Revealed Willpower

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Motivation

Self-regulation (self-control) is the ability to control or override one’s thoughts, emotions, urges, and behavior.

“Marshmallow Test” (Mischel): Self-regulation is closely a broad range of desirable outcomes:

- healthier inter-personal relationships,
- greater popularity,
- better mental health,
- more effective coping skills,
- reduced aggression,
- less susceptibility to drug and alcohol abuse, criminality, and eating disorders,
- and superior academic performance,
Psychologists say

- Self-control seems to rely on WILLPOWER.
- Willpower is more than metaphor or folk concept
- Willpower is a limited resource
Our Goal

★ Suggest a simple model of willpower
★ Provide a choice theoretic foundation for willpower as a cognitive resource model.
★ Illustrate that the model is tractable (an IO-application)
Let $X$ be a finite set of alternatives.

Two pieces information available: $(\succsim, c)$

- $\succsim$: preferences over alternatives (commitment (ex-ante) preferences)
- $c$: choices from any non-empty subsets of $X$
- richer than the standard choice data
- not as demanding as menu preferences
Example

- Going to the gym
- Reading a book
- Watching TV

Preferences:

\[ \text{gym} \succ \text{book} \succ \text{tv} \]

Choices:

\[ c(\text{book}, \text{tv}) = \text{book} \text{ and } c(\text{gym}, \text{tv}) = \text{tv} \]

\[ \text{gym} \] is not “choosable” over tv
Our formulation:

\[ c(A) = \arg\max_{x \in A} u(x) \]

subject to

\[ \max_{y \in A} v(y) - v(x) \leq w \]

where

- \( u \): ex-ante preferences (\( \succeq \))
- \( v \): temptation ranking
- the constant \( w \): the willpower stock
**Representation**

\[ c(A) = \arg\max_{x \in A} u(x) \quad \text{subject to} \quad \max_{y \in A} v(y) - v(x) \leq w \]

Two Extreme Cases

- \( w = \infty \) (Standard) \quad c(A) = \arg\max_{x \in A} u(x)

- \( w = 0 \) (Strotz) \quad c(A) = \arg\max_{x \in A} u(x) \quad \text{subject to} \quad v(x) \geq v(y) \text{ for all } y \in A

**Revealed Willpower**
For example,

\[
\begin{array}{c|cc}
& u & v \\
gym & 10 & 1 \\
book & 5 & 3 \\
tv & 0 & 5 \\
\end{array}
\]

Then from \(\{gym, book, tv\}\)

- Standard DM (cares \(u\)) would have chosen “gym”
- Strotz DM (cares \(v\)) would have chosen “tv”
- Our DM with \(w = 3\) would choose “book”
Road Map

- Representation
  - Without lotteries
  - With lotteries
- An IO application
  - Monopolistic contracting
  - Qualitatively different implications
Axioms

Axiom 1: $\preceq$ is complete and transitive.

Axiom 2: If $x \succ c(A \cup x)$ then $c(A) \sim c(A \cup x)$.

Axiom 3: $c(A) \preceq c(B) \Rightarrow c(A) \preceq c(A \cup B) \preceq c(B)$. 
Characterization

Theorem 0 \((\succsim, c)\) satisfies Axiom 1-3 if and only if there exist three functions \(u, v, w : X \to R\) such that

\[
c(A) = \arg\max_{x \in A} u(x)
\]

subject to

\[
\max_{y \in A} v(y) - v(x) \leq w(x)
\]

where \(u\) represents \(\succsim\).
Proof Sketch

- The proof is constructive,
- Uniqueness?
We can state the theorem in a different way:

**Theorem 0** \((\succeq, c)\) satisfies Axiom 1-3 if and only if there exists an interval order \(\succ\) such that

\[c(A) = \max(\succeq, \max(\succ, A))\]

- For every representation \((\succeq, \succ)\), we have \(\succ^* \subset \succ\), i.e. \(\succ^*\) is the one we construct in the proof.
An Additional Axiom

**Axiom 4** Suppose $y \succ c(y, z)$ and $c(t, z) = t$. If $x \succ c(x, y)$ then $c(x, t) = t$.

- $t$ is more tempting than $y$,
- $x$ is not choosable over $y$,
- Then $x$ is also not choosable $t$. 
Theorem 1 ($\succapprox$, $c$) satisfies Axiom 1-4 if and only if there exist two functions $u, \nu : X \to R$ and a scalar $w$ such that

$$c(A) = \arg\max_{x \in A} u(x)$$

subject to

$$\max_{y \in A} \nu(y) - \nu(x) \leq w$$

where $u$ represents $\succapprox$. 

