*Choice by Iterative Search**

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Abstract

In many real-life decisions, either information about alternatives is missing, or decision makers have limited cognitive capacity to go through all alternatives. To capture these situations, we propose and axiomatically characterize a new descriptive search model in which a decision maker explores her budget set (unlike the standard model) and has a stable preference (as in the standard model). In this boundedly rational model, she does not compare one option with all available alternatives, instead she proceeds through a path-dependent sequence of "consideration sets" until she no longer wishes to change her mind. Our model provides a unifying explanation for a number of seemingly irrational behavior without introducing changing preferences. In addition, unlike other boundedly rational choice procedures, our method is amenable to welfare analysis.

JEL Classification: D11, D81.

Keywords: Search, Satisficing, Bounded Rationality, Consideration Set, Endogenous Reference-Dependent Choice, Revealed Preferences.

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1 Introduction

Classical choice theory assumes that a decision maker chooses the best option among *all* available alternatives. This might be practically impossible, especially in situations where (i) a decision maker does not have all alternatives in view, and she must go out and search for alternatives, as when she buys a house or a car (incomplete information), or (ii) there is a menu in front of her but the menu is either too long or her time is too short (limited cognitive capacity).[1] In these situations, the budget set must be explored by the decision maker. As Herbert Simon pointed out many years ago, exploring the budget set is one of the most important aspects of real world decision making, which is neglected in the standard theory where the whole budget set is assumed to be taken in at once.

This paper proposes a new descriptive model of decision making in which a decision maker explores her budget set (unlike the standard model) and has a stable preference (as in the standard model). Our procedure is dynamic and incorporates the idea of limited search into a model of decision making. Consider a house search, for instance. The search starts with a particular house, which is the initial contemplation point.[2] In each step, search is limited to some "neighborhood" of the currently contemplated house, from which she picks her most preferred one. Then she continues the search within the neighborhood of that house. She stops her search when she can no longer find a better house within the neighborhood of the house she is currently contemplating. We call such a behavior *Choice by Iterative Search* (CIS). Borrowing a term from the marketing literature, we refer to a neighborhood as *a consideration set*: the set of elements with which she compares a contemplation point.[3]

Our model can be viewed as a dynamic version of the anchoring and adjustment heuristic. While this phenomenon is initially observed in value estimations (Tversky and Kahneman

[1]Although choice overload is usually attributed to the number of options presented to decision makers, another source of choice overload could be the number of attributes. Therefore, even with small number of alternatives, one may not compare all available alternatives.

[2]We assume that a starting point does not affect the rational assessment of the alternatives.

[3]Consideration sets are extensively studied in marketing literature, see Alba [1985] and Roberts and Lattin [1991]. Lehmann and Pan [1994] and Chakravarti and Janiszewski [2003] experimentally illustrate that the composition of consideration sets are affected by past experiences and anchors. Moreover, Hoeffler et al. [2006] reports that early experiences (anchors) affect the range of search.

[1974]), the experiments by Hoeffler et al. [2006] illustrate a dynamic anchoring and adjustment behavior in consumption choices. As in the anchoring and adjustment heuristic, our decision maker begins her search by a certain alternative (anchor), and then changes the anchor (adjustment) until she reaches a final decision. Therefore, the CIS model seems to be somewhat specific. On the contrary, our model is more general than it appears to be. For instance, the reservation-based search (RBS) model, which is often cited in labor and macroeconomics, is a particular type of the CIS model. In the RBS model, the decision maker keeps the current alternative until she encounters something better and stops searching when she finds an alternative that is above some reservation level. This is equivalent to say that her consideration set includes something better than the current alternative if and only if it is below the reservation level.

The stopping rule is the most important component of any search model. In our model, search is terminated by a simple stopping rule, which is based on the idea of choosing an alternative that is "reasonably good". The current contemplation point can be viewed as an aspiration level. This aspiration level is adjusted when the decision maker finds an alternative better than the contemplation point. The final aspiration level will be reached whenever there are no more adjustments. Similar to satisficing (Simon [1955]), decision makers do not conduct an exhaustive search of all available alternatives, but rather choose the alternative that is most preferred and meets the final aspiration level. As in the aspiration adaptation theory (Selten [1998]), the aspiration level evolves during the course of search.[4]

We point out that our search model accommodates a variety of seemingly irrational behaviors. Depending on a starting point, one could end up with a local maximum instead of a global one because the range of search is limited - a biased search. Even with a fixed starting point, the inclusion or exclusion of irrelevant alternatives into a budget set might cause a choice reversal.[5] In addition, our search model generates several other phenomena:

[4]In our model, the aspiration level never falls during the course of exploration. In this respect, ours differs from satisficing, where it is assumed that aspiration level falls as well as rises.

[5]This means that the main principle of the classical choice theory, which is called the independence of irrelevant alternatives (IIA) is violated. Considerable evidence has been provided to demonstrate that the IIA property does not hold in many real life situations (Tversky [1972], Tversky and Sattath [1979], Heath and Chatterjee [1995], Huber et al. [1982], Tversky and Simonson [1993], and Tversky and Kahneman [1991]).

(i) decision avoidance, (ii) importance of "irrelevant" information, (iii) less is more, and (iv) the attraction effect. In sum, the CIS model provides a unified explanation for different phenomena.

Although a decision maker who follows the CIS procedure is often considered as irrational because of these anomalies, we argue that she is procedurally rational, as in Simon [1998]. He emphasizes the distinction between "substantive" and "procedural" rationality. While the former concerns only *what* is finally chosen, the latter focuses on the process *how* the final choice is made. Our decision maker may be substantively irrational but is procedurally rational: Although she cannot always choose her best item, she *tries* to pick a better one given limitations she faces.

Our first result identifies choice behaviors that are consistent with the CIS model. We show that choice data is compatible with the CIS model if and only if it satisfies two simple behavior postulates: the Anchor Bias and the Dominating Anchor axioms. The first axiom states that if a decision maker ever chooses an alternative from a set, then she does not change her mind when this alternative is the starting point. The second axiom dictates that for every feasible set, there is some alternative preventing all other alternatives from being chosen whenever it is the starting point. The key feature of our approach is that our assumptions are stated in terms of choice experiments so a revealed preference type analysis can be used to test the CIS model.

In this characterization, we consider the most general version of CIS models where consideration sets depend not only on the current contemplation point but also on her budget set in an arbitrary manner. Nevertheless, we maintain the assumption of stable preference. Therefore, seemingly irrational behaviors can be explained without introducing changing preference (for instance, reference-dependent preference) as long as our postulates are satisfied.

Our second result illustrates how we can infer the preference of a decision maker only from choice data, which is important particularly for a welfare analysis. As Bernheim and Rangel [2008] point out, it is typically difficult to identify preferences from boundedly rational behavior. Our result shows that we can partially pin down preferences in the CIS model.[6] We show x is revealed preferred to y if x is chosen from *some* set with y being the starting point. The Dominating Anchor Axiom guarantees the revealed preference has no contradictions (no cycle).

We also study a special case of CIS models where consideration sets are not affected by a budget set (context-free consideration sets). This is plausible especially when the decision maker has no clear perception of what is available for her. We also axiomatize it so that we can distinguish behavioral patterns that are consistent with context-free consideration sets from others. Furthermore, this model provides richer information about how the decision maker reaches the final choice as well as preference. Indeed, one can pin down uniquely the path followed during the course of search from her choice data.

So far we have assumed that the available data about her choice is richer than in the standard model: it includes not only what the decision maker chooses but also which alternative she *initially* contemplates. One can imagine the case where her starting point is not observable (although the explosion of data mining technologies makes such data often available). Is there any way to identify people who follow an underlying CIS procedure from such restricted data? The answer is yes. We provide a simple postulate to test CIS models by using the standard choice data. Therefore, even with such limited data, it is possible to distinguish CIS decision makers from others.

The main advantage of CIS over other models of bounded rationality is its simplicity. Ironically, some models that are introduced to capture the limitations of human minds are so complicated that decision makers who follow those models should have essentially unlimited time and knowledge. Therefore, as Todd and Gigerenzer [2000] note, these models let the idea of perfect rationality to sneak in through the back door. In contrast, the consideration set is the *only* source of bounded rationality in our model, which may come from an external decision aid, for instance, the use of e-commerce sites. If this is the case, our agent does not

[6] A partial inference of preferences is also studied by Ambrus and Rozen [2008], Caplin and Dean [2008], Chambers and Hayashi [2008], Cherepanov et al. [2008], Green and Hojman [2008], Manzini and Mariotti [2008, 2009], Masatlioglu et al. [2008] in the presence of boundedly rationality.

need to spend time and knowledge to find the relevant consideration set. Therefore, unlike models referred to in Todd and Gigerenzer [2000], our model exhibits bounded rationality without imposing any extra cognitive load on the decision maker.

In addition to its generality and congruence with real world, our model has a wide range of applications. A recent paper, Eliaz and Spiegler [2007], introduces a particular version of the CIS model and investigates its implications in an industrial organization setting where two profit maximizing firms face boundedly rational consumers. They consider a market with two firms, each of which offers a variety of products. Each consumer has a default firm and considers products of the rival firm if and only if the rival firm offers some products which are similar to the best item of the default firm. The behavior of such a consumer satisfies our axioms, and thus it can be represented by the CIS model. Therefore, their paper illustrates tractability of the CIS model.

