A Regulated Oligopoly Multi-Market Model for Trading Smart Data

Shang-Pin Sheng, Yang Liu, Mingyan Liu
Electrical Engineering and Computer Science
University of Michigan, Ann Arbor
{shangpin, youngliu, mingyan}@umich.edu

***Abstract*—We present a price competition model for the trading of data products. Standard results suggest that under full competition the equilibrium only exists when all sellers have zero profit. We introduce a regulator which can also be thought of as the sellers forming a coalition/association, whose role is to enable money transfer based on partial observations of the sellers' actions. We show that by proper design of the transfer mechanism, efficient equilibrium (profit-maximizing or profit-positive) can be achieved, thereby providing an alternative to other competition-voiding mechanisms such as product differentiation and market split.**

I. Introduction

With the emergence of a myriad of data services and products, pricing of data products has also become more important then ever, as it can significantly influence demand, usage, quality of service, and ultimately the profitability and viability of businesses offering these data services. In this paper we focus on the competitive relationship among multiple providers/sellers of different data services/products, each catering to a different type of buyer/consumer, with a goal of extracting profit.

When considering competition in markets, the often used models are the Bertrand and Cournot competition models [1], [2]. The Bertrand model shows that with just two sellers, the market reaches perfect competition and both sellers sell at the marginal price. Specifically, it assumes that competing firms produce a homogeneous product; thus products from different firms are interchangeable, the result being that customers will always purchase from the firm that sets the lowest price. It follows that the only equilibrium point is when all firms set their price at the unit cost of production. Under the Cournot model, firms compete by choosing the amount of output they produce. Although they can choose their production quantity at will, the total amount all firms produce affects the market price of that product, the result being that the price approaches the marginal price as the number of sellers increases.

In reality we do not often see perfect competition where firms sell at marginal prices. Modification of these models thus typically aim to reflect the real market. For example, Bertrand-Edgeworth model [3] assumes a production limit of firms in the Bertrand model. Various other factors can also be incorporated to avoid perfect competition such as product differentiation, transport and search costs. Firms can also avoid competing with each other by colluding/side contracting, which has often been shown to improve the outcome, i.e., the sellers' collective profit. Examples include Coase [4], which showed that bargaining leads to an efficient (profit-maximizing) outcome in a trade with fully symmetric information and no transaction cost, Jackson [5], which considered a two stage game whereby firms first agree on utility transfers that effectively rewrite the payoff functions, and then play the price competition game in the second stage, and Ferreia[6], which considered cross-ownership as a form of side contracting. Other examples can be found in [7], [8], [9].

Following this line of thought, in this paper we consider the setting where multiple data service providers (also referred to as sellers) compete with each other over multiple data services/products (referred to as products or data products below). This market in its unregulated form is inefficient: all sellers will sell at marginal prices as discussed above. We introduce a regulator who coordinates the sellers so that they avoid competition by each focusing on different products. This may be equivalently viewed as forming an alliance/association of sellers who agree to abide by certain rules without violating privacy and individual rationality. Specifically, we propose a money transfer scheme where the seller transfers part of its profit each time it completes a transaction to other sellers, resulting in a profit sharing mechanism. The regulator is not assumed to fully observe the sellers' strategies (the details of the transactions that have occurred such as the price, the amount or duration of data sold, etc.), but only assumed to know each time that a transaction has occurred. In other words, its role is to simply to register/certify each transaction and facilitate the money transfer that follows. Under this model, we will discuss the conditions under which such a money exchange scheme could enforce efficient equilibria, i.e., profit maximizing or ensuring positive profit. We will specify the equilibrium region for the case of two sellers and extend to the more general case of multiple sellers. In the first case we also identify the fairness region (in terms of profit sharing) within the equilibrium region.

