) = \inf\{a \geq 0 : u(w_o + a - w_o) + v(0 - 1) \geq 0\}$$

(since $u(0) = 0 = v(0)$), and therefore, since u is continuous and strictly increasing, it follows that $u(\mathit{wta}(\mathbf{c})) = -v(-1)$. Similarly,

$$\mathit{wtp}(\mathbf{c}) = \sup\{w_o \geq a \geq 0 : u(w_o - (w_o + a)) + v(1 - 0) \geq 0\},$$

[27] Within the formalism of the present illustration, α can be any real number greater than or equal to $2w_o$.

and it follows that $u(-wtp(\mathbf{c})) = -v(1)$. Consequently,

$$u(wta(\mathbf{c})) = -v(-1) > v(1) = -u(-wtp(\mathbf{c})) > u(wtp(\mathbf{c})),$$

where the strict inequalities follow from the hypothesis of loss aversion. Since u is strictly increasing, we conclude that

$$wta(\mathbf{c}) > wtp(\mathbf{c})$$

must hold if $\mathbf{c}$ is induced by the Tversky-Kahneman model. It follows that the recent empirical evidence that shows that, at least for experienced traders, there is no significant gap between WTP and WTA, *refutes* this model.

Let us now consider the case where $\mathbf{c}$ is instead modeled as in Theorem A, where $U : X \to \mathbb{R}$ is assumed to be strictly increasing in both components. In that case, we readily find that

$$\begin{aligned} U^{\mathbf{c}}(w_o, 1) &= \inf\{U(w_o + a, 0) : a \geq 0 \text{ and } (w_o + a, 0) \in \mathbf{c}(\{(w_o + a, 0), (w_o, 1)\}, (w_o, 1))\} \\ &= U(w_o + wta(\mathbf{c}), 0) \end{aligned}$$

by definition of $U^{\mathbf{c}}$ and $wta(\mathbf{c})$. But Proposition C ensures us that $U(w_o, 1) = U^{\mathbf{c}}(w_o, 1)$, and hence $U(w_o, 1) = U(w_o + wta(\mathbf{c}), 0)$, holds iff $(w_o, 1)$ is an accumulation point of the mental constraint set $\mathcal{Q}^2(w_o, 1)$. Similarly, using Proposition D, we find $U(w_o, 1) = U_{\mathbf{c}}(w_o, 1) = U(w_o + wtp(\mathbf{c}), 0)$, provided that $\mathcal{Q}^2$ is a convex correspondence. It follows that, not only is that $\mathbf{c}$ is consistent with the absence of a gap between WTP and WTA, but we have an easy way of telling when this gap does not exist:

Fact. *If $X := I \times [0, 1]$ and $\mathbf{c}$ is as derived in Theorem A (with strictly increasing U and convex $\mathcal{Q}^2$), then*

$$wta(\mathbf{c}) = wtp(\mathbf{c})$$

if and only if $(w_o, 1)$ is an accumulation point of $\mathcal{Q}^2(w_o, 1)$.

Therefore, depending on the specifications of its basic ingredients, the present model is suitable for modeling the behavior of both experienced and inexperienced traders. It is a boundedly rational choice model that allows for status quo bias and reference-dependent choice, yet it does not necessitate the presence of the endowment effect.[28]

[28] It may be worth noting that the latter property does not depend on not modeling payments as "losses."

7 Conclusion

In this paper, we adopted the revealed preference method to derive a boundedly rational individual decision making model that allows for an agent not only to exhibit status quo bias, but also to make use of her default option as a reference point. The resulting model contains elements of the classical rational choice model, but can also be viewed as arising from a three-step choice procedure that involves multiple objectives. Moreover, while it permits status quo bias, it does not necessarily lead to the overvaluation of one's endowment, and hence it is duly consistent with the *absence* of a gap between one's willingness to sell and buy a given choice alternative.

Insofar as we can (objectively) assess, the model proposed here seems promising from an initial theoretical standpoint. Yet, obviously, its usefulness needs to be determined by means of its direct experimental testing and its predictive and explanatory performance in economic applications.

8 Proofs

8.1 Proof of Theorem A

Let $\mathbf{c}$ be a rational choice correspondence on $\mathcal{C}(X)$ that satisfies WSQB, D and SQI. Define the binary relation $\succcurlyeq$ on X by

$$y \succcurlyeq x \quad \text{if and only if} \quad y \in \mathbf{c}(\{x, y\}, \lozenge).$$

The same conclusion holds if instead one chooses to model the act of giving up money in exchange of goods as a loss as well (cf. Bateman, et al (1997, 2005)). To say this precisely, let s_o and b_o stand for the initial wealth of the seller and buyer, respectively, and to achieve comparability of initial standings, suppose that $U(s_o, 1) = U(b_o, 0)$. We define the minimum selling price of the mug as

$$S(\mathbf{c}) := \inf\{a \geq 0 : (s_o + a, 0) \in \mathbf{c}(\{(s_o + a, 0), (s_o, 1)\}, (s_o, 1))\},$$

and the maximum buying price as

$$B(\mathbf{c}) := \sup\{b_o \geq a \geq 0 : (b_o - a, 1) \in \mathbf{c}(\{(b_o - a, 1), (b_o, 0)\}, (b_o, 0))\}.$$

Under the conditions of Fact above, then, $S(\mathbf{c}) = S(\mathbf{c})$ iff $(s_o, 1)$ is an accumulation point of $\mathcal{Q}^2(s_o, 1)$.

Standard arguments, based on the α- and β-Axioms and part (a) of UHC, verify that $\succcurlyeq$ is a continuous and complete preorder on X. By the Debreu Representation Theorem, therefore, there exists a continuous map $U : X \to \mathbb{R}$ such that $y \succcurlyeq x$ iff $U(y) \geq U(x)$, for any $x, y \in X$. Applying the α- and β-Axioms, then, we find

$$\mathbf{c}(S, \Diamond) = \arg\max\{U(x) : x \in S\} \quad \text{for any } S \in \Omega_X. \tag{5}$$

Next we define the self-correspondence $\mathcal{Q}^2$ on X by

$$\mathcal{Q}^2(x) := \{y \in X : y \in \mathbf{c}(\{x, y\}, x)\},$$

and note that, by WSQB, $y \in \mathcal{Q}^2(x)$ implies $\{y\} = \mathbf{c}(\{x, y\}, x)$ so that, in view of the α-Axiom,

$$\mathcal{Q}^2(x) = \{y \in X : y \text{ is } \mathbf{c}\text{-superior to } x\}.$$

Evidently, $x \in \mathcal{Q}^2(x)$ for each $x \in X$, and hence $\mathcal{Q}^2$ is well-defined.

Claim 1. $\mathcal{Q}^2$ is injective and closed-valued. Moreover, for any $x \in X$, if $y \in \mathcal{Q}^2(x)$ and $U(y) \leq U(x)$, then $y = x$.

Proof of Claim 1. To prove the injectivity assertion, note that if $x \in \mathcal{Q}^2(y)$, then $\{x\} = c(\{x, y\}, x)$ by WSQB, while $y \in \mathcal{Q}^2(x)$ means $y \in c(\{x, y\}, x)$, which is possible in this case only if $x = y$. It follows that $\mathcal{Q}^2(x) = \mathcal{Q}^2(y)$ implies $x = y$.