Finally, we point out that the CIS model provides a foundation for a general class of *endogenous* reference-dependent choice models based on the concept of the personal equilibrium proposed in Köszegi and Rabin [2006].[7] An alternative is a personal equilibrium, if it is chosen whenever the decision maker expects to choose it. In other words, it is a self-fulfilling plan (rational expectation). If a starting point represents what the decision maker expects to buy, in CIS models, the final choice is always a self-fulfilling plan since it must be optimal within its consideration set. More importantly, our model describes an underlying mechanism how expectations evolve during the course of search.

The outline of this paper is as follows: We (i) illustrate the model by means of an example, then introduce the basic notations and definitions, (ii) illustrate implications of the CIS model, (iii) provide two nested characterizations for the CIS model, (iv) investigate the case of unobservable starting points and provide a behavioral test, and finally (v) conclude the paper. All proofs are given in the Appendix.

[7] Ok et al. [2008] axiomatically develop another endogenous reference-dependent choice model that accounts for attraction effect. In their model, there exists an underlying reference dependent choice based on Masatlioglu and Ok [2005], and a reference point is endogenously induced by a budget set.

Related Literature

Before we proceed to the model, we review the models related to our model and discuss their relationships.

Caplin and Dean [2008] and our paper are only ones which study search by employing the revealed preference approach. Caplin and Dean [2008] propose two nested models of search, "alternative-based search (ABS)" and "reservation-based search (RBS)," in which a decision maker goes through alternative sequentially and, at any given time, chooses the best one among those she has searched. Similar to our model, the most essential component of these models is the fixed preference. Unlike our model, the entire path followed during the course of search is the input of their models, rather than the output: their "choice process" data includes not only her choice without time limit, but also what she would choose if she was suddenly forced to quit the search at any given time.

On one hand, the ABS model is agnostic about how and why search is terminated. On the other hand, while the RBS model provides a simple stopping rule based on a static utility threshold as in Simon's satisficing; the stopping rule of the CIS model can be interpreted as satisficing with dynamically evolving thresholds. In sum, both papers investigate the same issues by utilizing different sets of choice data and of descriptive models.

Salant and Rubinstein [2008], Masatlioglu et al. [2008] and Manzini and Mariotti [2007, 2008] provide different models of decision making, all of which utilize the idea of limited consideration. Contrary to our model, all of them follow the two-stage choice process: in the first stage, a decision maker focuses on a small set of alternatives, and in the second stage she maximizes her preference among the alternatives surviving after the first stage. On the other hand, the consideration sets in our model dynamically evolve depending on a starting point and the subsequent contemplation points. The intuitive difference is that in these models decision makers shrink their budget set in the first stage, whereas in our model, decision makers explore the budget set by going through different consideration sets.

Our CIS model is also closely related to the literature on reference-dependent preferences, specifically Tversky and Kahneman [1991], Masatlioglu and Ok [2005], and Salant and Rubinstein [2008], where the reference point is exogenously given.[8] A major difference of the present work from these studies is that the CIS model is a sequential choice procedure, as opposed to their static models. Our model provides a complete description of a dynamic search model- the reference point dynamically evolves. A second major difference is that we do not assume our decision maker's behavior can be expressed by a preference ordering, even when the reference point (starting point) is fixed.[9]

2 The Model

An Illustration

We illustrate our model by means of the following example. Ms. Dema is buying a house. Her search starts with a specific house, say x_0, recommended by a colleague of hers. Since this purchase is by far the most expensive in her life, she wonders if there is a better house that she can afford. Because there are so many houses on the market, her task is formidable if she compares x_0 with all of them. Instead, she utilizes a real estate website to get a list of properties that are comparable to x_0 in several aspects. For example, they are similar to x_0 in terms of size and price and are located within a certain distance from x_0. Let us call this list her consideration set of x_0. If there is no property which is better than x_0 within the consideration set of x_0, she convinces herself to buy house x_0. If not, she picks the best house within the list, say x_1. However, since x_1 may be in a different neighborhood, she wants to check if it is also the best within its consideration set (i.e. properties comparable to x_1). This new list might include new houses that were not in the initial consideration set. If x_1 is indeed the best option within its consideration set, she chooses x_1. Otherwise, she continues this process by picking the best house in the new list. Her search stops when she finds a house that is optimal given its consideration set. Figure 1 summarizes this choice

[8]Other theoretical works in this field include, but not limited to, Munro and Sugden [2003], Sugden [2003], Sagi [2006], Bleichrodt [2007], Apesteguia and Ballester [forthcoming].

[9]Dean [2008] provides another static reference-dependent model in which IIA can be violated even for a fixed reference point. His model is also a special case of our general CIS model.

procedure.

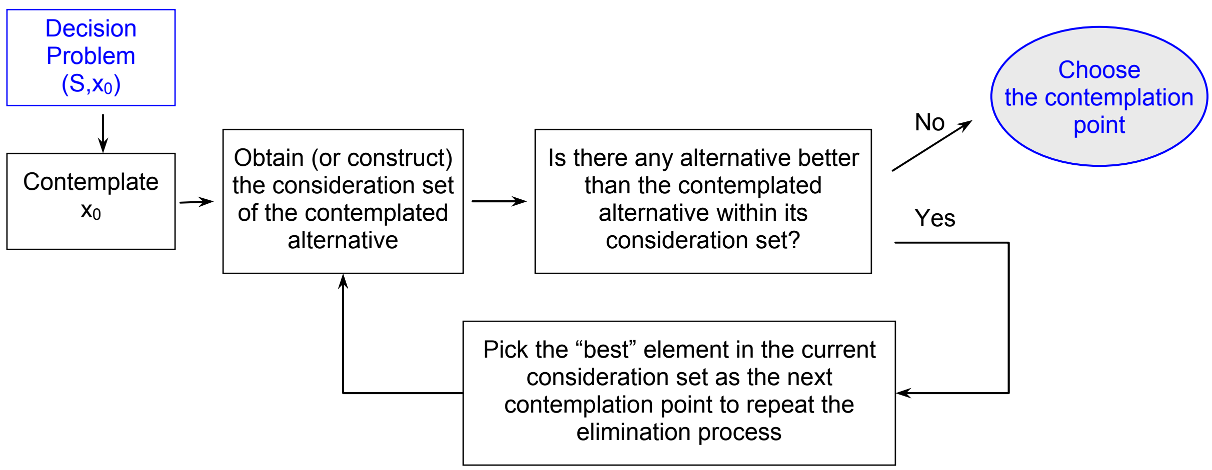

Figure 1: The Flowchart for the Choice by Iterative Search

Choice by Iterative Search is built on three components: a starting point, consideration sets, and the stopping rule. We explain them in turn. A starting point is the alternative that the decision maker initially pays attention to. In other words, it is the point where search starts. There could be different reasons for having a certain starting point. For example, (i) it could be her default option or status quo, (ii) it is what she expects to buy in the market, (iii) she might hear about it from someone in her social network, or (iv) she is exposed to advertisements for it. The starting point captures the idea that the initial stages have an impact on the later stages of the process and on the final decision - path dependence. For example, starting search with a very expensive item, as opposed to a very cheap item, can make a moderately expensive item choosable (e.g., Kardes [1986], Parducci and Fabre [1995], Sherman et al. [1978]). Nevertheless, the starting point does not change the ranking of the alternatives.

The notion of consideration sets is the most important component in our model. If a consumer does not know what items are available, she must first look for options that are comparable to the one currently contemplated, which constitute her consideration set. When her difficulty comes from her limited time or cognitive capacity, she must use a consideration set to reduce a complex problem into a much more manageable one. For example, "Comps," an abbreviation for "comparable properties," is used for comparative purposes in real estate.

In the above example, the consideration set consists of the comps. Consideration sets may be provided by external aids, such as web-sites or the recommendation of a friend. However, it is also possible that the consideration set is a reflection of a psychological or cognitive constraint in the mind of a decision maker. For example, both loss aversion and regret are two important psychological factors, which influence the composition of consideration sets (Han and Vanhonacker [1999], Zhao et al. [2008], Lin and Huang [2006]).[10]

Reference dependence plays an important role in the consideration set formation (Han and Vanhonacker [1999]). Chakravarti and Janiszewski [2003] found that consumers use their past experiences to construct consideration sets by including alternatives that have alignable attributes or overlapping features with their past experiences. For recommendation purposes, online merchants commonly provide links such as *Similar Items*, *Customers Who Viewed This Also Viewed*, and *What Do Customers Ultimately Buy After Viewing This Item?* Therefore, different starting points may induce different consideration sets and thus different final choices.

The final component is the stopping rule. Since our decision maker does not have enough time to go through all alternatives, she needs a sound way to end her search. Our stopping rule uses the idea of choosing an alternative that is "reasonably good." That is, the final choice is always optimal within its consideration set at least. Analogously, if she finds a locally optimal alternative, then she stops her search. Hence this stopping rule saves resources when maximizing would be too costly and enables decision makers choose at least one of the local maximizers.

As we mention in the introduction, this rule can be thought as a reflection of Simon's satisficing behavior, where a current contemplation point corresponds to a current aspiration level. The rule also captures the general wisdom "Before buying a house you fall in love with, do yourself a favor and just look at a few more, so you will never question if you made the right decision." This suggests that before buying a particular house one should consider its

[10]The consideration set could be generated by a combination of an external aid and a cognitive constraint. For example, the decision maker uses a web search engine but she compares x only with the alternatives in the first page of the search results. In this case, her consideration set depends on both the current contemplation point and her budget set.

comparable properties. In other words, the final choice must be optimal among those of which she is aware.