The above model provides an alternative to current practices by service providers to avoid direct competition, which tend to heavily rely on product differentiation, sometimes at the detriment of consumers because excessive differentiation makes choices difficult, and difficult choices can lead to poor decisions. Consider for instance the variety of smart phone plans being offered; each typically involves a very large number of parameters: a consumer has to decide the amount of data she needs, whether voice should be purchased per call or at a fixed price up to a certain limit, or whether it should be purchased per call during off peak time and unlimited otherwise; similar considerations are needed for text messages, the right contract terms combined with discount on the device, and so on. When a consumer shops for a service plan it is very hard to directly compare plans offered by different operators, which is by design because direct comparison hurts profit and can lead to price competition. This decision is made even harder when the consumer is simultaneously shopping for a device/phone because the additional possible combinations of service/contract plans and device offerings. In principle the finer the differentiation the more likely a consumer can find one best suited to her needs, but in practice this can be very challenging to achieve.

In this scenario, the sellers/operators are seeking to maximize profit but do so by making products (look) different/unique so as to essentially become the sole/monopoly provider/seller of each unique product. This paper takes a different view and presents an alternative to (excessive) product differentiation. Under our scheme, product differentiation is replaced by profit sharing through a regulator, whose existence essentially separates the products offered by each seller (i.e., at the positive-profit equilibrium, each seller is offering product(s) different from the others'). In other words, instead of trying to add differentiation, we show that the sellers can be made to differentiate along existing set of services/products.

Due to the generality of our model and abstraction, a "product" considered in this paper is not limited to data services and products. Thus this scheme is not limited to selling smart data plans, but its benefit is more pronounced when either product differentiation is hard to achieve, or differentiation becomes excessive and burdensome for consumers. This is the case in some geographic regions with multiple simultaneous service providers and intense competition, which leads to difficult decision-making and less than desirable decisions; this ultimately goes against the consumer's utility. It should also be mentioned that our scheme can be used to regulate only a subset of the market, e.g., providers can agree to be regulated and profit-share in some main categories of services, but free to compete or service differentiate in other categories.

This paper does not take social welfare as an explicit objective in evaluating the proposed scheme, but rather only the sellers' profit. This was done intentionally, the idea being that someone selling for profit (even if maximum profit) something useful is still better than sellers not making a profit thus having to exit the market; as long as buyers find the products useful the former situation poses higher social welfare than does the latter. In this sense the social welfare, while not the stated goal, is a positive by-product. It must be admitted that given that the market exists, it is important to further consider how to design additional mechanisms so that social welfare is maximized. How to do so within the proposed framework is further discussed in subsequent sections.

There is an interesting connection between our model and the class of coalition games, see e.g., [10] in the context of collaborative spectrum sensing that showed that through coalition, secondary users can greatly reduce the average miss probability. Under our model, the presence of the regulator may be viewed as forcing a coalition, though ours is a non-cooperative game while coalition games belong to the family of cooperative games. Moreover, since any kind of competition will result in zero profit for all sellers, there is no other efficient equilibrium other than the grand coalition in our context.

In the remainder of this paper we introduce the model for the market in Section II and show how an efficient equilibrium with fairness can be achieved with two sellers in Section III, and with multiple sellers in Section IV.

II. Model

Our model is similar to the Bertrand model but extended to multiple products catered to different buyer types with different products needs. Under the assumption that sellers all have sufficient supply, similar to the Bertrand model the result is price competition and the only equilibrium point is when all sellers sell at their marginal prices. In order to move the equilibrium to a more efficient (profitable) point, we introduce a regulator who can force money transfers among sellers. This transfer is only based on the occurrence of each transaction but not on the details of the transaction; thus the resulting game is one of partial information.

A. Sellers and Buyers

There are K sellers, each with sufficient resource to supply all products if they want to. We assume there is a set of N products that effectively partitions the market, each catering to a different type of buyer, giving rise to N distinct buyer types. This set need not be unique; for instance, the market can be partitioned by service plans with clear differentiation in the type of data product they offer, e.g., plans that offer a large amount of text messages but strict limitation on voice calls and vice versa. It can also be partitioned by service plans with clear differentiation in the duration of the contracts, e.g., month-to-month vs. one-year minimum, and so on. One can also partition the market by certain combination of features like these.