Now pick any $x \in X$, and take any sequence (y_m) in $\mathcal{Q}^2(x)$ with $y_m \to y$ for some $y \in X$. If $y_m = x$ for infinitely many m, then $y = x \in \mathcal{Q}^2(x)$. If $y_m = x$ for only finitely many m, then it is without loss of generality to assume that $y_m \neq x$ for each m. In that case, applying part (b) of UHC readily yields $y \in \mathbf{c}(\{x, y\}, x)$, that is, $y \in \mathcal{Q}^2(x)$. Since x was arbitrarily chosen in X, this proves that $\mathcal{Q}^2$ is closed-valued.

Finally, pick any $x, y \in X$ such that $U(y) \leq U(x)$. By (5), we have $x \in \mathbf{c}(\{x, y\}, \Diamond)$, so WSQB implies $\{x\} = \mathbf{c}(\{x, y\}, x)$. But, unless $y = x$, this means that $y \notin \mathcal{Q}^2(x)$. The proof of Claim 1 is now complete. □

Claim 2. For any $(S, x) \in \mathcal{C}_{\text{sq}}(X)$,

$$\mathbf{c}(S, x) \subseteq \mathbf{c}(S \cap \mathcal{Q}^2(x), x).$$

Proof of Claim 2. Take any $(S, x) \in \mathcal{C}_{\text{sq}}(X)$, and let $y \in \mathbf{c}(S, x)$. By the α-Axiom, $y \in \mathbf{c}(\{x, y\}, x)$, that is, $y \in S \cap \mathcal{Q}^2(x)$. Applying the α-Axiom again, therefore, we find $y \in \mathbf{c}(S \cap \mathcal{Q}^2(x), x)$. □

The converse containment in Claim 2 can be established only under a certain condition. To specify this condition we need to introduce two further notions. First we define the self-correspondence $\mathcal{E}$ on X by

$$\mathcal{E}(x) := \{y \in \mathcal{Q}^2(x) : y \text{ is } \mathbf{c}\text{-essential in reference to } x\},$$

and second, the self-correspondence $\mathcal{Q}^1$ on X by

$$\mathcal{Q}^1(x) := \mathcal{E}(x) \cup \{\omega \in \mathcal{Q}^2(x) : \{\omega\} = \mathbf{c}(\{x, w, \omega\}, x) \text{ for some } w \in \mathcal{E}(x)\backslash\{x\}\}.$$

Since any x is $\mathbf{c}$-essential in reference to itself, we have $x \in \mathcal{E}(x)$ for each $x \in X$ – both $\mathcal{E}$ and $\mathcal{Q}^1$ are thus well-defined. Moreover, it is obvious that $\mathcal{Q}^1(x) \subseteq \mathcal{Q}^2(x)$ for all $x \in X$. Using this fact, one can show that $\mathcal{Q}^1$ is injective and $\mathcal{L}_U(x) \cap \mathcal{Q}^1(x) = \{x\}$ in exactly the same way we proved the corresponding versions of these assertions in Claim 2.

Claim 3. For any $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ with $S \cap \mathcal{Q}^1(x) = \{x\}$,

$$\mathbf{c}(S \cap \mathcal{Q}^2(x), x) = \mathbf{c}(S \cap \mathcal{Q}^2(x), \Diamond).$$

Proof of Claim 3. Take any $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ with $S \cap \mathcal{Q}^1(x) = \{x\}$. Let $T := S \cap \mathcal{Q}^2(x)$, and pick any $y \in T\backslash\{x\}$. Then, by Claim 1, $U(y) > U(x)$ so (5) implies $\{y\} = \mathbf{c}(\{x, y\}, \Diamond)$. In turn, since y is $\mathbf{c}$-inessential in reference to x, SQI implies $\mathbf{c}(\{x, y\}, \Diamond) = \mathbf{c}(\{x, y\}, x)$, so we find $\{y\} = \mathbf{c}(\{x, y\}, x)$. It follows that T is $\mathbf{c}$-superior to x. Moreover, $S \cap \mathcal{Q}^1(x) = \{x\}$ means that $S \cap \mathcal{E}(x) = \{x\}$, that is, each alternative in $S\backslash\{x\}$ is $\mathbf{c}$-inessential relative to x. Then, by SQI, we have $\mathbf{c}(T, x) = \mathbf{c}(T, \Diamond)$. $\square$

Claim 4. For any $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ with $S \cap \mathcal{Q}^1(x) = \{x\}$,

$$\mathbf{c}(S, x) = \mathbf{c}(S \cap \mathcal{Q}^2(x), \Diamond).$$

Proof of Claim 4. Take any $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ with $S \cap \mathcal{Q}^1(x) = \{x\}$. Let $y \in \mathbf{c}(S \cap \mathcal{Q}^2(x), \Diamond)$. Then $y \in \mathbf{c}(S \cap \mathcal{Q}^2(x), x)$ by Claim 3. Take any $z \in \mathbf{c}(S, x)$. Then $z \in S \cap \mathcal{Q}^2(x)$, so $z \in \mathbf{c}(S \cap \mathcal{Q}^2(x), x)$ by Claim 2. Thus, by the β-Axiom, $y \in \mathbf{c}(S, x)$. It follows that $\mathbf{c}(S, x) \supseteq \mathbf{c}(S \cap \mathcal{Q}^2(x), \Diamond)$. Combining this with Claims 2 and 3 completes the argument. $\square$

Combining Claim 4 with (5), we conclude: For any $(S, x) \in \mathcal{C}_{\text{sq}}(X)$,

$$\mathbf{c}(S, x) = \underset{y \in S \cap \mathcal{Q}^2(x)}{\arg\max}\, U(y),$$

provided that $S \cap \mathcal{Q}^1(x) = \{x\}$.

We now turn to the analysis of the choice problems (S, x) with $S \cap \mathcal{Q}^1(x) \neq \{x\}$.

