

STATS 413:
Efficient Markets

US Treasury Bills

- A treasury bill is a short-term loan to the US government.
 - The government sells T-Bills to raise money for various projects.
 - A T-Bill is an IOU where the government promises to pay the bill holder a specified amount (the face value) on a specified date (the date of maturity).
 - For simplicity, we will only consider T-Bills that mature monthly.

US Treasury Bills

- Investors make money by buying T-Bills at a discount (a price less than the face value) and then holding it until it reaches maturity.
 - Eg. Buy a T-Bill at the beginning of the month for \$990 and receive the face value of \$1000 at the end of the month.
- These are low-risk investments, but they also have low returns.

Notation

- v_t is the price of the T-Bill at the start of month t (\leq face value).
- R_t is the nominal rate of return on the T-Bill over month t (unadjusted for inflation).
- P_t is the Consumer Price Index (CPI) at the start of month t .
 - CPI tracks changes in the price of a collection of consumer goods to measure inflation.

Notation

- π_{t+1} is the inflation rate over month t :

$$\pi_{t+1} = \frac{\mathbf{P}_{t+1} - \mathbf{P}_t}{\mathbf{P}_t}.$$

- $\widehat{\pi}_{t+1}$ is the forecast of the inflation rate over month t .
- ϵ_{t+1} is the forecast error: $\pi_{t+1} - \widehat{\pi}_{t+1}$.

Notation

– \mathbf{r}_{t+1} is the real rate of return over month t :

$$\mathbf{r}_{t+1} = \frac{\frac{1}{\mathbf{P}_{t+1}} - \frac{\mathbf{v}_t}{\mathbf{P}_t}}{\frac{\mathbf{v}_t}{\mathbf{P}_t}} = \frac{1 + \mathbf{R}_t}{1 + \pi_{t+1}} - 1 \approx \mathbf{R}_t - \pi_{t+1}.$$

– $\hat{\mathbf{r}}_{t+1}$ is the forecast of the real rate of return:

$$\hat{\mathbf{r}}_{t+1} = \frac{1 + \mathbf{R}_t}{1 + \hat{\pi}_{t+1}} - 1 \approx \mathbf{R}_t - \hat{\pi}_{t+1}.$$

Notation

- The subscript indicates at the start of which month the variable is known.
 - R_t only depends on the price at the start of month t and the face value – known at the start of month t .
 - r_{t+1} depends on CPI at the start of month t and at the start of month $t + 1$ – not known until the start of month $t + 1$.

Efficient Market Hypothesis

- **Rational expectations:**

- The forecast of the inflation rate uses all available information known at the start of month t (I_t).

$$\hat{\pi}_{t+1} = \mathbb{E}[\pi_{t+1} | I_t], \quad I_t := \{\mathbf{R}_t, \mathbf{R}_{t-1}, \dots, \pi_t, \pi_{t-1}, \dots\},$$

- The conditional expectation gives a predictor with the minimum MSE depending only on I_t .

- **Constant real rate of return forecasts:**

$$\hat{\mathbf{r}}_{t+1} = r' \text{ for all } t$$

Efficient Market Hypothesis

- **Goal:**
 - We want to test the efficient market hypothesis.
 - To do this, we will derive a simple, testable relationship under the efficient market hypothesis, which we will test (later) with a linear model.

Efficient Market Hypothesis

- **Conclusion:**
 - There is a linear relationship (with slope = 1) between the interest rate and the nominal rate of return.
 - We have data on both these variables, so we can test if the data support this linear relationship.