Specifically, given these and only these N choices, a buyer of type i will always choose to buy from the seller who offers the lowest price for product i given that it is below some M_i. This amount M_i reflects type i's price tolerance/upper limit, beyond which the buyer will simply walk away. M_i can also be viewed as the monopoly price for if there is only one seller, then the optimal price for the seller would be M_i. There is a cost c_i for each product i sold, the monopoly profit from buyer type i is defined as $\Theta_i = M_i - c_i$. Let r_i denote the number of buyers of type i among the buyer population. Equivalently r_i can also be the probability of a randomly arriving buyer being of type i; this will not affect our analysis. Note that the values M_i and r_i are market information assumed known to the seller prior to entering the market (this would be part of the market research done by the seller mentioned earlier).

Given any such set of N products that partitions the market, there may also be other products that cater to multiple buyer types induced by such a partition, but less profitable. These will be referred to as "secondary" products. Consider for instance the earlier example on texting vs. voice calls: some segment of the population uses predominately one or the other. Suppose product 1 is a service plan very favorable in terms of voice calls (unlimited, etc.) but expensive for texting; product 2 is the opposite and favorable to texting users. When only these two products are offered they partition the market into two types of buyers. Now consider the introduction of a product 3 that offers a combination: it is cheaper overall but offers less than 1 and 2 do in each category respectively. So product 3 may attract both types but be the least profitable (per customer): since it is less useful to either type compared to product 1 (or 2), the price they are willing to pay is less. Nevertheless, such a product may have higher total profit because it attracts a larger number of customers.

A seller's strategy consists of a set of products it chooses to sell and the prices to sell them at. If a product i is sold at price p_i, then its profit is $p_i - c_i$ where c_i is the unit cost of the i-th product. Our goal is to design a mechanism such that the induced game for the sellers has an efficient equilibrium where they collectively stay with the N primary products and charge monopoly prices at the same time. Throughout the paper we will also often refer to a particular buyer type as a distinct "market" featured with a distinct product, whenever there is no ambiguity. This should not be confused with the more generic use of the word "market" as in smart data market.

B. Regulator

We define a third party in the sellers' game, referred to as the regulator. This regulator need not be imposed by entities outside the group of sellers; it could be self-imposed by an alliance or coalition of sellers sharing the common goal of profit maximization, i.e., it can also be viewed as a collusion among the sellers. The regulator can enforce money transfer based on partial information of the actions of the sellers. Specifically, the regulator observes a signal each time a transaction takes place (a buyer completing the purchasing of a product from a seller). This signal contains no information of which product was sold and what price it was sold at. The money transfer takes the following form. When seller k sells a product, he has to give another seller l an amount t_{kl} (e.g., in dollars), for each $l \neq k$. This amount t_{kl} is a real nonnegative number. Note that the regulator's role is simply to transfer, but not to make profit. In essence, the introduction of such a regulator facilitates profit-sharing, which in turn helps sustain an efficient equilibrium.

C. Efficiency

The intention in introducing the regulator is to force the sellers to avoid competition and attain higher profits. In this context, efficiency is measured by the total profit of all sellers. Accordingly, at an efficient equilibrium the price that a buyer pays for is the same as one commanded in a monopoly market. Thus, an efficient equilibrium in our formulation maximizes the total profit of all sellers. As we discuss later, however, the key to the mechanism is profit-sharing, but not necessarily maximum profit-sharing. With additional conditions imposed on the regulator the mechanism could also achieve solutions with higher social welfare and not just net profit.

III. 2 SELLERS, 2 BUYER TYPES

A. Unregulated

We begin with a simple scenario with only 2 sellers and 2 primary products, which partitions the market into two buyer types, of population r_1, r_2, and monopoly profit Θ_1, Θ_2, respectively. As already discussed in relation to the Bertrand model, under perfect competition, the market will not exist with both sellers driven to selling for zero profit. We next show that with some constraint on the sellers' strategy space we can achieve efficient (positive profit) equilibria but the problem space for this to happen is highly limited.