Claim 5. For any $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ with $S \cap \mathcal{Q}^1(x) \neq \{x\}$,

$$\mathbf{c}(S, x) \subseteq S \cap \mathcal{Q}^1(x).$$

Proof of Claim 5. Take any $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ with $S \cap \mathcal{Q}^1(x) \neq \{x\}$, and let $y \in \mathbf{c}(S, x)$. Evidently, by the α-Axiom, so $y \in \mathcal{Q}^2(x)$. Now assume first that $S \cap \mathcal{E}(x) \neq \{x\}$. Let z be an element of $S \cap \mathcal{E}(x)$ distinct from x. By the α-Axiom, we have $y \in \mathbf{c}(\{x, y, z\}, x)$. If $\{y\} = \mathbf{c}(\{x, y, z\}, x)$, then we trivially have $y \in \mathcal{Q}^1(x)$. If, on the other hand, $\{y\} \neq \mathbf{c}(\{x, y, z\}, x)$, then WSQB implies $\{y, z\} = \mathbf{c}(\{x, y, z\}, x)$. So, in this case, by the β- and β^*-Axioms, we have $z \in \mathbf{c}(S, x)$ and $\{y, z\} = \mathbf{c}(\{y, z\}, \Diamond)$, that is, by (5),

$$y, z \in \mathbf{c}(S, x) \quad \text{and} \quad U(y) = U(z). \tag{6}$$

But since $z \in \mathcal{E}(x)$, there exists a $w \in \mathcal{Q}^2(x)$ such that

$$U(w) > U(z) \quad \text{and} \quad \{z\} = \mathbf{c}(\{x, w, z\}, x). \tag{7}$$

The second part of (7) and the α-Axiom yield $w \notin \mathbf{c}(S \cup \{w\}, x)$. Then, given that $y \in \mathbf{c}(S, x)$, applying the α- and β-Axioms again, we find $y \in \mathbf{c}(S \cup \{w\}, x)$. It follows that $y \in \mathbf{c}(\{x, w, y\}, x)$ by α- and β-Axioms. Moreover, $w \notin \mathbf{c}(\{x, w, y\}, x)$, for otherwise the α- and β-Axioms would entail $w \in \mathbf{c}(S \cup \{w\}, x)$ which we know to be false. In sum, $\{y\} = \mathbf{c}(\{x, w, y\}, x)$, while by (6) and (7), we have $U(w) > U(y)$. Thus $y \in \mathcal{E}(x) \subseteq \mathcal{Q}^1(x)$, as desired.

We now turn to the case where $S \cap \mathcal{E}(x) = \{x\}$. In this case, because $S \cap \mathcal{Q}^1(x) \neq \{x\}$, there exists a z in $S \cap (\mathcal{Q}^1(x) \backslash \mathcal{E}(x))$. (Since $x \in \mathcal{E}(x)$, z is distinct from x.) By definition of $\mathcal{Q}^1$, then, there must exist a $w \in \mathcal{E}(x) \backslash \{x\}$ such that $\{z\} = \mathbf{c}(\{x, w, z\}, x)$. Now, by the α-Axiom, w cannot be an element of $\mathbf{c}(\{x, y, w, z\}, x)$. So, since $y \in \mathbf{c}(\{x, y, z\}, x)$ by the α-Axiom, we are assured by the β-Axiom that $y \in \mathbf{c}(\{x, y, w, z\}, x)$. It follows that $y \in \mathbf{c}(\{x, y, w\}, x)$ by the α-Axiom. Moreover, since $w \notin \mathbf{c}(\{x, y, w, z\}, x)$, the β-Axiom implies $w \notin \mathbf{c}(\{x, y, w\}, x)$, so $\{y\} = \mathbf{c}(\{x, y, w\}, x)$. Since $w \in \mathcal{E}(x)$, this means that $y \in \mathcal{Q}^1(x)$, as is sought. $\square$

Claim 6. For any $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ with $S \cap \mathcal{Q}^1(x) \neq \{x\}$,

$$\mathbf{c}(S, x) = \mathbf{c}\left(\text{cl}(S \cap \mathcal{Q}^1(x)), x\right).$$

Proof of Claim 6. Take any $(S,x) \in \mathcal{C}_{\mathrm{sq}}(X)$ with $S \cap \mathcal{Q}^1(x) \neq \{x\}$. If $y \in \mathbf{c}(S,x)$, then, by Claim 5, $y \in S \cap \mathcal{Q}^1(x)$, so, obviously, $y \in \mathrm{cl}(S \cap \mathcal{Q}^1(x))$. By the α-Axiom, therefore, $y \in \mathbf{c}\left(\mathrm{cl}(S \cap \mathcal{Q}^1(x)), x\right)$. Conversely, let $y \in \mathbf{c}\left(\mathrm{cl}(S \cap \mathcal{Q}^1(x)), x\right)$. Pick any $z \in \mathbf{c}(S,x)$, and observe that $z \in \mathbf{c}\left(\mathrm{cl}(S \cap \mathcal{Q}^1(x)), x\right)$ by the $\subseteq$ part of the present assertion. So, applying the β-Axiom, we find $y \in \mathbf{c}(S,x)$, and the claim is proved. □

Claim 7. If $y \in \mathcal{Q}^1(x) \backslash \{x\}$, then $\{y\} = \mathbf{c}(T,x)$ for some $(T,x) \in \mathcal{C}_{\mathrm{sq}}(X)$ such that $T \cap \mathcal{E}(x) \neq \{x\}$.

Proof of Claim 7. Take any $y \in \mathcal{Q}^1(x)$ with $y \neq x$. Since $y \in \mathcal{Q}^2(x)$, we have $\{y\} = \mathbf{c}(\{x,y\},x)$ by WSQB. Thus, if $y \in \mathcal{E}(x)$, we are done upon setting $T := \{x,y\}$. If, on the other hand, $y \in \mathcal{Q}^1(x) \backslash \mathcal{E}(x)$, then, by definition of $\mathcal{Q}^1$, there exists a $w \in \mathcal{E}(x) \backslash \{x\}$ such that $\{y\} = \mathbf{c}(\{x,w,z\},x)$. In this case setting $T := \{x,y,w\}$ completes the proof. □

Claim 8. For any $(S,x) \in \mathcal{C}_{\mathrm{sq}}(X)$ with $S \subseteq \mathcal{Q}^1(x)$,

$$\mathbf{c}(S,x) = \mathbf{c}(S,\diamondsuit).$$

Proof of Claim 8. Take any $(S,x) \in \mathcal{C}_{\mathrm{sq}}(X)$ with $S \subseteq \mathcal{Q}^1(x)$. Note that the claim is trivial if $S = \{x\}$, so assume $S \neq \{x\}$.

Let $z \in \mathbf{c}(S,x)$. Note that if $z = x$, then WSQB and the α-Axiom imply that $S \cap \mathcal{Q}^2(x) = \{x\}$. Then, since $S \subseteq \mathcal{Q}^1(x) \subseteq \mathcal{Q}^2(x)$, we find $S = \{x\}$, a contradiction. So we have $z \neq x$, and hence by WSQB, $U(z) > U(x)$. To derive a contradiction, assume $U(y) > U(z)$ for some $y \in S \backslash \{x\}$. Then $\{y\} = \mathbf{c}(\{x,y,z\},\diamondsuit)$ by (5). Moreover, since $S \subseteq \mathcal{Q}^1(x)$, we have $y \in \mathcal{Q}^1(x) \backslash \{x\}$, so by Claim 7, $y = \mathbf{c}(T,x)$ for some $(T,x) \in \mathcal{C}_{\mathrm{sq}}(X)$ such that $T \backslash \{x\}$ contains an alternative that is $\mathbf{c}$-essential in reference to x. By D, therefore, we have $y \in \mathbf{c}(\{x,y,z\},x)$. By the α-Axiom, then, $\{y,z\} = \mathbf{c}(\{x,y,z\},x)$. But then, by the β^*-Axiom, $\{y,z\} = \mathbf{c}(\{y,z\},\diamondsuit)$, that is, $U(y) = U(z)$, a contradiction. We conclude that $U(z) \geq U(y)$ for all $y \in S$. In view of (5), this means that $z \in \mathbf{c}(S,\diamondsuit)$. That is, $\mathbf{c}(S,x) \subseteq \mathbf{c}(S,\diamondsuit)$.