Notations and Definitions

Throughout this paper X will stand for an arbitrary non-empty finite set, with each element of X a potential choice alternative. Let $K(X)$ denote the set of all nonempty subsets of X. An *extended choice problem* is a list (S, x_0) where $S \in K(X)$ is a feasible set (budget set) and $x_0 \in S$ is a starting point. The interpretation is that the individual is confronted with the problem of choosing an alternative from the feasible set S and initially contemplating alternative x_0.[11]

An *extended choice function* assigns a single chosen element to each extended choice problem. That is, $c(S, x_0) \in S$ for every extended choice problem. Before we introduce our main concept, "Choice by Iterative Search (CIS)", we formally define its two basic components: a preference and consideration sets.

A preference, which is typically denoted by $\succ$, is a strict order over X as in the standard theory.[12] Therefore, our decision maker would be perfectly rational if she compared all feasible alternatives. If $x \succ y$ for all $y \in S \setminus \{x\}$, we call x is the $\succ$-best in S.

A *consideration set* of x under choice problem S, denoted by $\Omega(x, S) \subset S$, consists of elements which the decision maker considers when she currently contemplates x in choice problem S.[13] In other words, $\Omega(x, S)$ is the set of alternatives that are taken seriously by consumers when x is the contemplation point. Naturally we assume that $x \in \Omega(x, S)$. We call $\Omega(\cdot, \cdot)$ a consideration set mapping.

We consider a special class of consideration sets in which, given a contemplation point,

[11] Masatlioglu and Ok [2005], Salant and Rubinstein [2008] and Bernheim and Rangel [2008], use similar frameworks in which x_0 is interpreted as a status quo, a frame, and ancillary conditions, respectively.

[12] We assume $\succ$ has no indifference for simplicity.

[13] While it is a useful simplification that the consideration sets are only affected by the contemplation point and (possibly) the budget set. The model can be extended cases where the consideration sets are affected by more general framing effects (such as presentation). In this case, the consideration set can be written as $\Omega(f)$, where f is a frame.

whether an alternative is considered is independent of other alternatives. We call this particular class *context-free* consideration sets. Formally, for each x, there is a set of alternatives which are comparable to x no matter what the budget set is, denoted by $\Omega^*(x)$. Then the consideration set of x under choice problem S consists of alternatives that are both comparable to x and feasible, i.e. $\Omega(x, S) = \Omega^*(x) \cap S$ where $x \in \Omega^*(x)$ for all x.

Given these components, we are ready to define choice by iterative search. An extended choice function c is a *Choice by Iterative Search* (CIS) if there exist a ranking $\succ$ and a consideration set mapping Ω such that, for every extended choice problem (S, x_0), there exists a sequence of contemplation points $(x_0, x_1, \dots, x_n, y)$ with $n \geq 0$ such that

i) x_k is the $\succ$-best element in $\Omega(x_{k-1}, S)$ for $k = 1, \dots, n$

ii) $y = c(S, x_0)$ is the $\succ$-best element in both $\Omega(x_n, S)$ and $\Omega(y, S)$.[14]

If so, we say c is represented by $(\succ, \Omega)$. Occasionally, we mention that $\succ$ represents c, which means that there exists some Ω such that $(\succ, \Omega)$ represents c.

Let c be a choice by iterative search. If $y = c(S, x_0)$ and $y \neq x_0$, then there is a sequence of contemplation points starting from x_0 and ending at y. In the first step, the decision maker contemplates x_0, but discards it since there are some alternatives in $\Omega(x_0, S)$ which are better (with respect to $\succ$) than x_0. Then she picks the $\succ$-best element in $\Omega(x_0, S)$ denoted by x_1 (note that x_1 must be better than x_0) to start the next stage of the process. This process stops when the decision maker reaches y where there is no better alternative within its own consideration set. We can think of y as the fixed point of this decision process.

Under the assumption that consideration sets are context-free, we can provide a graphical illustration for our model. In Figure 2, each node corresponds to a particular alternative, and the position of nodes represents preferences: the higher the position is, the better the alternative is, $(x \succ y \succ z)$. A connection between two alternative indicates that each of

[14]We should point out that the decision maker moves forward and regards $\Omega(x_k, S)$ without reference to $\Omega(x_{k-1}, S)$. This might be interpret as if she forgot whatever she had considered in the prior stage of the dynamics. Actually, this is not important since she remembers the best alternative in $\Omega(x_{k-1}, S)$, which is x_k.

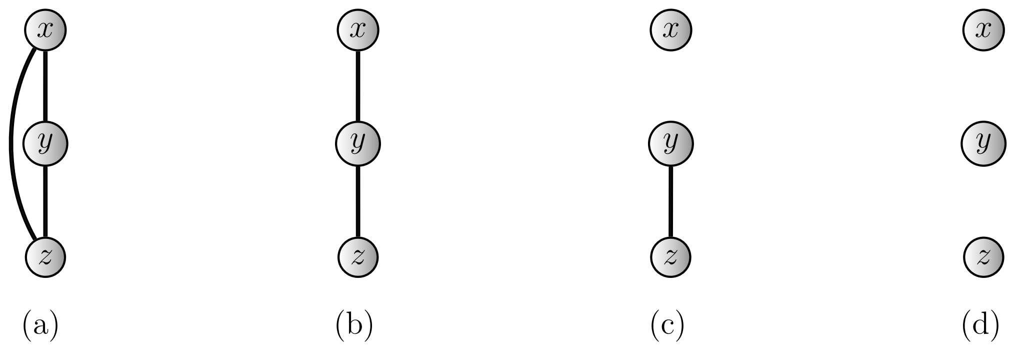

Figure 2: Graphical Representations for Context-Free Consideration Sets

them is the consideration set of the other, such as relations "similar to" or "is near".[15] We illustrate four cases from one extreme to the other with three elements in Figure 2(a)-(d): while (a) represents a standard decision maker (independent of a starting point, the decision maker considers all feasible alternatives, hence always chooses the best alternative), in (d), she does not consider anything other than the starting point, hence she sticks to the starting point no matter what the budget set is. The interesting one is Figure 2(b): Her best option, x is visible from y but not from z. Therefore, she cannot reach x when z is the starting point unless y is available. In other words, to choose x, she needs to follow the path of contemplation points, $z \to y \to x$.

Anomalies

Before we proceed to the representation result, we will discuss several interesting implications of choice by iterative search. When a starting point is interpreted as a reference point, the model explains the status quo bias and the endowment effect (reference-dependence choices in general). In addition, even with fixed starting point, it accommodates choice reversals. We also illustrate three particular versions of that: (i) less is more and (ii) exogenously provided irrelevant information can influence choices and (iii) decision avoidance. Finally,

[15] A directed graph can be used to illustrate asymmetric membership of consideration sets: only one of them is the consideration set of the other.

we demonstrate the attraction effect in CIS model with random starting point. To sum it up, our model provides a unifying explanation for different phenomena.

All of following examples involve a small number of alternatives. One can easily imagine some alternative is not considered even in such a decision problem because the complexity may come from natures of alternatives. For instance, a standard digital camera, nowadays, has more than fifty attributes, which makes comparison difficult even among two alternatives. Hence, the source of choice overload is not only the number of options presented to decision makers, but also the number of attributes.

Choice Reversals

Choice reversals refer to situations where x is sometimes chosen when y is available but y is selected over x in some other occasion. We show that even when the decision maker starts the search with the same item, she may exhibit choice reversals. To see this in a simple example, we employ a CIS model with context-free consideration sets.

Assume that Ms. Dema faces three houses x, y, and z. Her preference among the three houses is $x \succ y \succ z$. While x is comparable to y and y is comparable to z, x is not comparable to z. Her consideration sets include *only* comparable items (see Figure 2(b)).[16] We focus on her extended choice problems where the starting point is z. Then, she sticks with z unless y is available. Hence, if only x and z are available, her choice will be z, but she will choose x if all of the three items are available by following the path of contemplation points, $z \to y \to x$. Therefore, we observe a choice reversal:

$$x = c(\{x, y, z\}, z) \text{ and } z = c(\{x, z\}, z)$$

Here, even with a fixed starting point, our extended choice function does not necessarily satisfy the assumption of standard theory - Independence of Irrelevant Alternatives, so such a behavior cannot be represented by any preference (even by a starting-point dependent

[16] Formally, her consideration sets are: $\Omega^*(x) = \{x, y\}$, $\Omega^*(y) = \{x, y, z\}$ and $\Omega^*(z) = \{y, z\}$.

preference).

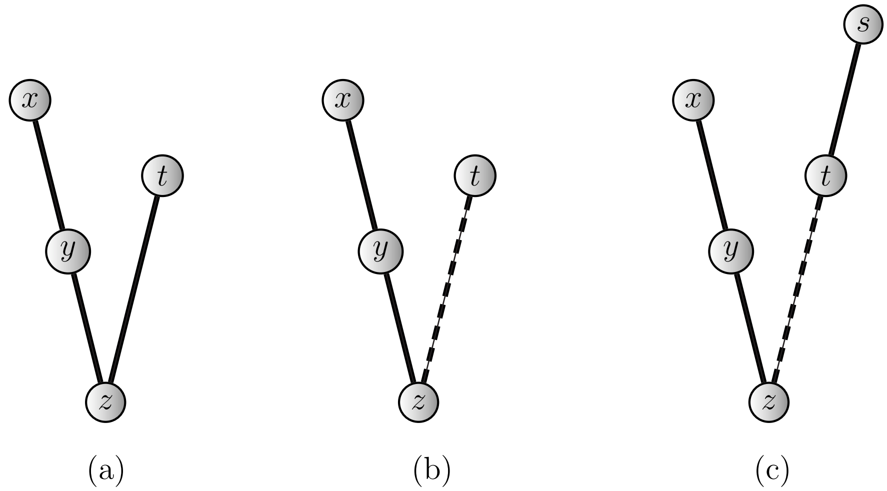

Figure 3: Graphical Illustrations: (a) Less is more, (b) Irrelevant information hurts and (c) Irrelevant information benefits.