Specifically, assume that each seller can only sell one of the products, and let seller 1 (2) be assigned to take product 1 (2). Let's also include a third (secondary) product that attracts both buyer types with less profit Θ_3, i.e., $(r_1 + r_2)\Theta_3 < r_1\Theta_1 + r_2\Theta_2$. In this case, deviating from the assigned market/product is not profitable for seller 1 if,

$$r_1\Theta_1 \geq r_2\Theta_2 \tag{1}$$
$$r_1\Theta_1 \geq (r_1 + r_2)\Theta_3 \ . \tag{2}$$

Equation (1) is for seller 1 to not take the market of seller 2, Eqn. (2) is for seller 1 to not choose to acquire both types of buyers by the third product. Here deviation only refers to a seller moving into a product market not "assigned" to it (or its equilibrium market); it does not refer to deviating from the monopoly pricing. This is because we already know that deviation from the monopoly pricing will lead to zero profit and collapse of the market. Similarly,

$$r_2\Theta_2 \geq r_1\Theta_1 \tag{3}$$
$$r_2\Theta_2 \geq (r_1 + r_2)\Theta_3 \tag{4}$$

would ensure seller 2 does not deviate. We don't have to consider other cases because if these conditions are satisfied, any price lower will not be beneficial to offer. For these 4 equations to be satisfied, the set of parameters must satisfy the following condition:

$$r_1\Theta_1 = r_2\Theta_2 \geq (r_1 + r_2)\Theta_3 \ . \tag{5}$$

If we assume $r_1 = r_2 = 0.5$, $\Theta_3 = 1$ and plot it on the Θ_1-Θ_2 plane, then the only values for Θ_1, Θ_2 that satisfy this condition lie on the 45 degrees line starting from $\Theta_1 = \Theta_2 = 2$. This will also be referred to as the *stability or stable region* of these parameters. This example suggests that when each seller is limited to selling only one product, it is possible for the market to exist whereby the sellers make non-zero profit. However, such existence depends on very restrictive selections of the problem parameters, e.g., a line out of a 2D plan in this example. In other words, it is all but impossible for sellers to not compete, or to make a profit, in an unregulated environment. Note that if the third, secondary product doesn't exist, then the conditions (2), (4) and the RHS of (5) simply go away and this doesn't alter the basic conclusion reached here, which is that positive-profit equilibrium only exists under very limited parameter choices.

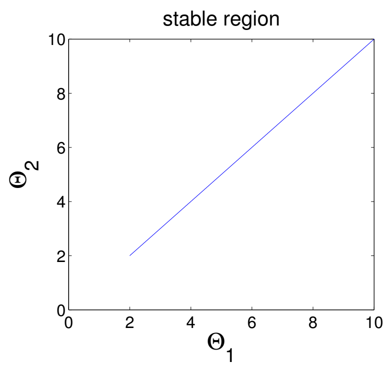

Fig. 1. Stable region without regulation

B. With Regulation

Suppose that a seller, upon each completed sale, pays a certain amount of money to its rival. Let's denote by t_1 (t_2) the money given from seller 1 to 2 (2 to 1) when seller 1 (2) sells. Then the incentive compatibility condition for staying with its assigned market/product is as follows for seller 1:

$$r_1(\Theta_1 - t_1) + r_2 t_2 \geq r_2(\Theta_2 - t_1) \quad (6)$$
$$r_1(\Theta_1 - t_1) + r_2 t_2 \geq r_1(\Theta_1 - t_1) + r_2(\Theta_2 - t_1) \quad (7)$$
$$r_1(\Theta_1 - t_1) + r_2 t_2 \geq (r_1 + r_2)(\Theta_3 - t_1) \quad (8)$$
$$r_1(\Theta_1 - t_1) + r_2 t_2 \geq r_2 t_2 \quad (9)$$

and for seller 2:

$$r_2(\Theta_2 - t_2) + r_1 t_1 \geq r_1(\Theta_1 - t_2) \quad (10)$$
$$r_2(\Theta_2 - t_2) + r_1 t_1 \geq r_1(\Theta_1 - t_2) + r_2(\Theta_2 - t_2) \quad (11)$$
$$r_2(\Theta_2 - t_2) + r_1 t_1 \geq (r_1 + r_2)(\Theta_3 - t_2) \quad (12)$$
$$r_2(\Theta_2 - t_2) + r_1 t_1 \geq r_1 t_1 \quad (13)$$