To establish the converse containment, take any $y \in \mathbf{c}(S,\diamondsuit)$. If $y = x$, then the α-Axiom and WSQB imply that $\{x\} = \mathbf{c}(\{x,z\},x)$ for all $z \in S$. Since $S \subseteq \mathcal{Q}^1(x) \subseteq \mathcal{Q}^2(x)$, this is possible only if $S = \{x\}$, a contradiction. So we have $y \in \mathcal{Q}^1(x) \backslash \{x\}$. Then, by Claim 7, $y = \mathbf{c}(T,x)$ for some $(T,x) \in \mathcal{C}_{\mathrm{sq}}(X)$ such that $T \backslash \{x\}$ contains an alternative that is $\mathbf{c}$-essential in reference to x. By D, therefore, we have $y \in \mathbf{c}(S,x)$. That is, $\mathbf{c}(S,\diamondsuit) \subseteq \mathbf{c}(S,x)$.
□

The present development establishes the main assertion in the case of finite choice problems.

Claim 9. For any $(S, x) \in \mathcal{C}_{\text{sq}}(X)$ with $|S| < \infty$ and $S \cap \mathcal{Q}^1(x) \neq \{x\}$,

$$\mathbf{c}(S, x) = \arg\max\{U(y) : y \in S \cap \mathcal{Q}^1(x)\}.$$

Proof of Claim 9. Since S is a finite set, Claim 6 says that $\mathbf{c}(S, x) = \mathbf{c}\left(S \cap \mathcal{Q}^1(x), x\right)$. But $S \cap \mathcal{Q}^1(x) \subseteq \mathcal{Q}^1(x)$, so $\mathbf{c}\left(S \cap \mathcal{Q}^1(x), x\right) = \mathbf{c}\left(S \cap \mathcal{Q}^1(x), \Diamond\right)$ by Claim 8. Applying (5) completes the proof. $\square$

The final step of the argument is given next.

Claim 10. $\mathcal{Q}^1(x)$ is a closed set for any $x \in X$.

Proof of Claim 10. Let (y_m) be a sequence in $\mathcal{Q}^1(x)$ which converges to some $y \in X$. Since $\mathcal{Q}^1(x) \subseteq \mathcal{Q}^2(x)$ and $\mathcal{Q}^2(x)$ is closed, $y \in \mathcal{Q}^2(x)$. We wish to show that $y \in \mathcal{Q}^1(x)$. The argument proceeds by distinguishing between two cases.

Case 1. $y_m \in \mathcal{E}(x)$ for infinitely many m.

Passing to a subsequence, if necessary, we may assume in this case that $y_m \in \mathcal{E}(x)$ for each $m = 1, 2, \ldots$. Then, for each m, there exists a $z_m \in \mathcal{Q}^2(x)$ such that

$$U(z_m) > U(y_m) \quad \text{and} \quad \{y_m\} = \mathbf{c}(\{x, y_m, z_m\}, x). \tag{8}$$

By Claim 9, this is possible only if $z_m \notin \mathcal{Q}^1(x)$ for each m. It follows that

$$\{y_m\} = \mathbf{c}(\{x, y_m, z_k\}, x) \quad \text{for each } m, k \in \mathbb{N},$$

so part (b) of UHC yields

$$y \in \mathbf{c}(\{x, y, z_k\}, x) \quad \text{for each } k \in \mathbb{N}. \tag{9}$$

Now, to derive a contradiction, assume $y \notin \mathcal{Q}^1(x)$. Then, by Claim 9, (9) implies $U(y) \geq U(z_k)$ for each $k = 1, 2, \ldots$. By the first part of (8), therefore, $U(y) > U(y_k)$ for each $k = 1, 2, \ldots$. But since U is continuous, we have $\lim U(y_k) = U(y)$, so there must exist a strictly increasing sequence (m_k) of positive integers such that $U(y_{m_1}) < U(y_{m_2}) < \cdots$. Define $T := \{x, y, y_{m_1}, y_{m_2}, \ldots\}$ which is a closed subset of X. Note that if $y_{m_k} \in \mathbf{c}(T, x)$ for some k, then $y_{m_k} \in \mathbf{c}(\{x, y_{m_k}, y_{m_{k+1}}\}, x)$ by the α-Axiom, but this contradicts Claim 9.

Thus $y_{m_k} \notin \mathbf{c}(T, x)$ for each k. Since $y \in \mathcal{Q}^2(x)$, therefore, $y \in \mathbf{c}(T, x)$. But then, by the α-Axiom, $y \in \mathbf{c}(\{x, y, y_{m_1}\}, x)$. Yet, given that $y_{m_1} \in \mathcal{Q}^1(x)$ and $y \notin \mathcal{Q}^1(x)$, this contradicts the representation obtained in Claim 9.

Case 2. $y_m \notin \mathcal{E}(x)$ for infinitely many m.

Passing to a subsequence, if necessary, we may assume in this case that $y_m \in \mathcal{Q}^1(x) \backslash \mathcal{E}(x)$ for each $m = 1, 2, ...$ Then, for each m, there exists a $z_m \in \mathcal{E}(x) \backslash \{x\}$ such that $\{y_m\} = \mathbf{c}(\{x, y_m, z_m\}, x)$. By Claim 9, we have $U(y_m) > U(z_m)$ for each m. On the other hand, since X is compact, (z_m) has a convergent subsequence – we denote this subsequence again by (z_m), and write $z := \lim z_m$. From the analysis of Case 1, we know that $z \in \mathcal{Q}^1(x)$, while continuity of U ensures that $U(y) \geq U(z)$. Suppose there exists a strictly increasing sequence (m_k) of positive integers such that $U(y_{m_1}) \geq U(y_{m_2}) \geq \cdots$. Then, $U(y_{m_k}) \geq U(z)$ for each k, so

$$y_{m_k} \in \mathbf{c}(\{x, y_{m_k}, z\}, x), \quad k = 1, 2, ...$$

Thus, part (b) of UHC yields $y \in \mathbf{c}(\{x, y, z\}, x)$. But since $z \in \mathcal{Q}^1(x)$, Claim 9 shows that this is possible only if $y \in \mathcal{Q}^1(x)$, as desired. Now suppose we cannot find a strictly increasing sequence (m_k) in $\mathbb{N}$ such that $U(y_{m_1}) \geq U(y_{m_2}) \geq \cdots$. Then there is an integer $M > 0$ such that $U(y_M) < U(y_{M+1}) < \cdots$. So,

$$y_m \in \mathbf{c}(\{x, y_m, z_M\}, x) \quad \text{for all } m \geq M,$$

and hence by part (b) of UHC, $y \in \mathbf{c}(\{x, y, z_M\}, x)$. Since $z_M \in \mathcal{Q}^1(x)$, Claim 9 then entails $y \in \mathcal{Q}^1(x)$, as we sought. □

Combining Claims 6, 8, 10 and (5), we conclude: For any $(S, x) \in \mathcal{C}_{\text{sq}}(X)$,

$$\mathbf{c}(S, x) = \underset{y \in S \cap \mathcal{Q}^1(x)}{\arg\max}\, U(y),$$

provided that $S \cap \mathcal{Q}^1(x) \neq \{x\}$. The proof of the "only if" part of Theorem A is thus complete.