Characterization
Proof Sketch

- We need to show that willpower stock is independent of the chosen alternative, i.e. \( w(x) = w \)
- The constraint can be written

\[
\left\{ x \in S : \max_{y \in S} v(y) - v(x) \leq w \right\}
\]

Here \( \succ \) must be a semiorder (a special class of interval order)
- Using the interval order (from the proof of Theorem 1), construct a semiorder, \( \succ^{**} \).
- After illustrating the representation holds, done.
**Theorem 1** \((\succeq, c)\) satisfies Axiom 1-4 if and only if there exists a semiorder \(\triangleright\) such that \(c\) is represented by

\[c(A) = \max(\succeq, \max(\triangleright, A))\]

- For every representation \((\succeq, \triangleright)\), we have \(\triangleright^{**} \subset \triangleright\),
Non-Uniqueness of $\nu$

$\nu$ is ordinally non-unique!!!

\[
\begin{align*}
    g & \succ b \succ t \\
    c(g, b) & \succ c(b, t) \\
    c(g, t) & \\
    c(g, b, t)
\end{align*}
\]

- $\nu(g) = 4$, $\nu(t) = 3$, and $\nu(b) = 0$, and $w = 2$
- $\nu'(g) = 2$, $\nu'(t) = 3$, and $\nu'(b) = 0$, and $w = 2$
- Both $(\nu, w)$ and $(\nu', w)$ represent.
- $\nu$ are $\nu'$ are different even in ordinal sense.

To be able get uniqueness result for $\nu$, we need a richer structure!!!
Let $X$ be the finite set of potentially available alternatives.

Let $\Delta$ be the set of all lotteries on $X$.

Menus are non-empty finite subsets of $\Delta$.

$\succsim$: ex-ante preferences on $X$

$c$: choices on $2^X$
Represenation

\[ c(A) = \arg\max_{p \in A} u(p) \]

subject to

\[ \max_{q \in A} v(q) - v(p) \leq w \]

where

- \( u, v \) are linear functions
- \( w \) is a positive scalar.
New Axioms

**Axiom A** There exist $p$ and $q$ such that $p \succ q = c(p, q)$.

**Axiom B** $\succsim$ is represented by EU.

**Axiom C** Suppose $p_n \rightarrow p$ and $q_n \rightarrow q$ with $p_n \succ q_n$ for all $n$. If $c(p_n, q_n) = p_n$ then $c(p, q) = p$. 
New Axioms

**Axiom D** (Temptation Independence) Suppose \( p \succ q \) and \( p' \succ q' \). Then for all \( \alpha \in [0, 1] \),

(i) If \( c(p, q) = p \) and \( c(p', q') = p' \) then \( c(p\alpha p', q\alpha q') = p\alpha p' \),

(ii) If \( c(p, q) = q \) and \( c(p', q') = q' \) then \( c(p\alpha p', q\alpha q') = q\alpha q' \).

**Axiom E** (Invariance to Replacement) If \( c(p\alpha r, q\alpha r) = p\alpha r \) then \( c(p\alpha r', q\alpha r') = p\alpha r' \) for any \( r' \).
Theorem 2 \((\succeq, c)\) satisfies Axiom 1-3 and A-E if and only if there exist two linear functions \(u, v : \Delta \rightarrow R\) and a scalar \(w\) such that

\[
c(A) = \arg\max_{p \in A} u(p)
\]

subject to

\[
\max_{q \in A} v(q) - v(p) \leq w
\]

where \(u\) represents \(\succeq\).

Both \(u\) and \(v\) are “unique.”
Characterization

**Theorem 1** $\succsim$ satisfies Axiom 1-3 and Axiom 4 if and only if there exist two functions $u, v : X \to R$ and a scalar $w$ such that

$$c(A) = \arg\max_{x \in A} u(x)$$

subject to

$$\max_{y \in A} v(y) - v(x) \leq w$$

where $u$ represents $\succsim$. 
More Willpower

When can say $(\succ_1, c_1)$ has more willpower than $(\succ_2, c_2)$?