Note that we removed the constraint that each seller can only sell one product. Thus, there are 4 different scenarios for each seller: (i) to switch to the other seller's market (Eqn. (6)); (ii) to take the other seller's market (Eqn. (7)); (iii) to switch to the third product (Eqn. (8)); and (4) to give up its own market and just receive money from the other seller (Eqn. (9)). We want to show that by choosing appropriate t_1 and t_2, we can make staying with the assigned market the best strategy of both sellers. Consider the extreme case where $t_1 = \Theta_1$ and $t_2 = \Theta_2$, then Eqns. (6), (7), (10) and (11) are satisfied.

$$r_1(\Theta_1 - \Theta_1) + r_2\Theta_2 \geq r_2(\Theta_2 - \Theta_1) \quad (14)$$
$$r_2(\Theta_2 - \Theta_2) + r_1\Theta_1 \geq r_1(\Theta_1 - \Theta_2) \quad (15)$$

Also, note that Eqns. (8) and (12) can be rearranged as follows where maximizing t_1 and t_2 makes the inequality the least binding/restrictive.

$$r_2(t_1 + t_2) \geq (r_1 + r_2)\Theta_3 - r_1\Theta_1 \quad (16)$$
$$r_1(t_1 + t_2) \geq (r_1 + r_2)\Theta_3 - r_2\Theta_1 \quad (17)$$

Θ_1 and Θ_2 are the largest values t_1 and t_2 can be. This is because they would rather not give any contract if $t_i > \Theta_i$ and Eqns. (9) and (13) will not be satisfied. This means that if setting $t_1 = \Theta_1, t_2 = \Theta_2$ cannot allow all equations be satisfied, any other values of t_1, t_2 cannot allow the equations be satisfied. By setting $t_1 = \Theta_1, t_2 = \Theta_2$, we know that the following equations are the conditions to check whether it is possible to have any money transfer to cause both sellers to follow the assignment:

$$r_1\Theta_1 - (r_1 + r_2)\Theta_3 + r_2(\Theta_1 + \Theta_2) \geq 0 \quad (18)$$
$$r_2\Theta_2 - (r_1 + r_2)\Theta_3 + r_1(\Theta_1 + \Theta_2) \geq 0 \quad (19)$$

Solving for these two inequalities we have,

$$\Theta_2 \geq max(\Theta_3 - \frac{r_1}{r_1 + r_2}\Theta_1, \frac{r_1 + r_2}{r_2}(\Theta_3 - \Theta_1)) \quad (20)$$

Taking the same example as in the previous subsection, $r_1 = r_2 = 0.5$ and $\Theta_3 = 1$, we get $\Theta_2 \geq max(1 - 0.5\Theta_1, 2(1 - \Theta_1))$ as shown in Fig. 2. The stable region now contains all points above both lines.

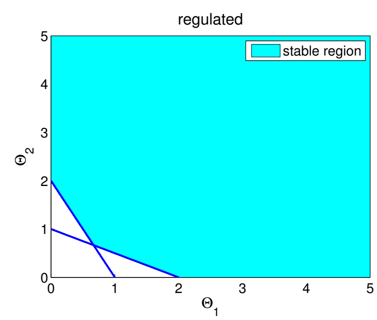

Fig. 2. Stable region under regulation

Comparing Figures 1 and 2, we observe that the stable region expanded from a line to a plane, not to mention the elimination of the one-product constraint. In Figure 1, only the Θ values exactly on the line were possible for the market to exist with positive profit, significantly limiting the type of products the market can profitably sustain. By contrast, Figure 2 suggests that for a majority of the products there exists a transfer mechanism that can separate the markets between the sellers and enable positive and indeed, maximum profit.

C. Fairness

We have shown that the stable region can be expanded by introducing money exchange without knowing what the sellers actually did. However, we have not specified any constraints on the resulting profit share. It is conceivable that sellers will only agree to this money transfer scheme if the profit earned is fair in some sense. In what follows we consider not only the stability region but also the region where fairness is achieved. Without loss of generality, we will assume $r_1 \geq r_2$. Let's consider the additional fairness condition where both sellers obtain the same profit under the money transfer t_1, t_2:

$$r_1(\Theta_1 - t_1) + r_2 t_2 = r_2(\Theta_2 - t_2) + r_1 t_1 \ . \tag{21}$$

Similar as before, if we maximize both t_1 and t_2 then Eqns. (6-13) become less restrictive. Previously the maximums were Θ_1, Θ_2; however, now we cannot simply use the maximum because t_1 and t_2 are coupled. We consider 2 cases.