We now turn to the "if" part of Theorem A. Take any continuous map $U : X \to \mathbb{R}$ and any closed-valued injective self-correspondences $\mathcal{Q}^1$ and $\mathcal{Q}^2$ on X such that

$$\mathcal{L}_U(x) \cap \mathcal{Q}^1(x) = \{x\} = \mathcal{L}_U(x) \cap \mathcal{Q}^2(x) \;\text{ and }\; \mathcal{Q}^1(x) \subseteq \mathcal{Q}_2(x)$$

for every $x \in X$. Next, define $\mathbf{c} : \mathcal{C}(X) \to 2^X$ as stated in Theorem A. Given that $\mathcal{Q}^1$ and $\mathcal{Q}_2$ are closed-valued, and U is continuous, $\mathbf{c}(S,x)$ is easily checked to be a nonempty closed subset of S for any $S \in \Omega_X$. Thus $\mathbf{c}$ is a choice correspondence on $\mathcal{C}(X)$. It is also straightforward to verify that $\mathbf{c}$ satisfies the α-, β- and β^*-Axioms, so to conclude that $\mathbf{c}$ is rational, it is enough to show that it satisfies UHC.

That $\mathbf{c}$ satisfies part (a) of UHC is an almost immediate consequence of the Berge Maximum Theorem. To prove part (b) of UHC, take any $(S,x) \in \mathcal{C}_{\text{sq}}(X)$ and any sequence (y_m) in X such that $y_m \in \mathbf{c}(S \cup \{y_m\}, x)$ for each m, and $y_m \to y$ for some $y \in X$. We wish to show that $y \in \mathbf{c}(S \cup \{y\}, x)$. Of course, this is trivially true if $y_m = x$ for infinitely many m, for then $S = \{x\}$ and $y = x$. We thus focus on the case where $y_m \neq x$ for each m. Assume next that $y_m \in \mathcal{Q}^1(x)\backslash\{x\}$ for infinitely many m. Then, since $\mathcal{Q}^1(x)$ is a closed subset of X, we have $y \in \mathcal{Q}^1(x)$. Moreover, there is a subsequence (y_{m_k}) of (y_m) such that $U(y_{m_k}) \geq \max U(S \cap \mathcal{Q}^1(x))$. By continuity of U, then, $U(y) \geq \max U(S \cap \mathcal{Q}^1(x))$, and it follows that $y \in \mathbf{c}(S \cup \{y\}, x)$, as we sought. Now suppose $y_m \in \mathcal{Q}^1(x)\backslash\{x\}$ for only finitely many m. Then, it is without loss of generality to posit that $y_m \in \mathcal{Q}^2(x)\backslash\mathcal{Q}^1(x)$ for each m. In this case, because $y_m \in \mathbf{c}(S \cup \{y_m\}, x)$, we have $S \cap \mathcal{Q}^1(x) = \{x\}$ and $U(y_m) \geq \max U(S \cap \mathcal{Q}^2(x))$, $m = 1, 2, \ldots$. It follows that $y \in \mathcal{Q}^2(x)$ (because $\mathcal{Q}^2(x)$ is closed in X) and $U(y) \geq \max U(S \cap \mathcal{Q}^2(x))$ (because U is continuous). Thus, $y \in \mathbf{c}(S \cup \{y\}, x)$, as desired.

We now know that $\mathbf{c}$ is a rational choice correspondence on $\mathcal{C}(X)$. It is also evident that $\mathbf{c}$ satisfies WSQB. To prove that it also satisfies D, take any $S \in \Omega_X$ and $(T,x) \in \mathcal{C}_{\text{sq}}(X)$ such that $y \in \mathbf{c}(S, \diamondsuit)$ and $\{y\} = \mathbf{c}(T,x)$, and assume that there is a $z \in T\backslash\{x\}$ which is $\mathbf{c}$-essential in reference to x. By definition, then, we have

$$U(w) > U(z) \quad \text{and} \quad \{z\} = \mathbf{c}(\{x,w,z\}, x)$$

for some $w \in X$ which is $\mathbf{c}$-superior to x. (Notice that $U(w) > U(z) \geq U(x)$, so $w \in \mathcal{Q}^2(x)\backslash\{x\}$). The representation of $\mathbf{c}$ entails that this is possible only if $z \in \mathcal{Q}^1(x)\backslash\{x\}$. Since $z \in T$ and $y \in \mathbf{c}(T,x)$, therefore, $y \in \mathcal{Q}^1(x)$. But $U(y) \geq \max U(S)$, and hence $y \in \mathbf{c}(S,x)$, establishing that $\mathbf{c}$ satisfies D.

It remains to show that $\mathbf{c}$ satisfies SQI. To this end, take any $(S,x) \in \mathcal{C}_{\text{sq}}(X)$ such that S is $\mathbf{c}$-superior to x, which means $S \subseteq \mathcal{Q}^2(x)$, and assume that every $y \in S\backslash\{x\}$ is $\mathbf{c}$-inessential in reference to x. Let $y \in \mathbf{c}(S,x)$. To derive a contradiction, suppose $U(z) > U(y)$ for some $z \in S$. Since $S \subseteq \mathcal{Q}^2(x)$, we have $z \in \mathcal{Q}^2(x)$. Since $y \in \mathbf{c}(S,x)$, then, we must have $z \in \mathcal{Q}^2(x)\backslash\mathcal{Q}^1(x)$ and $y \in \mathcal{Q}^1(x)\backslash\{x\}$. So $\{y\} = \mathbf{c}(\{x,y,z\},x)$. But this means that y is $\mathbf{c}$-essential in reference to x, a contradiction. Conclusion:

$$\mathbf{c}(S,x) \subseteq \underset{y\in S}{\arg\max} U(y) = \mathbf{c}(S,\Diamond).$$

Conversely, let $z \in \mathbf{c}(S,\Diamond)$, but assume that $z \notin \mathbf{c}(S,x)$. This is possible only if $z \notin \mathcal{Q}^1(x)$. Now pick any $y \in \mathbf{c}(S,x)$, and notice that we must have $y \in \mathcal{Q}^1(x)$ by the nature of the representation of $\mathbf{c}$. It follows that $U(z) \geq U(y)$ while $\{y\} = \mathbf{c}(\{x,y,z\},x)$. But this means that y is $\mathbf{c}$-essential, a contradiction. Conclusion: $\mathbf{c}(S,\Diamond) \subseteq \mathbf{c}(S,x)$. This establishes that $\mathbf{c}$ satisfies SQI.

The proof of Theorem A is now complete.