Less is More

Next we show that having more options might cause a decision maker to make a worse choice. Consider Ms. Dema's choice problem with an additional house t that is comparable only to z (see Figure 3(a)).[17] Her preference is $x \succ t \succ y \succ z$. Let z be the starting point. If all the alternatives are available, she will pick the second best alternative, t, since t is the $\succ$-best element both in comps of z and comps of t. On the other hand, if t is removed from the feasible set, she will choose x by following the path of contemplation points, $z \to y \to x$. Hence, having fewer options makes it possible to choose the first best, x. That is, "less is more."[18]

[17]Her consideration sets are $\Omega^*(x) = \{x, y\}$, $\Omega^*(y) = \{x, y, z\}$, $\Omega^*(z) = \{y, z, t\}$ and $\Omega^*(t) = \{z, t\}$.

[18]Even though our illustration is based on the knowledge of the decision maker's true preference, one can show the welfare loss due to the revealed preference. In other words, it can be shown that our model allows us to both infer that x is welfare enhancing relative to t and that removing t therefore leaves the decision maker better off.

Importance of Irrelevant Information

Suppose the source of consideration sets is incomplete information. We illustrate that irrelevant information can influence choices in our model. Contrary to the standard theory, making the decision maker aware of the availability of an inferior option might be beneficial or harmful for decision makers in the CIS model, even if such information does not affect the starting point.

Here, we assume that a third party can influence the consideration sets of a decision maker, so it can be interpreted as a comparative statics exercise on consideration sets to make welfare statements. Consider a slight modification of the above example where the decision maker is not aware of t when the starting point is z, i.e. $t \notin \Omega^*(z)$ (see Figure 3(b): the dotted line indicates that the decision maker is not aware of t initially). When all alternatives are available, she will pick x by following the path of contemplation points, $z \to y \to x$. Now imagine that she becomes aware of t because of either her buyer agent or a successful advertisement (now $t \in \Omega^*(z)$, i.e., the dotted line becomes a straight line). This exogenous information would be regarded as irrelevant in the standard theory because x is better than t according to her preference. However, in our model, the decision maker now chooses t since t now is the best element both in $\Omega^*(z)$ and $\Omega^*(t)$. Therefore, providing seemingly irrelevant information might hurt decision makers.

On the contrary, irrelevant information might be beneficial. To see this, we add another house s. Her preferences are $s \succ x \succ t \succ y \succ z$ and her consideration sets are $\Omega^*(x) = \{x, y\}$, $\Omega^*(y) = \{x, y, z\}$, $\Omega^*(z) = \{y, z\}$, $\Omega^*(t) = \{t, s\}$, and $\Omega^*(s) = \{s, t\}$ (see Figure 3(c)). Again z is the starting point and all alternatives are available. She then picks x, which is the second best alternative. However, she would choose s if she is made aware of t (so t is now within $\Omega^*(z)$) by following the path $z \to t \to s$. Even though providing information on t is again seemingly irrelevant, it affects her choice and improves her welfare.

Since the source of bounded rationality in our framework is the limited comparison (consideration sets), one may think that having a larger consideration set (weakly) improves her welfare. This is not true in general. Our first observation above illustrates the possibility that it makes her worse off. Therefore, the comparative static exercise in terms of consideration sets are not straight forward in our framework.

Decision Avoidance

The existence of empirical studies shows that people tend to choose the default option when they face an extensive selection of products (Iyengar and Lepper [2000], Iyengar et al. [2004] and Dean [2008])[19]. Iyengar et al. [2004], for instance, illustrates this phenomena in choices of 401(k) plans: the fraction of workers who participate in 401(k) was significantly larger when faced a limited selection than when faced an extensive selection. In sum, when people are presented with too many choices, the natural tendency is to stick the default option to avoid the decision making.

These studies suggest that whether a particular option is considered depends not only on the default (the current contemplation point) but also on the presence of other alternatives - context dependence. The general CIS model can accommodate this phenomena by assuming that consideration sets shrink as more alternatives are present.

The Attraction Effect

The attraction effect refers to a phenomenon where a dominated entry will increase the likelihood that the dominating alternative is chosen (Heath and Chatterjee [1995] and Huber et al. [1982]). For instance, suppose that there are two options x and y between which trade-off is close, and a third item, say d_x, is clearly dominated by x but not by y (see Figure 4). It is known that the presence of d_x boosts the chance of x being chosen.

Lehmann and Pan [1994] experimentally show that introducing new products causes the attraction effect by affecting particularly the composition of consideration sets. Our model provides an explanation for the attraction effect in line with their findings. The decision maker has a complete preference over alternatives, but some of comparisons are very clear

[19]Dean [2008] provides not only experimental evidence but also an axiomatic model which can accommodate decision avoidance.

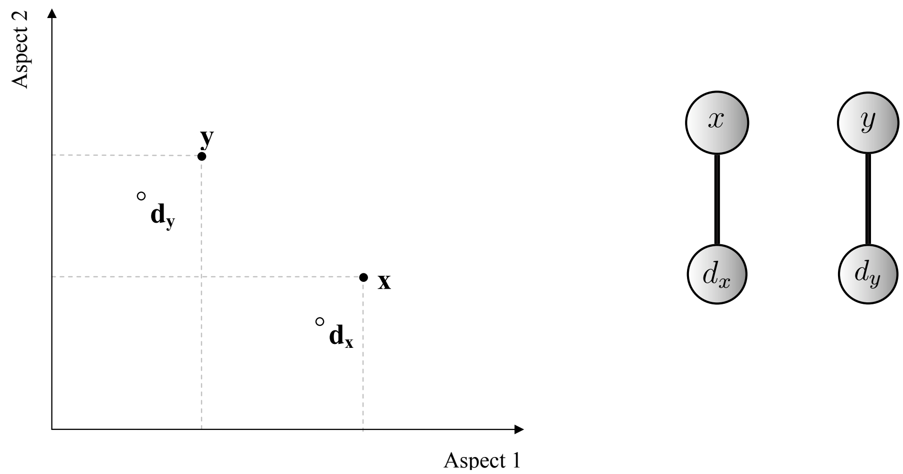

Figure 4: The Attraction Effect

and some are not (cognitively costly). Her consideration set of an alternative consists of those that are easily comparable to it. As in Figure 4, suppose that x clearly dominates d_x, and y clearly dominates d_y. Then, the presence of d_x will increase the chance of x being chosen, but it goes down when d_y is added. This is because x (y) is chosen when either x or d_x (y or d_y) is her starting point.[20]

3 A Characterization

We first consider the most general version of CIS model where a consideration set depends not only on the contemplation point but also on the budget set in a arbitrary manner. This is because we would like to identify the largest class of choice behaviors that result from either incomplete information or limited cognitive capacity, rather than changing preferences. Moreover, this model can be thought as a benchmark model which can accommodate a number of anomalies observed both within economics and psychology. For example, the decision avoidance cannot be explained by context-free consideration sets.

[20]We implicitly assume that the chance that any given item is initially considered goes down when more items are available. Note that under this assumption, a two-step version of our model will be enough to explain the attraction effect, such as Masatlioglu and Ok [2005].

Since we are working with a really general model and we have demonstrated that our model can accommodate many different (seemingly irrational) behaviors, one may argue that the CIS model can explain any choice behavior. This claim is false. For example, this model rules out situations in which an agent chooses y starting her search with x, z starting with y, and x starting with z. This particular type of cycles is not permitted since, in our model, one never chooses an item that is worse than the starting point.[21] Hence, we must have $x \succ y \succ z \succ x$ which contradicts that $\succ$ is a preference.

We now identify the class of choice behaviors compatible with the general version of CIS model. The first axiom is imposed to exclude the cyclical behaviors described above. We say x blocks y if y cannot be chosen whenever x is the starting point.

> THE DOMINATING ANCHOR AXIOM: *For any budget set S, there is some alternative $x^* \in S$ that blocks all other alternatives in S: $c(T, x^*) \notin S \setminus \{x^*\}$ for all T.*[22]

This axiom states that any budget set S has at least one alternative x^* which will prevent all other alternatives in S from being chosen in any choice problem whenever x^* is the starting point. Particularly, as long as x^* is initially considered, x^* must be chosen from any subset of S (including S itself). When a budget set is not a subset of S, either x^* or another alternative which does not belong to S is chosen whenever x^* is initially contemplated. Normatively, this axiom can be thought as the condition for the decision maker to be immune to the classical money-pumping argument.[23]

To understand the axiom better, it is useful to see an example which does not satisfy it. Indeed, the example at the beginning of this section is the one. What is a dominating anchor

[21] Although there has been a plenty of experimental evidence pointing to choice cycles in certain situations, there is no known market (experimental) evidence to this particular type of cycles. Indeed, our model can accommodate other types of choice cycles. For instance, the observation of $c(xya, a) = x$, $c(yza, a) = y$, and $c(zxa, a) = z$ does not contradict our axiom: the Dominating Anchor Axiom.