1) $\Theta_2 \leq \frac{r_1}{r_2}\Theta_1$: In this case, if we set $t_2 = \Theta_2$, then $t_1 = \Theta_1/2 + \frac{r_2}{2r_1}\Theta_2 \leq \Theta_1$ from Eqn. (21). Since $t_1 \leq \Theta_1$, this is a valid choice that maximizes t_1 and t_2. Using Eqns. (6) and (10), the following has to be satisfied:

$$\frac{1 + r_1/r_2}{1 - r_2/r_1}\Theta_1 \geq \Theta_2 \geq \frac{1}{(2 + r_1/r_2)}\Theta_1 \ . \tag{22}$$

Because we assumed $r_1 \geq r_2$, we have $\frac{1+r_1/r_2}{1-r_2/r_1}\Theta_1 \geq \frac{r_1}{r_2}\Theta_1$. The region is thus given by the following:

$$\frac{r_1}{r_2}\Theta_1 \geq \Theta_2 \geq \frac{1}{(2 + r_1/r_2)}\Theta_1 \ . \tag{23}$$

2) $\Theta_2 \geq \frac{r_1}{r_2}\Theta_1$: Similarly we set $t_1 = \Theta_1$ and obtain $t_2 = \Theta_2/2 + \frac{r_1}{2r_2}\Theta_1 \leq \Theta_2$ from Eqn. (21). Since $t_2 \leq \Theta_2$, this is a valid choice that maximizes t_1 and t_2. Using Eqns. (6) and (10), we find the following condition:

$$(2 + r_1/r_2)\Theta_1 \geq \Theta_2 \geq \frac{1 - r_1/r_2}{1 + r_2/r_1}\Theta_1 \tag{24}$$

Because $\frac{1-r_1/r_2}{1+r_2/r_1} \leq 0$, the right hand side of the inequality is always satisfied. We can conclude that fairness can be achieved in the region

$$(2 + r_1/r_2)\Theta_1 \geq \Theta_2 \geq \frac{r_1}{r_2}\Theta_1 \ . \tag{25}$$

Combining Eqns. (23) and (25), we conclude that the region where fairness is achievable is,

$$(2 + r_1/r_2)\Theta_1 \geq \Theta_2 \geq \frac{1}{(2 + r_1/r_2)}\Theta_1 \ . \tag{26}$$

Next consider the condition given by Eqns. (8) and (12).

1) $\Theta_2 \leq \frac{r_1}{r_2}\Theta_1$: Let $t_1 = \Theta_1/2 + \frac{r_2}{2r_1}\Theta_2$, $t_2 = \Theta_2$,

$$\Theta_2 \geq \frac{(r_1 + r_2)\Theta_3 - r_1\Theta_1/2}{3r_2/2 + r_1} \tag{27}$$

$$\Theta_2 \geq \frac{(r_1 + r_2)\Theta_3 - (r_1 + r_2/2)\Theta_1}{r_2 + r_2^2/(2r_1)} \tag{28}$$

2) $\Theta_2 \geq \frac{r_1}{r_2}\Theta_1$: Let $t_1 = \Theta_1$, $t_2 = \Theta_2/2 + \frac{r_1}{2r_2}\Theta_1$,

$$\Theta_2 \geq \frac{(r_1 + r_2)\Theta_3 - (3r_1/2 + r_2)\Theta_1}{r_2/2} \tag{29}$$

$$\Theta_2 \geq \frac{(r_1 + r_2)\Theta_3 - (r_1 + r_1^2/(2r_2)\Theta_1}{r_2 + r_1/2} \tag{30}$$

Eqns. (26) and (27-30) characterize the entire region where fairness is achievable by at least one (t_1,t_2) pair that also