8.2 Proof of Theorem B

Given Theorem A, the "if" part of Theorem B is fairly straightforward, so we solely concentrate on the "only if" part of the latter result. Let $\mathbf{c}$ be a rational choice correspondence on $\mathcal{C}(X)$ that satisfies WSQB, D, SQI and TRE. Define the map U and self-correspondences $\mathcal{E}$, $\mathcal{Q}^1$ and $\mathcal{Q}^2$ as in the proof of Theorem A. The implications of TRE on the behavior of $\mathcal{Q}^1$ and $\mathcal{Q}^2$ are discerned in the following claim.

Claim 11. $\mathcal{Q}^1 \circ \mathcal{Q}^1 \subseteq \mathcal{Q}^1$ and $\mathcal{Q}^2 \circ \mathcal{Q}^2 \subseteq \mathcal{Q}^2$.

Proof of Claim 11. Take any $x, y, z \in X$ with $y \in \mathcal{Q}^1(x)$ and $z \in \mathcal{Q}^1(y)$. We wish to show that $z \in \mathcal{Q}^1(x)$. We may assume that x, y and z are distinct, otherwise $z \in \mathcal{Q}^1(x)$ obtains trivially. Then $y \in \mathcal{Q}^1(x)\backslash\{x\}$ and $z \in \mathcal{Q}^1(y)\backslash\{y\}$, so by Claim 9, we have $\{y\} = \mathbf{c}(\{x,y\},x)$ and $\{z\} = \mathbf{c}(\{y,z\},y)$. Now, if $y \in \mathcal{Q}^1(x)\backslash\mathcal{E}(x)$, then, by definition of $\mathcal{Q}^1(x)$, there must exist a $w_1 \in \mathcal{E}(x)\backslash\{x\}$ such that $\{y\} = \mathbf{c}(\{x,y,w_1\},x)$. Similarly, if $z \in \mathcal{Q}^1(y)\backslash\mathcal{E}(y)$, then, there exists a $w_2 \in \mathcal{E}(y)\backslash\{y\}$ such that $\{z\} = \mathbf{c}(\{x,z,w_2\},y)$. Define

$$\omega_1 := \begin{cases} y, & \text{if } y \in \mathcal{E}(x) \\ w_1, & \text{otherwise} \end{cases} \quad \text{and} \quad \omega_2 := \begin{cases} z, & \text{if } z \in \mathcal{E}(y) \\ w_2, & \text{otherwise} \end{cases},$$

and let $S := \{x,y,\omega_1\}$ and $T := \{y,z,\omega_2\}$. Then, $\{y\} = \mathbf{c}(S,x)$ and $\{z\} = \mathbf{c}(T,y)$, while $S \cap \mathcal{E}(x) \neq \{x\}$ and $T \cap \mathcal{E}(y) \neq \{y\}$. Therefore, by TRE, we have $z \in \mathbf{c}(S \cup \{z\},x)$. Since $S \cap \mathcal{Q}^1(x) \neq \{x\}$ (for $x \neq y \in S \cap \mathcal{Q}^1(x)$), Claim 9 entails that $z \in \mathcal{Q}^1(x)$, as we sought.

Next, take any $x, y, z \in X$ with $y \in \mathcal{Q}^2(x)$ and $z \in \mathcal{Q}^2(y)$. We wish to show that $z \in \mathcal{Q}^2(x)$. Again, we may assume that x, y and z are distinct, otherwise $z \in \mathcal{Q}^2(x)$ obtains trivially. Then $y \in \mathcal{Q}^2(x) \backslash \{x\}$ and $z \in \mathcal{Q}^2(y) \backslash \{y\}$, so given that $\mathcal{L}_U(x) \cap \mathcal{Q}^2(x) = \{x\}$ and $\mathcal{L}_U(y) \cap \mathcal{Q}^2(y) = \{y\}$, we have $U(z) > U(y) > U(x)$. The representation of $\mathbf{c}$ then implies $\{y\} = \mathbf{c}(\{x, y\}, x)$ and $\{z\} = \mathbf{c}(\{y, z\}, y)$. In turn, TRE ensures that $z \in \mathbf{c}(\{x, z\}, x)$, so $z \in \mathcal{Q}^2(x)$. □

Now define the binary relation $\trianglerighteq^2$ on X as follows:

$$y \trianglerighteq^2 x \quad \text{if and only if} \quad y \in \mathcal{Q}^2(x).$$

Since $\mathcal{Q}^2$ is injective and $x \in \mathcal{Q}^2(x)$ for each $x \in X$, $\trianglerighteq^2$ is antisymmetric and reflexive. Moreover, Claim 11 implies that it is transitive, so we conclude that $\trianglerighteq^2$ is a partial order on X. Let $\mathcal{N}$ denote the set of all linear orders on X that include $\trianglerighteq^2$. It is a trivial matter to show that $\mathcal{N} \neq \emptyset$ and $\trianglerighteq^2 = \bigcap \mathcal{N}$. Of course, since X is a finite set, $\mathcal{N}$ is a finite set as well. Moreover, each linear order in $\mathcal{N}$ can be represented by means of a real map on X. Therefore, letting $n := |\mathcal{N}|$, we see that there are n maps $u_i : X \to \mathbb{R}$, $i = 1, ..., n$, such that $y \trianglerighteq^2 x$ iff $u_i(y) \geq u_i(x)$ for each $i = 1, ..., n$. Defining $\mathbf{u} : X \to \mathbb{R}^n$ by $\mathbf{u}(\omega) := (u_1(\omega), ..., u_n(\omega))$, then, we have $y \trianglerighteq^2 x$ iff $\mathbf{u}(y) \geq \mathbf{u}(x)$, for any $x, y \in X$. Noting that each u_i, and hence $\mathbf{u}$, is injective, we have

$$\mathcal{Q}^2(x) = \mathcal{U}_{\mathbf{u}}(x) \cup \{x\}, \quad x \in X.$$

By an analogous reasoning, we find a positive integer m and injection $\mathbf{v} : X \to \mathbb{R}^m$ such that

$$\mathcal{Q}^1(x) = \mathcal{U}_{\mathbf{v}}(x) \cup \{x\}, \quad x \in X.$$

Since $\mathcal{Q}^1(x) \subseteq \mathcal{Q}^2(x)$ for each $x \in X$, the statement $\mathbf{v}(y) \geq \mathbf{v}(x)$ implies $\mathbf{u}(y) \geq \mathbf{u}(x)$, for any $x, y \in X$. We may then write

$$\mathcal{Q}^1(x) = \mathcal{U}_{(\mathbf{u},\mathbf{v})}(x) \cup \{x\}, \quad x \in X.$$

Finally, let $A := \{(\mathbf{u}(x), \mathbf{v}(x)) : x \in X\}$, and define $f : A \to \mathbb{R}$ by $f(a) := U((\mathbf{u}, \mathbf{v})^{-1}(a))$. Finally, we extend f to $\mathbb{R}^{m+n}$ arbitrarily, say, by setting $f(b) := 0$ for all $b \in \mathbb{R}^{m+n} \backslash A$. It is now a trivial matter to check that the representation of $\mathbf{c}$ obtained in Theorem A reduces to that asserted in Theorem B.