[22] This axiom is equivalent to the acyclicity of the revealed preference, which we will define shortly.

[23] Suppose this axiom is violated for some S. That is, for all $\alpha \in S$, there exists T_α including α such that $\alpha \neq \beta = c(T_\alpha, \alpha)$ and $\beta \in S$. Hence, she is willing to pay a small amount of her wealth whatever item x in S she currently owns by offering corresponding budget set T_x to exchange x with $y(= c(T_x, x)) \in S$. Because S is finite, she will eventually end up with the same element she owned before but with less money.

(or x^*) for the set $\{x, y, z\}$? It cannot be x since x does not block y. Similarly, neither y nor z can be. Therefore, the axiom is violated.[24]

In the CIS model, the final choice is always optimal in its consideration set. Therefore, if she started with this item, she would stick with it, which is a necessary requirement for our decision process.[25]

> THE ANCHOR BIAS AXIOM: *If a decision maker ever chooses y from S, then she does not change her mind when she first considers y in S: If $y = c(S, x_0)$, then $y = c(S, y)$.*

One interpretation of this axiom is that the decision maker will not regret her choice if one replaces the starting point with her original choice given a fixed feasible set. If the starting point is the past consumption, it can also interpreted as no variety-seeking behavior.

It turns out that these two axioms are also the sufficient conditions for the choice by iterative search.

Theorem 1 *An extended choice function c obeys the Anchor Bias and Dominating Anchor Axioms if and only if c is a CIS.*

The novelty of Theorem 1 is that our procedural choice behavior is captured by two simple behavioral postulates. This means that the idea of Simon's procedural rationality can be tested by the revealed preference approach in our framework. However, questions regarding the determinants and/or sources of Ω are beyond the scope of this paper.

[24] The Dominating Anchor Axiom invokes an existential quantification within S, which may be undesirable in terms of testing of the model. Here, we show that only three observations can falsify the axiom. Indeed, even two observations might be enough, for instance, $c(T, x) = y$ and $c(T', y) = x$. This is because the axiom applies to *any* set S, which can be as small as a doubleton, i.e. $\{x, y\}$.

[25] Different versions of this axiom have been named "Status Quo Bias" (Masatlioglu and Ok [2005], "No Regret" (Sagi [2006]), and "Strict Exchange Aversion" (Sugden [2003]).

Revealed Preference

Identifying the preferences of a decision maker from choice data is a crucial issue, particularly for the purpose of welfare analysis. However, it is a difficult job especially with boundedly rational models, like ours. Nevertheless, we provide the inference of the preference as much as possible.

Suppose we observe $c(S, x_0) = x_0$. The standard theory interprets it as x_0 is the best alternative. However, in our framework, it is possible that x_0 is the worst element but the consideration set of x_0 consists of only x_0. Therefore, her choice does not reveal any information about her preference.

Nevertheless, if a decision maker chooses x from *some* set with y being the starting point, we must conclude x is better than y. This is said x is *directly* revealed preferred to y.[26] More interestingly, we may be able to infer her preference between x and y even when we never observe such a choice. For example, suppose she chooses x from some set when α is the starting point and while α is chosen from some other set when y is the starting point. Then we can infer x is better than α, which is better than y. Therefore, we conclude that x must be better than y. The stable preference assumption is crucial for the conclusion. In such a case, we say x is *indirectly* revealed to be preferred to y.[27] Formally, let $x \succ_c y$ if x is directly or indirectly revealed to be preferred to y.

Notice that such an indirect inference is not possible with a reference-dependent preference model, where it is natural to consider that x is better than y whenever x is chosen when y is the reference point, i.e., $x \succ_c y$ if $x \succ_y y$. Suppose we observe the choices described in the previous paragraph. They imply that we have $x \succ_\alpha \alpha$ and $\alpha \succ_y y$. Since preferences are changing with respect to reference points, we cannot conclude that $x \succ_y y$.[28] Therefore, we cannot compare x and y with a reference-dependent preference model without additional strong requirements.

[26]Caplin and Dean [2008] obtain a revealed preference in a similar way. They assume that an item a decision maker would choose if the search is terminated at any given time is observable and x is revealed to be preferred to y if x would be chosen at earlier time and y would be selected if she could search more.

[27]There may be more than one element between x and y.

[28]See Masatlioglu and Ok [2008] for a detail discussion.

It is clear that if she follows a CIS then her true preference $\succ$ must include $\succ_c$. The converse is also true, any preference including $\succ_c$ can explain her choice with appropriately chosen consideration sets. Therefore, $\succ_c$ is the revealed preference in our framework, which can be naturally used for welfare analysis.

Proposition 1 *(Revealed Preference) $\succ_c$ is the revealed preference whenever c is a CIS: $\succ$ represents c if and only if $\succ$ includes $\succ_c$.*

Finally, we would like to see if it is possible to infer some information about the consideration sets of a decision maker or the path she follows to reach the final decision. The first observation is that y *must* be included in the consideration set of x under feasible set S if and only if (i) y is the final choice and (ii) there is no third alternative z in S such that $y = c(S, z)$ and x is *not* revealed to be preferred to z (revealed consideration). The second observation is that y *must not* be included in the consideration set of x under S if and only if y is revealed to be preferred to her final choice, i.e. $y \succ_c c(S, x)$ (revealed inconsideration). Therefore, we can elicit very little information about consideration sets. Similarly, the inference about the actual path is hard: any choice can be explained as if it were reached in two steps with the doubleton consideration set $\{x, c(S, x)\}$ followed by the singleton set $\{c(S, x)\}$.

This is because of the huge freedom of the representation. Therefore, one may expect more inferences are possible if we know her consideration sets have a certain structure. Indeed, one can infer not only more about the consideration sets but also the actual path with context-free consideration sets, which will be discussed in the next section.

4 Context-Free Consideration Sets

In the last section, we provide the largest class of choice functions in which the stable preference assumption can be sustained in our framework. To do this, we study a general model of choice by iterative search without putting any structure on consideration sets, which depend not only on the contemplation point but also on the budget set in an arbitrary manner. The dependence of budget set is questionable especially when the decision maker has no clear perception of what is available for her. Hence, we would like to consider a special case where consideration sets are context-free. We should note that most of the examples we study in Section 2 belong to this class.

We provide several examples of context-free consideration sets.

- The set $\Omega^*(x)$ includes alternatives that have alignable attributes or overlapping features as x, for instance, comps of a house.
- Internet shopping sites provide Ω^* under "Related Items" or "Similar Items." Sometimes, they suggest several items such as "Items Customers Who Viewed This Also Viewed," that are not necessarily similar items to the one a shopper is currently contemplating but are very likely interesting to her.
- Depending on a contemplation point, some cues (or aspects) might look more important than others. Then the decision maker considers the items which dominate the current one in *these* cues. Hence $\Omega^*(x)$ excludes alternatives which incurs loss relative to x in some of these cues.
- Some binary comparisons are difficult because of either trade-off or unnoticeable difference. To avoid these difficulties, the decision maker only considers items which are easy to compare to x.

We now provide a characterization for the CIS model with context-free consideration sets. To do that, we first point out that we can infer more about decision maker's preference when consideration sets are known to be context-free. Consider a decision maker who chooses y from $\{y, z\}$ and x from S including y when z is the starting point. With context-dependent consideration sets, we cannot conclude $x \succ y$.[29] However, if her consideration set is known to be context-free, then she must have $x \succ y$.

[29]Of course, this claim is false when $y = z$. In this case, we have $x = c(S, y)$ so it must be $x \succ y$ by Proposition 1.

To see this, note that $y = c(\{y,z\},z)$ implies that she must have compared y and z to discard z and choose y, so it must be not only $y \succ z$ but also $y \in \Omega^*(z)$. Given this observation, there are only two possibilities at the decision problem (S,z): either y is the next contemplation point (y is the $\succ$-best in $\Omega^*(z) \cap S$) or not. If so, she has chosen x afterwards ($c(S,y) = x$), which immediately implies that x must be preferred to y by Proposition 1. If not, the next contemplation point, say t, must be better than y. Since x is the final choice, either x is equal to t or better than t. In either case, x is preferred to y. Hence any $\succ$ representing c must include the following order: for any distinct x and y,

$$x \rhd_c y \text{ if } x = c(S,z) \text{ and } y = c(\{y,z\},z) \text{ for some } z \neq x \ \text{ and } S \ni y.$$

Notice that this definition does not exclude the possibility that y is equal to z. Hence x is revealed to be preferred to y when $x = c(S,y)$ as in the general CIS model.

Since $\rhd_c$ is the "revealed preference" for the CIS model with context-free consideration sets, we require that this revealed preference information should be consistent with some underlying preference ordering.

A 1 (Consistency) *The revealed preference has no conflict:* $\rhd_c$ *is acyclical.*

Now consider a decision maker who is happy to stay with x under two decision problems. Then, the next axiom requires that she is willing to keep x when these two budget sets are combined.

A 2 (Expansion) *The starting point chosen from each of two sets is also chosen from their union: if* $x = c(S,x) = c(T,x)$*, then* $x = c(S \cup T, x)$*.*

It is easy to see that this is a necessary condition. $x = c(S,x) = c(T,x)$ implies that there exists no element in S or T such that it is both comparable to x and better than x. If this is the case, the decision maker sticks to her starting point under the decision problem $(S \cup T, x)$.