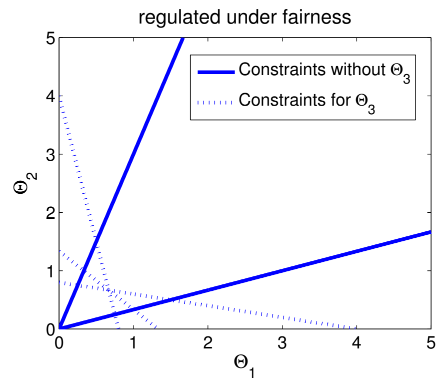

Fig. 3. Fairness region

results in an efficient equilibrium. Using the same example $r_1 = r_2$ and $\Theta_3 = 1$, we plot the region in Figure 3. Here the solid lines correspond to Eqn. (26). Between the two solid lines is the area where both fairness and efficiency can be achieved through money transfer. The dashed lines correspond to Eqns. (27-30). If there exists the third product Θ_3, then the Θ_1 and Θ_2 values have to be above these lines.

In Fig. 4 we further show how the regions compare between efficient equilibrium and fair and efficient equilibrium.

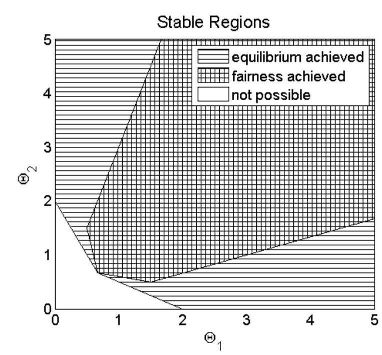

Fig. 4. Comparing different regions

D. Discussion

The above shows that an efficient market can exist by introducing a money transfer scheme, which provides the incentive for a seller not to steal the rival's market. The transfer amount can be chosen to split profit fairly (as shown above) or any amount agreed upon by the sellers, as long as it allows equilibrium to exist. We have shown that the stable/equilibrium region is maximized when we maximize the money transfer. For given problem parameters, if they fall within this region, then the problem can be easily reversed to find regions of choices for the desired transfer amount.

Compared to other competition-avoiding schemes such as market split or price fixing, the above mechanism is more flexible and can achieve equilibrium where the latter could fail. Consider for instance two products where one happens to be (much) more profitable than the other. In this case direct split between two sellers would not be stable, while splitting the buyer/consumer population could be hard to achieve. Similarly, price fixing does not necessarily lead to profit-sharing (two sellers both selling at monopoly price may get different shares of the market) so may not be as sustainable as the regulated scheme suggested above.

In addition, if the regulator is a neutral third party and has real regulatory power, it can also suggest less-than-monopoly prices ($< M_i$) for those willing to participate in the transfer, the idea being that without regulation there is zero profit so positive profit incentivizes participation; but the profit need not be maximum. The assumption of monopoly pricing means the regulator does not need to have the power to enforce certain pricing; it only certifies/authenticates transactions but does not need to know the details of a transaction. However, if we are willing to allow the regulator more power, then it can also facilitate social welfare maximization through better pricing.

IV. Extensions

We now discuss some extensions of our results to the more complicated scenario of multiple sellers and multiple buyer types. We again assume that in this case the sellers have agreed to divide up the products/buyer types so that each sells to a distinct and disjoint subset. We shall examine the conditions under which they will not deviate from this assignment.

Specifically, let the sellers be indexed by $k = 1, ..., K$, each assigned with N_k buyer types indexed by $k_i \in \{k_1, k_2, ..., k_{N_k}\}$. With regulation, a seller is forced to transfer t_{kl} to seller l if she completed a transaction. Accordingly, the profit of seller k can be written as follows:

$$\sum_{i=1}^{N_k} r_{k_i}(\Theta_{k_i} - \sum_{l\neq k} t_{kl}) + \sum_{l\neq k}\sum_{i=1}^{N_l} r_{l_i} t_{lk} \ . \tag{31}$$

The following is a key result that helps identify the conditions under which an equilibrium exists. The proof is omitted for brevity.