8.3 Proof of Proposition C

Take any $x \in X$ with $U^{\mathbf{c}}(x) < \infty$. Suppose x is an accumulation point of $\mathcal{Q}^2(x)$. Then, by definition, there is a sequence (y_m) in $\mathcal{Q}^2(x)\backslash\{x\}$ such that $y_m \to x$. Evidently, $x \neq y_m \in \mathbf{c}(\{x, y_m\}, x)$, so the definition of $U^{\mathbf{c}}$ entails $U(y_m) \geq U^{\mathbf{c}}(x)$, $m = 1, 2, ...$ By continuity of U, therefore, $U(x) \geq U^{\mathbf{c}}(x)$. On the other hand, it follows from the definition of $U^{\mathbf{c}}$ that there is a sequence (z_m) in X such that $x \neq z_m \in \mathbf{c}(\{x, z_m\}, x)$ for each m, and $U(z_m) \to U^{\mathbf{c}}(x)$. But then $U(z_m) \geq U(x)$, $m = 1, 2, ...,$ by definition of $\mathbf{c}$, so $U^{\mathbf{c}}(x) \geq U(x)$ by continuity of U. We conclude that $U^{\mathbf{c}}(x) = U(x)$, as desired.

Conversely, assume that $U^{\mathbf{c}}(x) = U(x)$. Then, by definition of $U^{\mathbf{c}}$, there exists a sequence (z_m) in X such that $x \neq z_m \in \mathbf{c}(\{x, z_m\}, x)$ for each m, and $U(z_m) \to U(x)$. Since X is a compact metric space, (z_m) must have a subsequence, say (z_{m_k}), that converges to some $z \in X$. Since $z_{m_k} \in \mathcal{Q}^2(x)$ for each k, and $\mathcal{Q}^2(x)$ is a closed subset of X, we have $z \in \mathcal{Q}^2(x)$. Moreover, $U(z) = \lim U(z_{m_k}) = U(x)$ by continuity of U. It follows that $z \in \mathcal{L}_U(x) \cap \mathcal{Q}^2(x) = \{x\}$, that is, $z = x$. We conclude that (z_{m_k}) is a sequence in $\mathcal{Q}^2(x)\backslash\{x\}$ that converges to x, that is, x is an accumulation point of $\mathcal{Q}^2(x)$.

8.4 Proof of Proposition D

Take any $x \in X$ with $U_{\mathbf{c}}(x) > -\infty$, and define $S(x) := \{y \in X : y \neq x \in \mathbf{c}(\{x, y\}, y)\}$. Since $U_{\mathbf{c}}(x) > -\infty$, we have $S(x) \neq \emptyset$. Moreover, the definition of $U_{\mathbf{c}}$ and the fact that $\mathcal{L}_U(y) \cap \mathcal{Q}^2(y) = \{y\}$ for all $y \in S(x)$, imply that $U(x) \geq U_{\mathbf{c}}(x)$. To prove the converse, we fix an arbitrary $z \in S(x)$. Since $x \in \mathcal{Q}^2(z)$ and $x \in \mathcal{Q}^2(x)$, the convexity of the correspondence $\mathcal{Q}^2$ entails that $x \in \mathcal{Q}^2((1 - \frac{1}{m})x + \frac{1}{m}z)$, that is, $(1 - \frac{1}{m})x + \frac{1}{m}z \in S(x)$, for each $m = 1, 2, ...$ This shows that x is an accumulation point of $S(x)$, so

$$U_{\mathbf{c}}(x) = \sup\{U(y) : y \in S(x)\} \geq U(x).$$

We conclude that $U(x) = U_{\mathbf{c}}(x)$, as was to be proved.

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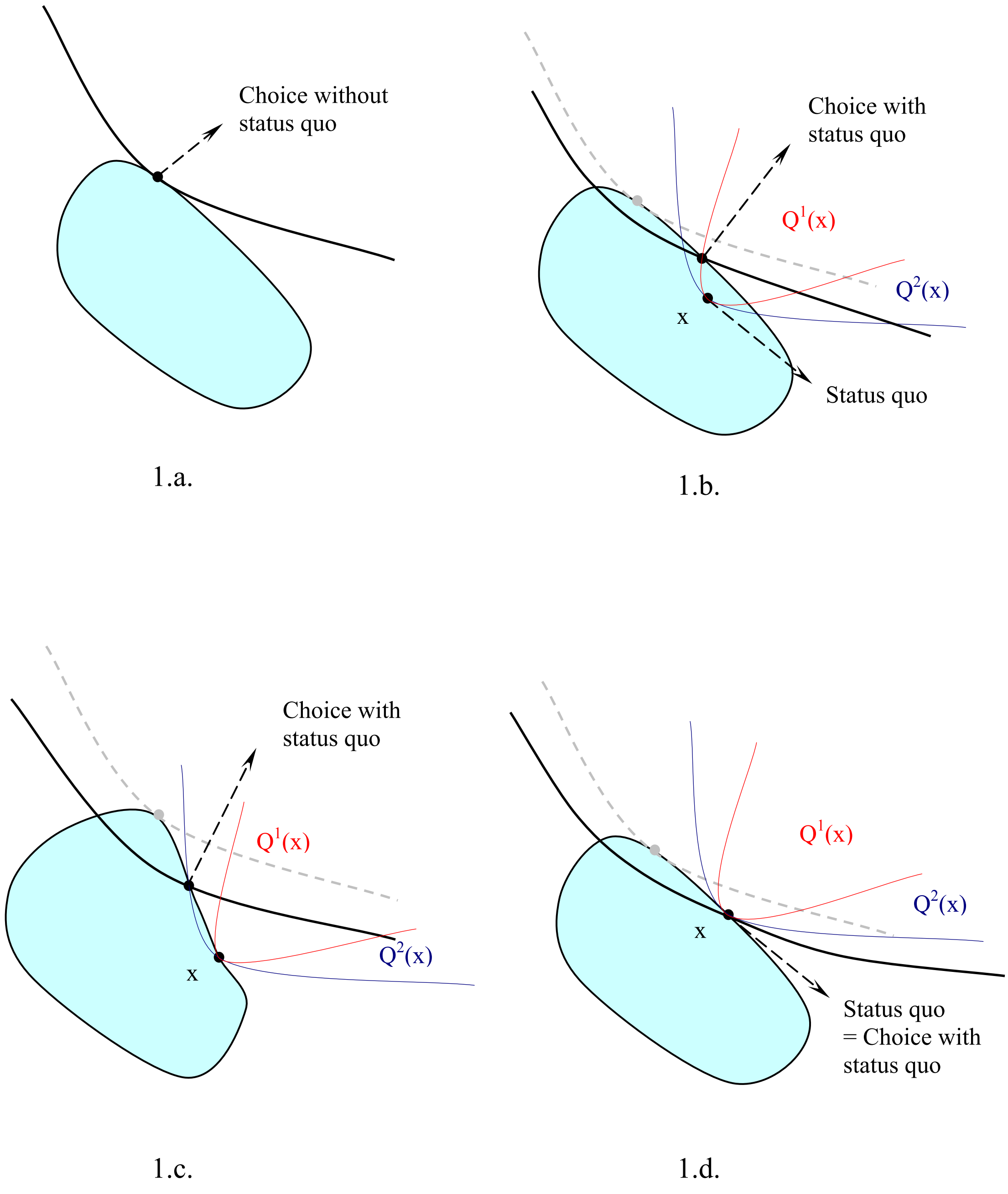
Choice without status quo
1.a.
Choice with status quo
Q1(x)
Q2(x)
x
Status quo
1.b.
Choice with status quo
Q1(x)
Q2(x)
x
1.c.
Q1(x)
Q2(x)
x
Status quo = Choice with status quo
1.d.