Now imagine that a decision maker is willing to move away from her starting point x for y not only in a binary comparison but in a larger budget set. Then it is reasonable to assume that she will continue to choose y when her budget set gets smaller as long as her starting point is still x. This is summarized by the following Sandwich Axiom.

A 3 (Sandwich Axiom) *An alternative must be chosen from any subset of S whenever it is chosen from S and against the starting point: if $c(S,x) = c(\{x,y\},x) = y$, then $c(T,x) = y$ whenever $\{x,y\} \subset T \subset S$.*

We show that this is a necessary condition. When a decision maker discards x for y when only these two items are available, we know that y is not only better than x but also comparable to x. Therefore, if she also chooses y starting with x when there are more items, then we can conclude that y is the best item in $\Omega^*(x) \cap S$ as well as in $\Omega^*(y) \cap S$. This implies that, for any smaller set T, y will be chosen as long as x is the starting point.

Finally, suppose $c(S,x) \neq x$. This means the decision maker finds a better alternative than the starting point in the course of search. Therefore, she must have abandoned the starting point and replaced it with something else, say y. In the CIS model, whether the starting point is x or y does not affect the final choice as long as the budget set is fixed. The next axiom imposes that the final choice will not be affected even when some alternatives become unavailable. We call such an alternative "replacement" and impose one additional condition. That is, a replacement should not be dominated by any item in S that dominates x. Formally, we say y is *a replacement for x in S* if (i) $c(T,x) = c(T,y)$ as long as T is a subset of S and (ii) if $z = c(\{x,z\},x)$ and $z \in S$, then $y = c(\{y,z\},y)$. Our final axiom requires the existence of a replacement whenever the starting point is discarded.

A 4 (Replacement of Dominated Anchor) *There is a replacement whenever the starting point is not chosen: if $c(S,x) \neq x$ then there exists a replacement for x in S.*

To see why this axiom is necessary. Suppose $x \neq c(S,x)$. Then, in the context-free model, a decision maker will pick the best among those that are comparable to x within S, say y.

We argue that y is indeed a replacement for x in S. Clearly, y is still the best item within x's comparable when some items becomes unavailable, so y satisfies the first requirement. Furthermore, if $z = c(\{x, z\}, x) \in S$, then z must be also comparable to x, so z must be worse than y. Therefore, the decision maker never switches from y to z, so y also satisfies the second condition. Notice that this axiom only assumes the existence of a replacement. It does not specify which alternative should be a replacement and does not even require the uniqueness of a replacement.

It turns out that these axioms, in addition to the Anchor Bias axiom, are not only necessary but also sufficient for an extended choice function to be represented by a CIS model with context-free consideration sets.

Theorem 2 *An extended choice function c satisfies Anchor Bias and A1-A4 if and only if c is a CIS with context-free consideration sets.*

Revealed Preference, Consideration Sets and Order of Search

We would like to point out that choice data produce more information about the decision maker's preference. As we explained, $\rhd_c$ must be a part of the preference of the decision maker with context-free consideration sets. Therefore, if we observe $x \rhd_c y$ or "$x \rhd_c z$ and $z \rhd_c y$" for instance, we can conclude that she prefers x over y unambiguously. It turns out the converse is true as well: we can infer x is preferred to y *only if* the transitive closure of $\rhd_c$ tells x is better than y. Therefore, it is the revealed preference when her behavior is known to be a CIS with context-free consideration sets.

Proposition 2 *(Revealed Preference) The transitive closure of $\rhd_c$ is the revealed preference if c is a CIS with context-free consideration sets.*

Now we can also provide a satisfactory revealed consideration and inconsideration from her choice behavior. To decide whether y is in the consideration set of x, first observe $c(\{x, y\}, x)$. Then we can conclude that y must be in the consideration set if and only if

$c(\{x, y\}, x) = y$. When this is not the case, we can infer that y is not considered if and only if y is revealed to be preferred to x.

Furthermore, we can now pin down uniquely the order of search the decision maker follows for each extended decision problem from her choice data. Consider a decision maker facing a budget set S and starting with x_0. Her next contemplation point can be identified in the following way. First, collect all items in S for which she is willing to abandon the starting point in a binary comparison (i.e. $y = c(\{x_0, y\}, x_0)$). Then her next contemplation point, x_1, is the choice from all of such items with x_0 being a starting point. Once we identify x_1, we can analogously identify the next point and so on recursively. Therefore, her choice data completely reveals how she reached the final choice.[30]

5 Unobservable Starting Points

Our revealed preference approach is based on the assumption that we can observe not only what the decision maker chooses from a budget set but also which alternative she initially contemplates. Nowadays, the internet makes this information accessible. For example, amazon.com offers data such as "What Do Customers Ultimately Buy After Viewing This Item?"

Nevertheless, we can still imagine a situation where we do not observe the starting point. This is because either any information other than the choices of the decision maker is ignored or not available at all. For example, if the starting point is what the decision maker expects to buy in the market, it is reasonable to assume that information about expectations is not available. Given this limited information, we investigate how to identify people who follow an underlying CIS model.

Suppose Ms. Dema actually follows a CIS model denoted by c, but we do not observe her starting point. Suppose we observe Ms. Dema chooses y from S - the only available data. Then there must exist a starting point which leads her to pick y. Of course, this starting

[30]Note that her choice is not affected by removing alternatives that are not on the identified path. In addition to our axioms, an experiment based on this prediction can also used to test this model.

point may not be unique, for example, different starting points might result in the same final choice, $y = c(S, x) = c(S, z)$. At the same time, given a fixed budget set, different final choices might be observed depending on starting points, $y = c(S, x)$ and $w = c(S, z)$. We define the choice correspondence induced by c, denoted by $C_c : K(X) \to K(X)$, as follows:

$$y \in C_c(S) \text{ if there exists } x_0 \text{ such that } y = c(S, x_0).$$

In other words, $y \in C_c(S)$ means "y is chosen from S for some starting point." This is in line with Sen [1993]: "... it may be useful to interpret $C(S)$ as the set of "choosable" elements - the alternatives that can be chosen."[31] Note that $x, y \in C_c(S)$ does not necessarily imply that x is indifferent to y in our framework: it simply means that both x and y are choosable from S depending on the starting point.

The induced choice correspondence, C_c, is what one can observe when the decision maker follows particular CIS, c, but her starting point is unobservable. In other words, C_c is the only available data to an outside observer who knows that the choices of the decision maker are affected by the starting point but does not have information on the starting points.

Now imagine that we observe a choice correspondence C. We wonder whether the decision maker actually follows some CIS or not. In other words, we would like to determine if there exists a CIS, say c, which induces the observed choice correspondence, i.e. $C = C_c$. The answer is yes if and only if her choice correspondence satisfies a simple axiom called the Bliss Point Axiom. This result makes it possible to identify decision makers following a CIS even with the limited data.

First, we point out that an induced choice correspondence may violate the Independence of Irrelevant Alternatives, which has been considered one of the weakest consistency requirements in the literature.

> Independence of Irrelevant Alternatives (IIA): *If option x is choosable from a set S, it must be choosable from any of its subsets: If $x \in C(S)$ then*

[31] Salant and Rubinstein [2008] follow this idea in defining a choice correspondence from a frame-dependent choice function.

$x \in C(T)$ *for all* $T \subset S$ *with* $x \in T$.

We now modify Example 6 in Rubinstein and Salant [2006] for our purpose. Consider a circular shopping mall as in Figure 5. Ms. Dema is committed to buy a particular commodity from one of the shops in the mall. The prices vary from shop to shop (but each shop's price is fixed) and depending on a day, some shops may be closed. She arrives at one of parking lots surrounding the mall and goes to the nearest opened shop, say shop b, to check the price. After that, she walks on the hallway clockwise to the next opened shop to compare its price with b's price. If b's price is cheaper, then she walks back to shop b and buy. Otherwise, she further investigates the next opened shop in the clockwise direction. She stops searching whenever she observes a higher price than the one in the shop she previously visited, and buys from the previous shop, which is indeed the cheapest shop among those she has checked. It is routine to check her behavior is a CIS.[32]

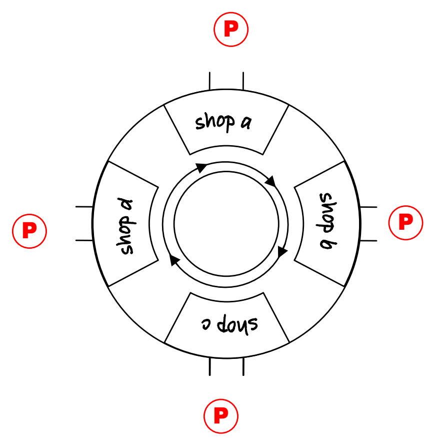

Figure 5: A Circular Shopping Mall

Suppose the prices of shops a, b, c, and d are p_a, p_b, p_c, and p_d respectively, and $p_c > p_a > p_b > p_d$. Assume that she found a parking place in front of shop b (so she visits shop b first). She will buy from shop b if shop c is open since $p_b < p_c$, that is, $b = c(\{b, c, d\}, b)$. Therefore, we have $b \in C_c(\{b, c, d\})$. However, if c is closed and d is open, she will visit shop d next and

[32]Let S be a set of currently opened shops. Her ranking $\succ$ over shops is based on their prices – cheaper is better. Her consideration set $\Omega(x, S)$ consists of shop x and the next opened shop in the clockwise direction. For example, when $S = \{a, b, d\}$, then $\Omega(b, S) = \{b, d\}$.

find out that shop d is cheaper than shop b so she will not buy from shop b, $d = c(\{b, d\}, b)$. That is, $b \in C_c(\{b, c, d\})$ and $b \notin C_c(\{b, d\})$, which is a violation of IIA.