Lemma 1. *A seller will follow the market assignment if and only if it is neither valuable to drop one of her own products nor valuable to add one of her rival's products to her own set.*

Based on Lemma 1 we only need to consider two types of deviation, i.e., adding or removing a product to/from one's assigned set. For seller k not to deviate, we must have:

$$\Theta_{k_i} - \sum_{l\neq k} t_{kl} \geq 0, \quad \forall i = 1, ..., N_k \tag{32}$$

$$t_{lk} \geq \Theta_{l_i} - \sum_{m\neq k} t_{km}, \quad \forall l \neq k, \forall i = 1, ..., N_l \tag{33}$$

The most restrictive condition in the first equation is given by the smallest Θ_{k_i}. Define $\underline{\Theta}_k = \min_{i=1,...,N_k} \Theta_{k_i}$ and observe that $\sum_{l\neq k} t_{kl} \leq \underline{\Theta}_k$ is a necessary condition. Substituting the maximum $\underline{\Theta}_k$ into the second equation, we have

$$t_{lk} \geq \Theta_{l_i} - \underline{\Theta}_k, \quad \forall l \neq k, \forall i = 1, ..., N_l \ . \tag{34}$$

Similarly, t_{lk} has to be at least $\overline{\Theta}_l - \underline{\Theta}_k$ for Eqn. (33) to be satisfied for all i, where we have used a similar definition $\overline{\Theta}_l = \max_{i=1,...,N_l} \Theta_{l_i}$. Re-checking the first equation, we have

$$\Theta_{k_i} - \sum_{l\neq k}(\overline{\Theta}_k - \underline{\Theta}_l) \geq 0, \quad \forall i = 1, ..., N_k \ . \tag{35}$$

Rearranging the above equation and accounting for all $i = 1, ..., N_k$, we conclude that,

$$\underline{\Theta}_k \geq \sum_{l\neq k}(\overline{\Theta}_k - \underline{\Theta}_l), \quad \forall k \tag{36}$$

is a necessary and sufficient condition such that there exist money transfers to ensure an efficient equilibrium corresponding to the market assignment. The money transfer from seller k to l, $t_{lk} = \overline{\Theta}_l - \underline{\Theta}_k$ will maximize the stable region. By condition Eqn (36) above, we notice a few factors that affect whether a stable money transfer is possible: (i) large $\underline{\Theta}_k$ is better; (ii) small $\overline{\Theta}_k$ is better; (iii) fewer number of sellers K is better. Combing (i) and (ii) we see that the it would be desirable for most of the products to have profits close to each other and the products assigned to each seller to have similar profits.

V. Conclusion

In this paper we introduce a pricing competition model that can be used in trading smart data products. We first show that the market will result in full competition where equilibrium only exists when all sellers have zero profit. We then introduce a regulator who can facilitate a set of money transfer based on partial observations of the sellers actions. We show that by the introduction of this regulator, we can induce the market to have efficient (profit maximizing) equilibria. The conditions for designing a stable money transfer were characterized for cases of two-seller and multiple-seller cases, and how to achieve fair profit share is also discussed.

References

[1] Joseph Bertrand. Review of Walras's Théorie Mathématique de la Richesse Sociale and Cournot's Recherches sur les Principes Mathématiques de la Théorie des Richesses. *J. des Savants*, 1883.

[2] Antoine-Augustin Cournot. Recherches sur les principes mathématiques de la théorie des richesses. 1838.

[3] Martin Shubik. Strategy and market structure. *The Yale Law Journal*, 1959.

[4] Ronald Harry Coase. The problem of social cost. *The Journal of Law and Economics*, 3:1–44, 1960.

[5] Matthew O. Jackson and Simon Wilkie. Endogenous games and mechanisms: Side payments among players. *The Review of Economic Studies*, 72(2):543–566, 2005.

[6] José Luis Ferreira and Roberts Waddle. Strategic profit sharing between firms. *International Journal of Economic Theory*, 6(4):341–354, 2010.

[7] Felix R. FitzRoy and Kornelius Kraft. Profitability and profit-sharing. *The Journal of Industrial Economics*, 35(2):113–130, 1986.

[8] Joseph Farrell and Carl Shapiro. Asset ownership and market structure in oligopoly. *The RAND Journal of Economics*, 21(2):275–292, 1990.

[9] David A. Malueg. Collusive behavior and partial ownership of rivals. *International Journal of Industrial Organization*, 10(1):27–34, 1992.

[10] Walid Saad, Zhu Han, Tamer Basar, Mérouane Debbah, and Are Hjorungnes. Coalition formation games for collaborative spectrum sensing. *Vehicular Technology, IEEE Transactions on*, 60(1):276–297, 2011.