It must be $\succ_1 = \succ_2$

**Theorem (?)**

$(\succ, c_1)$ has more willpower than $(\succ, c_2)$ if and only if

$p \succ c_1(p, q) \implies p \succ c_2(p, q)$
BE CAREFUL!!!

<table>
<thead>
<tr>
<th>Preferences</th>
<th>gym $\succ$ book $\succ$ tv</th>
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<tr>
<td></td>
<td>{gym, book} {gym, tv} {book, tv}</td>
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<tr>
<td>Will</td>
<td>gym</td>
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<tr>
<td>Tim</td>
<td>book</td>
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</table>

Does it mean Will have more willpower than Tim?

No!! There are no common $(u, v)$ with $w_W > w_T$

Proof: Suppose common $(u, v)$ and $w_W > w_T$

- $v(t) - v(b) > w_W$ and $v(t) - v(g) < w_W \Rightarrow v(b) < v(g)$.
- But then $v(b) - v(g) < 0 < w_T$, a contradiction.
Theorem

$(\succ, c_1)$ has more willpower than $(\succ, c_2)$ if and only if

1. $p \succ c_1(p, q) \implies p \succ c_2(p, q)$

2. Suppose $p \succ c_1(p, q)$ and $c_1(p, q') \succ q'$. Then for any $\beta \in (0, 1)$ whenever $p \succ c_2(p, p\beta q')$ it must be $p \succ c_2(p, p\beta q)$. 
A monopolist facing consumer(s) with limited willpower
A monopolist facing consumer(s) with limited willpower

- A two-period model of monopolistic contracting
- Firm offers a contract in period 1, which consists of
  - a set of services, $M$,
  - a price for each service, $p_s$ for all $s \in M$,
  - $c(s)$ the cost of producing service $s$

- Consumer can accept or reject it (outside option is 0)
- If accepted, both parties are committed to it.
- Consumer chooses a service in period 2
APPLICATION

\[ c(A) = \arg\max_{x \in A} U(x) \]

subject to

\[ \arg\max_{z \in A} V(z) - V(x) \leq w \]

Assume that both \( U \) and \( V \) are quasi-linear in money:

\[ U(s, p_s) = u(s) - p_s \]

\[ V(s, p_s) = v(s) - p_s \]
At the time of contracting,

- Sophisticated
  - aware of her limited willpower,
  - perfectly anticipates future behavior,

- Naive
  - unaware of it,
  - wrongly believes that he will behave according to $u$,

We focus on naive consumer.
Example: Services in a Hotel

Example

There are four possible services: $s_1$, $s_2$, $s_3$, $s_4$.

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What does firm offer?

• offer only **one service** (Commitment Contract)

\((x^*, p^*)\) solves

\[
\max_{x, p} p - c(x) \quad \text{s.t. } u(x) - p \geq 0
\]

The solution is

\[
x^* = \arg\max (u - c) \quad \text{and} \quad p^* = u(x^*)
\]
STROTZ \( w = 0 \)

\[ x^* = \text{argmax}(u - c) \quad \text{and} \quad p^* = u(x^*) \]

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Profit: \( 8 - 4 = 4 \)
Is there a better contract for the firm?

Consider \((s_1, 4; s_3, 16 - \epsilon)\)

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<td>16 - (\epsilon)</td>
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In period 1, the naive consumer believes that he will choose \(s_1\),
In period 2, he ends up choosing \(s_3\),
Profit: \(7 - \epsilon (\geq 4)\)
Offer **two services** (Indulging Contract)
- $y$: persuading the consumer to sign the contract
- $x$: actually sold

$$(x^*, p_x^*; y^*, p_y^*)$$ solves

$$\max_{x, p_x, y, p_y} p_x - c(x)$$

subject to

$$u(y) - p_y \geq 0$$
$$v(x) - p_x \geq v(y) - p_y$$
\[ u(y) - p_y = 0 \quad \text{and} \quad v(x) - p_x = v(y) - p_y \]

\[ \Rightarrow \]

\[ \max_{x; y} \quad v(x) - c(x) + u(y) - v(y) \]