FIGURE 1.

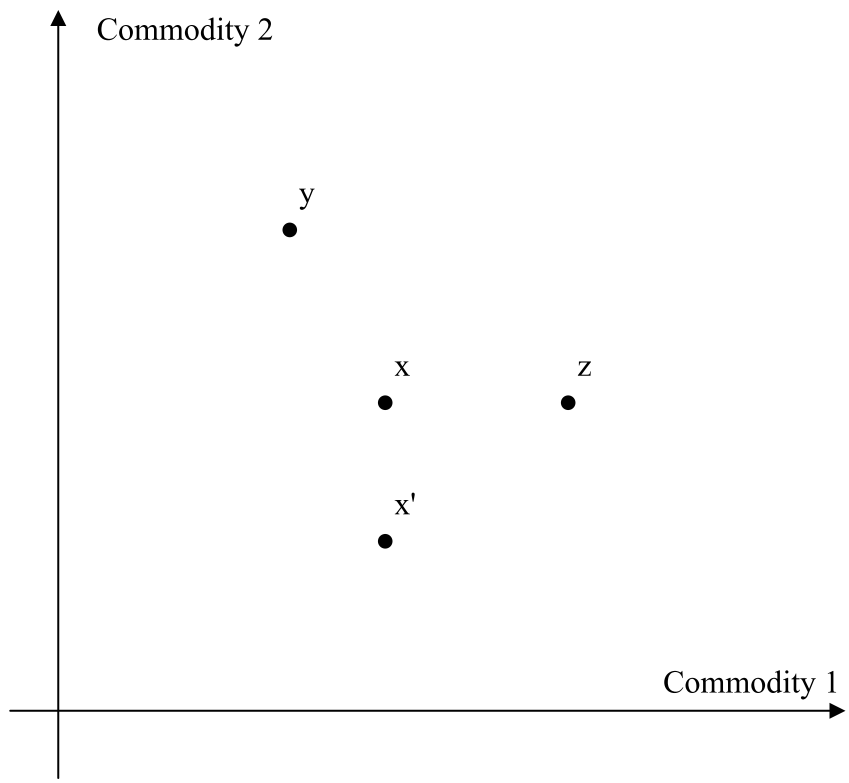

FIGURE 2.

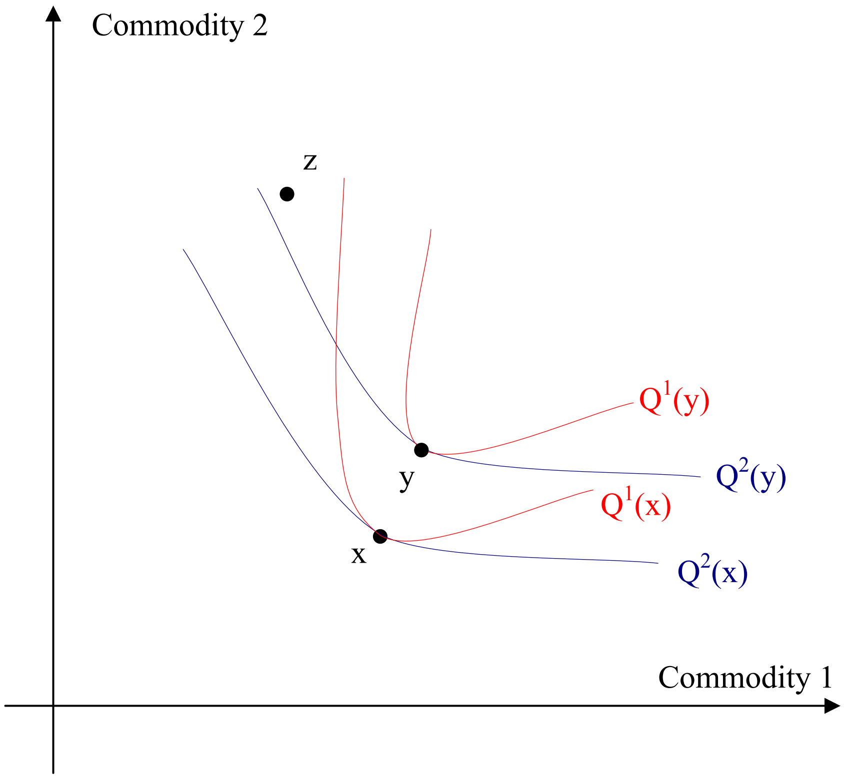

FIGURE 3.

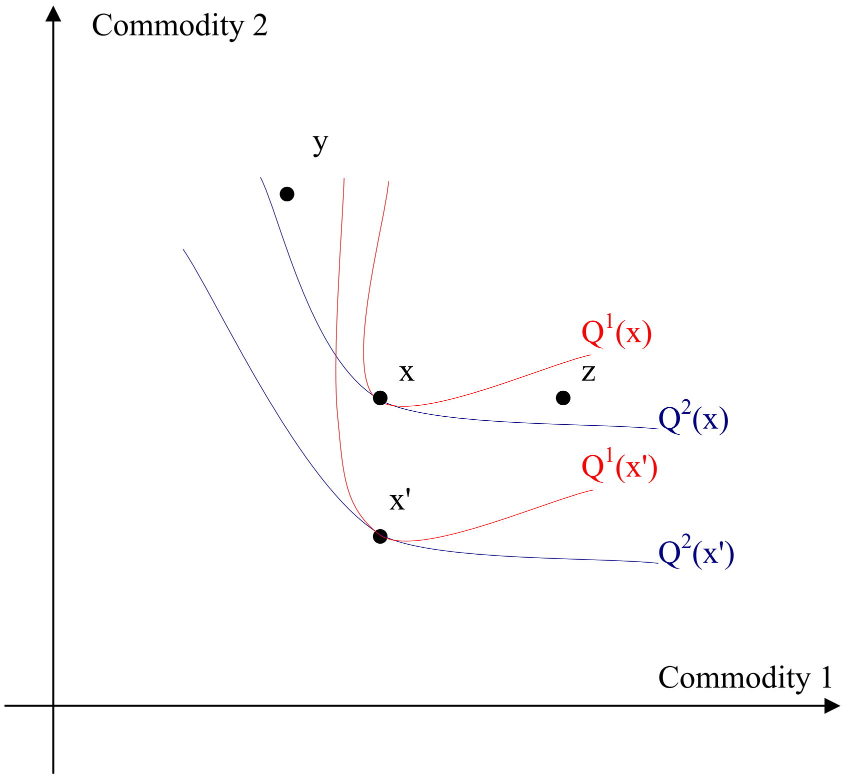

FIGURE 4.