In the example above, C_c violates IIA, as b is choosable from a bigger set $\{b, c, d\}$, but not from a smaller set $\{b, d\}$. Nevertheless, we claim that her induced choice behavior enjoys some kind of consistency in the flavor of IIA: the cheapest shop d is always choosable whenever it is open. This observation hints at our following axiom.

> The Bliss Point Axiom (BP): *For any set S, some choosable alternative from S must be always choosable from any smaller decision problem: There exists $x \in C(S)$ such that $x \in C(T)$ if $x \in T \subset S$.*[33]

We call such x a bliss point of S. By contrast, IIA dictates that *all* choosable alternatives should be choosable from any smaller choice problem (whenever they are available). Therefore, the BP axiom is weaker than IIA and provides an alternative foundation for choice theories, while still not allowing choice cycles. Similar to how the Dominating Anchor axiom prevents our decision maker from being money pumped, the Bliss Point axiom makes the decision maker immune to infinite money-pumping.

We next show that the BP axiom guarantees the existence of underlying choice by iterative search.

Theorem 3 *A choice correspondence C satisfies the Bliss Point Axiom if and only if there exists an underlying CIS that induces C.*

The power of Theorem 3 is that it connects choice patterns that are considered "irrational" in traditional choice theory to our choice by iterative search model which captures bounded rationality. It provides a very simple and intuitive postulate to identify CIS decision makers even with such limited data.

While the proof of Theorem 3 is provided in the Appendix, we illustrate that any induced choice correspondence must satisfy the Bliss Point Axiom. Note that the $\succ$-best element

[33] This axiom appears in Agaev and Aleskerov [1993] as "the fixed-point condition" to characterize interval choice models.

within S will be chosen from any subset of S whenever it is the starting point. Therefore, that element must be included in the induced choice correspondence for any subset of S. In other words, the $\succ$-best element is a bliss point of S since it can never be eliminated by any element within S.

One can provide a similar result for the CIS models with context-free consideration sets. A choice correspondence C is generated by an acyclical binary relation, P, if and only if there exists a CIS choice function with context-free consideration sets c such that $C_c = C$. To prove this, given P, one should have $x \succ y$ and $x \in \Omega^*(y)$ if xPy. For the other direction, define xPy if $x \succ y$ and $x \in \Omega^*(y)$.

Endogenous Reference Point

In this section, we show that the CIS model also provides a foundation for a general class of *endogenous* reference-dependent choice models based on the concept of the personal equilibrium proposed in Köszegi and Rabin [2006].

Köszegi and Rabin [2006] proposes a model of reference-dependence in which the reference point is determined endogenously. Given an underlying reference-dependent choice, c_{RD}, they endogenize reference point by utilizing the notion of a personal equilibrium: the expectation matches with the actual choice, i.e. $x = c_{RD}(S, x)$. In other words, a personal equilibrium is a self-fulfilling plan. Formally,

$$x \in PE_{c_{RD}}(S) \ \text{ if } x = c_{RD}(S, x)$$

where $PE_{c_{RD}}(S)$ is the set of personal equilibria for the budget set S. Therefore, any personal equilibrium is an endogenous reference point.

In their model, they use a particular reference-dependent choice model where, for a fixed reference point, $c_{RD}(\cdot, x)$ is represented by a preference. Moreover, they assume that c_{RD} satisfies the Anchor Bias axiom.[34]

[34]They define c_{RD} in terms of a reference-dependent utility $U(y|x)$ where $c_{RD}(S, x) = \{y \in S | \ U(y|x) \geq$

We keep the idea of the personal equilibrium as a mean to endogenize the reference point but use the CIS model for the underlying reference-dependent choice. As a corollary of Theorem 3, a choice correspondence satisfies the Bliss Point axiom if and only if there exists an underlying reference-dependent choice function c_{RD} which is a CIS such that

$$x \in C(S) \text{ if and only if } x = c_{RD}(S, x).$$

In other words, C is the set of personal equilibria which are induced by the reference-dependent choice function $c_{RD}(S, x)$.[35]

6 Concluding Remarks

This paper makes a contribution to an area of search theory by utilizing revealed preference techniques. We recognize that people must search for information about feasible alternatives especially in complicated decision problems. Therefore, we incorporate the idea of search into decision theory to get new insights how people make decisions. We illustrate how to infer decision maker's preference and her search process from the choice data.

To do this, we propose a new descriptive model in which a decision maker explores her budget set. The key feature of this boundedly rational model is limited search: she does not compare one option with all available alternatives, instead she proceeds through a path-dependent sequence of consideration sets until no longer wishing to change her mind. Our model provides a unified explanation for a number of seemingly irrational behavior without introducing changing preferences. In addition, we provide two clean characterizations for two nested CIS models. To infer preferences, we use a richer data set that also includes information about the starting point. As Salant and Rubinstein [2008] points out that such data should be the basis for richer economic modeling since the standard data might miss

$U(z|x)$ for all $z \in S\}$. And the anchor bias axiom is equivalent the condition: $U(y|y) > U(x|y)$ whenever $U(y|x) > U(x|x)$.

[35] Actually, Theorem 3 considers $C(S) = \bigcup_{y \in S} c_{RD}(S, y)$. Because of the Anchor Bias axiom, if c_{RD} is a CIS then C consists of the fixed points of c_{RD}. Therefore, C is equal to the set of personal equilibrium.

essential information about decision making.

Finally, we consider the case where the data about starting points is not available. Even with such limited data, a decision maker who follows a CIS model can be distinguished from one who does not by the Bliss Point Axiom, which is a slight relaxation of the well-known the Independence of Irrelevant Alternatives.

One interesting future direction is to investigate the optimal design of consideration sets. In internet commerce, consumers' consideration sets are often constructed through suggestions provided by the website. Since the effect of consideration sets are shown to be crucial, it is important for internet commerce sites to design the recommendation system so that consumers can easily find their desirable products.

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7 Appendix

7.1 Proofs of Theorem 1 and Proposition 1

The proof of the "if" part of Theorem 1 is left to readers so we shall prove its "only-if" part. Suppose c satisfies the Anchor Bias axiom and the Dominating Anchor axiom. Then, $\succ_c$ defined right before Proposition 1 is acyclic. To see this, notice that $\succ_c$ is the transitive closure of the directly revealed preference (let us denote it by P) so if $\succ_c$ has a cycle, P must have a cycle as well. If elements of S form a cycle in terms of P, this means that for every alternative $x \in S$, there exists another item $y \in S$ $\{x\}$ such that $y = c(S, x)$, which is a violation of the Dominating Anchor axiom. Hence, P cannot have a cycle, which implies that $\succ_c$ is acyclic.

Therefore, there exists a preference that includes $\succ_c$ Take one of such preferences arbitrarily, say $\succ$ and define the consideration sets as follows:

$$\Omega(x, S) = \{x, c(S, x)\}.$$

Now we show that $(\succ, \Omega)$ represents c. If $x = c(S, x)$, by construction, $\Omega(x, S)$ contains only x so the decision maker will stay with x. Suppose $c(S, x) = y \neq x$. Then $\Omega(x, S)$ is equal to $\{x, y\}$. By the construction of $\succ_c$, we have $y \succ_c x$ so it is $y \succ x$,. Therefore, y is the $\succ$-best element of $\Omega(x, S)$. This means x will be discarded and y will be the next contemplation point. Then we shall show that y will be chosen in this round. By the Anchor Bias axiom, $y = c(S, x)$ implies $y = c(S, y)$. By construction, it is $\Omega(y, S) = \{y\}$ so y is the $\succ$-best element of $\Omega(y, S)$. Therefore, $(\succ, \Omega)$ represents c.

Since $\succ$ can be any preference including $\succ_c$, this proves the "if" part of Proposition 1. The "only if" part is given in the main body. ■

7.2 Proof of Theorem 2

The proof of the "if" part is left to readers. Now suppose c satisfies Anchor Bias and A1-A4. By Consistency, $\rhd_c$ is acyclical, so we can find a strict ranking $\succ$ which includes $\rhd_c$. Define $\Omega^*(x) \equiv \{y | \, c(\{x, y\}, x) = y\}$ and note that $x \in \Omega^*(x)$ for all x in X. Then $(\succ, \Omega^*)$ generates a CIS with context-free consideration sets. Let us denote it by c^*, and we will show that $c^* = c$. First, we need to prove some claims.

Claim 1 $\Omega^*(x) \cap S = \{x\}$ if and only if $c(S, x) = x$.

Proof of Claim 1: $S \cap \Omega^*(x) = \{x\}$ implies that $x = c(\{x, y\}, x)$ for all $y \in S$. Then, by Expansion, we have $c(S, x) = x$. On the other hand, if $c(S, x) = x$ then, by Sandwich Axiom, $x = c(\{x, y\}, x)$ for all $y \in S$ since $c(\{x, x\}, x) = x$. Hence $S \cap \Omega^*(x) = \{x\}$. □

Claim 2 If $c(S, x) \neq x$ then any replacement for x in S must be in $\Omega^*(x) \setminus \{x\}$.