The solution is

\[ x^* = \arg\max (v - c) \quad \text{and} \quad y^* = \arg\max (u - v) \]
STROTZ \( w = 0 \)

\[ x^* = \text{argmax}(v - c) \quad \text{and} \quad y^* = \text{argmax}(u - v) \]

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- \( s_1 \): to attract the customer
- \( s_3 \): to be sold

**Revealed Willpower**
What about prices for $s_1$ and $s_3$?

$u(y) - p_y = 0$ and $v(x) - p_x = v(y) - p_y$

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$p_{s_1} = u(s_1) = 4$
$p_{s_3} = v(s_3) - v(s_1) + p_{s_1} = 16$

Profit: $16 - 9 = 7 > 4$
Contracting with dynamically inconsistent naive agents,

O’Donoghue and Rabin, 1999; Gilpatric, 2003; Sarafidis, 2004; DellaVigna and Malmendier, 2004; 2006; and especially Eliaz and Spiegler 2006,

Indulging Contract.
Our Model $w > 0$

- Offer one service (Commitment Contract)

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$\Leftarrow x^*$

Profit: $8 - 4 = 4$
Our Model \( w > 0 \)

- Offer **two services** (Indulging Contract)
  - \( y \): persuading the consumer to sign the contract
  - \( x \): actually sold

\[(x^*, p_x^*; y^*, p_y^*) \text{ solves} \]

\[
\max_{x, p_x; y, p_y} p_x - c(x)
\]

subject to

\[
\begin{align*}
  u(y) - p_y & \geq 0 \\
v(x) - p_x & \geq v(y) - p_y + w
\end{align*}
\]
What about prices for $s_1$ and $s_3$?

\[ u(y) - p_y = 0 \text{ and } v(x) - p_x = v(y) - p_y - w \]

- $p_{s_1} = u(s_1) = 4$
- $p_{s_3} = v(s_3) - v(s_1) + p_{s_1} - w = 16 - w$

Profit: $16 - w - 9 = 7 - w$
Our Model $w > 0$

Is there a better contract for the firm?

Assume $w = 2$ and consider a contract with three services

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In period 1, he believes that he will choose $s_1$,
In period 2, $s_4$ is so tempting that he cannot choose $s_1$,
he ends up choosing $s_3$.

Profit: $7 - \epsilon$
Our Model \( w > 0 \)

What happens when \( w = 4 \)?

The old contract does not work !!!

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In Period 2, the customer will choose \( s_1 \)
$w = 4$, Decrease the price of $s_4$

\[
\begin{array}{cccccc}
  & u & v & c & p & u - p & v - p \\
  s_1 & 4 & 6 & 1 & 4 & 0 & 2 \\
  s_3 & 12 & 18 & 9 & 16 - \epsilon & -4 + \epsilon & 2 + \epsilon \\
  s_4 & 16 & 24 & 16 & 18 - \epsilon & -2 + \epsilon & 6 + \epsilon \\
\end{array}
\]

In Period 2, the customer will choose $s_4$
\( w = 4 \), Decrease the price of \( s_3 \)

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In period 1, he believes that he will choose \( s_1 \),
In period 2, \( s_4 \) is soo tempting that he cannot choose \( s_1 \),
he ends up choosing \( s_3 \).
Profit: \( 5 - \epsilon \) (< 7)
Is there a better one?

An alternative contract selling \( s_1, s_2, \) and \( s_4 \)

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<tbody>
<tr>
<td>( s_1 )</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>8</td>
<td>12</td>
<td>4</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>( s_4 )</td>
<td>16</td>
<td>24</td>
<td>16</td>
<td>( 18 - \epsilon )</td>
<td>( -2 + \epsilon )</td>
<td>( 6 + \epsilon )</td>
</tr>
</tbody>
</table>
Now consider the following contract

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>$v$</td>
<td>$c$</td>
<td>$p$</td>
<td>$u - p$</td>
<td>$v - p$</td>
<td></td>
</tr>
<tr>
<td>$s_1$</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$s_2$</td>
<td>8</td>
<td>12</td>
<td>4</td>
<td>10 - $\epsilon$</td>
<td>-2 + $\epsilon$</td>
<td>2 + $\epsilon$</td>
</tr>
<tr>
<td>$s_4$</td>
<td>16</td>
<td>24</td>
<td>16</td>
<td>18 - $\epsilon$</td>
<td>-2 + $\epsilon$</td>
<td>6 + $\epsilon$</td>
</tr>
</tbody>
</table>

In period 1, he believes that he will choose $s_1$.
In period 2, $s_4$ is so tempting that he cannot choose $s_1$, he ends up choosing $s_2$.
Profit: $6 - \epsilon$ (better than selling $s_3$)
Our Model \( w > 0 \)

- Offer **three services** (Compromising Contract)
  - \( y \): persuading the consumer to sign the contract
  - \( x \): actually sold
  - \( z \): tempting the consumer not to choose \( y \)

\[(x^*, p_x^*; y^*, p_y^*; z^*, p_z^*)\] solves

\[
\max_{x, p_x; y, p_y; z, p_z} p_x - c(x)
\]

subject to

\[
\begin{align*}
  u(y) - p_y & \geq 0 \\
  v(z) - p_z & \geq v(y) - p_y + w \\
  v(x) - p_x + w & \geq v(z) - p_z \\
  u(x) - p_x & \geq u(z) - p_z
\end{align*}
\]
Our Model $w > 0$

\[ u(y) - p_y = 0 \text{ and } v(z) - p_z = v(y) - p_y + w \]

\[ \Rightarrow \]

\[ \max_{x,p_x,y,z} p_x - c(x) \]

subject to

\[ p_x \leq v(x) + u(y) - v(y) \]
\[ p_x \leq u(x) + [u(y) - v(y)] + [v(z) - u(z)] - w \]

Choose

\[ y^* = \arg\max (u - v) \text{ and } z^* = \arg\min (u - v) \]
Our Model $w > 0$

Define

$$Y = u(y^*) - v(y^*) \quad \text{and} \quad Z = u(z^*) - v(z^*)$$

$$\Rightarrow$$

$$\max_{x, p_x} p_x - c(x)$$

subject to

$$p_x \leq v(x) + Y$$

$$p_x \leq u(x) + Y - Z - w$$
Our Model $w > 0$

\[
\begin{align*}
\text{max}_{p_x} & \quad p_x - c(x) \\
\text{subject to} & \quad p_x \leq v(x) + Y \\
& \quad p_x \leq u(x) + Y - Z - w
\end{align*}
\]

We need to do a case analysis

- If $u(x) - v(x) \geq w + Z$, the first constraint is binding.
  - $\max_x v(x) - c(x) + Y$

- If $u(x) - v(x) \leq w + Z$, the second constraint is binding
  - $\max_x u(x) - c(x) + Y - Z - w$
COMPROMISING CONTRACT

For the middle range $w$

services are ordered w.r.t. $u-v$

$$u(x_v) - v(x_v) = w + Z$$

Max $u-c+Y-Z-w$

Max $v-c+Y$

$z^*$

$x_v$

$x_e$

$x_u$

$y^*$

In our example

$s_4$

$s_3$

$s_2$

$s_1$

REVEALED WILLPOWER
Compromising Contract

For low $w$

services are ordered w.r.t. $u-v$

Revealed Willpower
COMPROMISING CONTRACT

For high $w$

services are ordered w.r.t. $u-v$

In our example

$s_4$  $s_3$  $s_2$  $s_1$
**Optimal Contract**

![Diagram of an optimal contract](image)

- **Profit**
  - $v(x_v) - c(x_v) - Y$
  - $v(x_u) - c(x_u) - Y$
  - $u(x_u) - c(x_u)$

- **W**
  - $x_v$
  - $x_e$
  - $x_u$
  - $X_u$
  - $Y - Z$

- **Regions**
  - Compromising
  - Commitment

**Revealed Willpower**
The monopolist sells a service somewhere between $x_u$ and $x_v$.

Profit is weakly decreasing in consumer’s willpower.

The consumer’s welfare is weakly increasing in his willpower.

When $w$ is very small, the monopolist can earn the same amount of the profit when he has no willpower at all.

When $w$ is very high, no exploitation.
Summary

- Model is tractable
- More complicated contracts
- Qualitatively different results (Strotz or Costly Self-control)
- “Compromise Effect” as a market outcome

- Provide a model, which is psychologically motivated and tractable
- Our characterization is simple
- Temptation modeled as a constraint rather than a direct utility cost.
- Our application illustrates that the model has interesting implications
THANK YOU