Proof of Claim 2: Let x^* be a replacement for x in S. If $c(S, x) \neq x$ then there exists an alternative $y \neq x$ such that $c(\{x, y\}, x) = y$ by Claim 1. Hence $c(\{x^*, y\}, x^*) = x^*$, so x^* cannot be equal to x.

By the second condition of the definition of a replacement, we have $c(\{x^*, x\}, x^*) = x^*$ since $c(\{x, x\}, x) = x$. Since we can replace x with x^* as a starting point, $c(\{x^*, x\}, x) = x^*$. Therefore $x^* \in \Omega^*(x) \setminus \{x\}$. $\square$

Claim 3 $c(S \cap \Omega^*(x), x)$ is the unique replacement for x in S.

Proof of Claim 3: If $c(S, x) = x$ then $c(S \cap \Omega^*(x), x) = x$ because $S \cap \Omega^*(x) = \{x\}$ by Claim 1. For any $T \subset S$ (including x), $T \cap \Omega^*(x) = \{x\}$ so Claim 1 implies $c(T, x) = x$ for all $T \subset S$. Moreover, the second condition of the definition of a replacement is fulfilled since $S \cap \Omega^*(x) = \{x\}$. Hence $x = c(S \cap \Omega^*(x), x)$ is the unique replacement.

Now suppose $c(S, x) \neq x$. Then RDA implies that there exists a replacement for x in S, say y. We shall show that y must be equal to $x^* \equiv c(S \cap \Omega^*(x), x)$. Assume not (i.e. $y \neq x^*$). Since y is a replacement, we have $c(\{x, y, x^*\}, x) = c(\{x, y, x^*\}, y)$. Notice that $c(\{x, x^*\}, x) = x^*$ since $x^* \in \Omega^*(x)$. Then Sandwich axiom implies that $c(\{x, y, x^*\}, x) = x^*$ since $\{x, y, x^*\} \subset S \cap \Omega^*(x)$. Therefore $c(\{x, y, x^*\}, y) \neq y$, so there must exist a replacement for y in $\{x, y, x^*\}$ by RDA. It cannot be y by Claim 2. Since $y \in \Omega^*(x)$, we have $y \rhd_c x$ and $x \notin \Omega^*(y)$, hence x cannot be replacement. Therefore, x^* is the replacement for y in $\{x, y, x^*\}$, which implies $c(\{y, x^*\}, y) = c(\{y, x^*\}, x^*)$. By Claim 2, $x^* \in \Omega^*(y)$. Therefore, $x^* = c(\{y, x^*\}, x^*) = c(\{y, x^*\}, y)$. However, it must be $c(\{y, x^*\}, y) = y$ because $c(\{x, x^*\}, x) = x^*$ ($x^* \in \Omega^*(x)$) and y is a replacement for x in S (specifically the second part). This yields a contradiction, therefore $x^* = c(S \cap \Omega^*(x), x)$ is the unique replacement for x in S. $\square$

Claim 4 *$c(S \cap \Omega^*(x), x) = c^*(S \cap \Omega^*(x), x)$ for any (S, x).*

Proof of Claim 4: If $S \cap \Omega^*(x)$ includes only x, the statement is trivial so assume that it includes some elements other than x, hence $c(S \cap \Omega^*(x), x) \neq x$ by Claim 1. Let y be the $\succ$-best element in $S \cap \Omega^*(x)$. By definition of c^*, $y = c^*(S \cap \Omega^*(x), x)$. Suppose $c(S \cap \Omega^*(x), x) = z \neq y$. Since $c(S \cap \Omega^*(x), x) \neq x$ and , Claim 3 implies z is a replacement for x in both S and $S \cap \Omega^*(x)$ (since $(S \cap \Omega^*(x)) \cap \Omega^*(x) = S \cap \Omega^*(x)$). By the definition of a replacement, $c(\{x, y, z\}, x) = c(\{x, y, z\}, z)$. We also know that $c(S \cap \Omega^*(x), z) = z$ by Anchor Bias. By Sandwich Axiom, $c(\{x, y, z\}, z) = z$, so $c(\{x, y, z\}, x) = z$. On the other hand, $y = c(\{x, y\}, x)$ since $y \in \Omega^*(x)$. These two imply $z \rhd_c y$, i.e. $z \succ y$. This contradicts that y is the $\succ$-best element within $S \cap \Omega^*(x)$. $\square$

Claim 5 *$c(S, x) = c^*(S, x)$ for any (S, x).*

Proof of Claim 5: For any S, make its partition recursively as follows:

$$I_0^S = \{x \in S |\ S \cap \Omega^*(x) = \{x\}\}$$

and

$$I_k^S = \{x \in S \backslash (I_0^S \cup \cdots \cup I_{k-1}^S) |\ \Omega^*(x) \cap S \subset \{x\} \cup I_0^S \cup \cdots \cup I_{k-1}^S\}$$

We will prove the assertion of Claim 5 by induction. Take any $x \in I_0^S$. Then we have $x = c^*(S, x)$. By Claim 1, $c(S, x) = x$ as well.

Now, assume $c = c^*$ for any $x \in I^S_{k'}$ with $k' \leq k-1$. We shall show that $c(S,x) = c^*(S,x)$ for any $x \in I^S_k$.

Since $k > 0$, $S \cap \Omega^*(x)$ includes some element other than x. Therefore, by Claim 1, it cannot be $x = c(S,x)$. Let y be the $\succ$-best element in $S \cap \Omega^*(x)$. Then by definition of c^*, we have $y = c^*(S \cap \Omega^*(x), x)$ and $c^*(S,x) = c^*(S,y)$. Then

$$\begin{aligned} c(S,x) &= c(S, c(S \cap \Omega^*(x), x)) \text{ by Claim 3} \\ &= c(S, c^*(S \cap \Omega^*(x), x)) \text{ by Claim 4} \\ &= c(S,y) \end{aligned}$$

Finally, since $x \in I^S_k$ and $y \in S \cap \Omega^*(x)$, it must be $y \in I^S_l$ for some $l < k$. Therefore, by the inductive hypothesis we have $c(S,y) = c^*(S,y)$. Therefore, $c(S,x) = c^*(S,x)$. □

By Claim 5, we have reached the required result.

Since $\succ$ can be any ranking including $\rhd_c$, this proves the "if" part of Proposition 2. The "only if" part is given in the main body. ■

7.3 Proof of Theorem 3

Suppose choice correspondence C is induced by a CIS represented by $(\succ, \Omega)$. Then the $\succ$-best element in S will be a bliss point of S. This is because for any $T \subset S$, $\Omega(x,T)$ never includes an element which is $\succ$-better than x so $x = c(T,x)$ for all $T \subset S$ (as long as $x \in T$). Thus, we have $x \in C(T)$ for any $T \subset S$ including x.

Conversely, let C satisfy the BP axiom. To construct $\succ$ from the choice correspondence, we utilize the notion of bliss points. The BP axiom guarantees the existence of a bliss point in any choice problem, particularly in X. Since bliss points of X are never eliminated, it is natural to put them on the top of the ranking. Then in the next step, we will remove all the bliss points of X from X. Now consider bliss points of this strictly smaller set. They cannot be eliminated in the absence of bliss points of X, so we put them into the second layer. Formally, we recursively define $\succ$ starting from the grand set, X and construct a partition of X. Let $X = X_0$ and

$$I_0 = \{x \in X_0 | \ x \in C(T) \text{ for all } T \text{ s.t. } x \in T \subset X_0\}.$$

Note that I_0 is the set of bliss points of X_0. BP implies that I_0 is non-empty. Define $X_1 = X_0 \setminus I_0$. If it is non-empty, define

$$I_1 = \{x \in X_1 | \ x \in C(T) \text{ for all } T \text{ s.t. } x \in T \subset X_1\}.$$

BP also implies that I_1 is non-empty. Then define recursively,

$$I_k = \{x \in X_k | \ x \in C(T) \text{ for all } T \text{ s.t. } x \in T \subset X_k\}$$

where $X_k = X_{k-1} \setminus I_{k-1}$ until $\bigcup I_k = X$. Note that $\{I_n\}$ is a partition of X, and X_n's are nested. Given the partition $\{I_n\}$, define $x \succ y$ if $x \in I_k, \ y \in I_l$ and $k < l$. Since X is finite, this recursive process will end in finite time. Since it is incomplete as of now, take any completion of it. By construction, $\succ$ is a preference. Now define

$$\Omega(x,S)=\begin{cases}\{x\} & \text{if } x\in C(S)\\ \{x,y_{x,S}\} & \text{otherwise}\end{cases}$$

where $y_{x,S}$ is an arbitrary element in S with $y_{x,S}\succ x$. We need to prove that such an element exists whenever $x\notin C(S)$. Suppose such an element does not exist and $x\in I_k$. Then $S\subset X_k$ by definition of $\succ$. Since x is a bliss point of X_k, it must be $x\in C(S)$, which is a contradiction. Therefore, Ω is well defined.

Let c be a CIS represented by $(\succ,\Omega)$. We need to show that c induces C. If $x\in C(S)$ then $x=c(S,x)$ because $\Omega(x,S)$ is a singleton. By the definition of C_c, $x\in C_c(S)$. On the other hand, If $x\notin C(S)$ then $x\neq c(S,x)$ because $\Omega(x,S)$ contains an element which is $\succ$-better than x. By the contrapositive of the Anchor Bias axiom, $x\neq c(S,y)$ for any $y\in S$. Therefore, $x\notin C_c(S)$. ■