



## Product variety and manufacturing complexity in assembly systems and supply chains

S.J. Hu (2)\*, X. Zhu, H. Wang, Y. Koren (1)

Department of Mechanical Engineering, The University of Michigan, 2350 Hayward, Ann Arbor, MI 48109, USA

### ARTICLE INFO

**Keywords:**  
Assembly  
Customization  
Complexity

### ABSTRACT

Mixed-model assembly systems and modular supply chains are enablers to high product variety. However, as variety gets very high, the assembly and supply processes can become very complex. In assembly systems, the complexity may cause human errors and in turn impacts system performance. The complexity also impacts supply chain configuration and inventory control policy. This paper proposes a unified measure and models of complexity to assist in designing systems with robust performances. Complexity is defined as an entropy function of product variety and models are developed to describe the complexity propagation in multi-stage assembly systems and multi-echelon supply chains. Applications of the models are presented for complexity mitigation.

© 2008 CIRP.

### 1. Introduction

Mass customization has been the mantra for today's manufacturing [1]. It promises individualized products at near mass production cost. As a result of such paradigm change, the number of product variety offered by manufacturers has increased drastically. For example, BMW claims that "Every vehicle that rolls off the belt is unique" and the number of possible automobile variations in the BMW 7 Series alone could reach  $10^{17}$  [2]. Production systems and supply chains must be designed to handle such high variety while at the same time achieve mass production quality and productivity. Mixed-model assembly systems and modular supply chains have been recognized as major enablers to handle the increased variety.

Various industries are practicing mixed-model assembly systems since they bring various benefits. For example, a mixed-model auto assembly line as shown in Fig. 1 not only can save investment cost by sharing multiple products in the same line but also absorb demand fluctuation.

The concepts of modular assembly supply chain and traditional non-modular ones are shown in Fig. 2. In the modular configuration, the final assembler apportions product modules to intermediate sub-assemblers instead of doing all the assembly work itself. As a result, only a few assembled modules will be delivered to the final assembler, which reduces the complexity of the final assembly process while shifting risk and responsibility to the sub-assemblers. Modular assembly has found applications in many industries, such as automotive and aerospace.

The high number of variety or build-combinations undoubtedly presents enormous difficulties in the design and operation of the assembly systems and supply chains. It has been shown by both empirical data and simulations [3,4] that increased product variety

has significant negative impact on the performance (quality and productivity) in case of automotive vehicle production, including assembly and parts supply. One of the possible approaches to assess the impact of product variety on performance is to investigate how variety *complicates* the assembly process and supply chain operations. Some limited research has been done on assembly system and supply chain complexity. MacDuffie et al. [3] defined product mix complexity based on variety (product mix and its structure) and found significant negative correlation between complexity and manufacturing system performance through *empirical* study. Deshmukh et al. [5] defined an entropic complexity measure for part mix in job shop scheduling. Fujimoto et al. [6] introduced a complexity measure based on product structure using entropy for different stages of process planning. More recently, ElMaraghy et al. [7] applied entropy function to quantify the complexity of manufacturing systems and their configurations with examples in machining processes.

In supply chain, Frizelle and Woodcock [8] defined complexity as the variety and uncertainty associated with a system. Based on this definition, they classified the complexity of a supply chain system into structural complexity, which is associated with the variety embedded in the static system, and operational complexity, which is associated with the uncertainty of the dynamic system. Sivadasan et al. [9] developed an experimental methodology to study the operational complexity in a single supplier–customer system.

This paper proposes a unified measure of complexity by integrating both product variety and assembly process information, and then develops models for evaluating complexity in multi-stage mixed-model assembly systems and multi-echelon supply chains. The paper is organized as follows. In Section 2, we define complexity based on entropy and develop models for assembly systems and supply chains. In Section 3, system design methodologies based on the complexity models are discussed to enhance assembly system performance and determine optimal

\* Corresponding author.

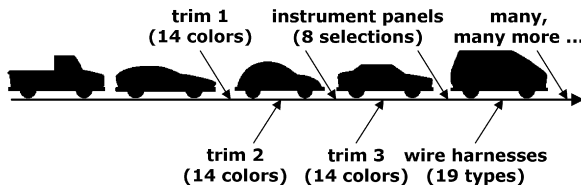


Fig. 1. Automobile mixed-model assembly line.

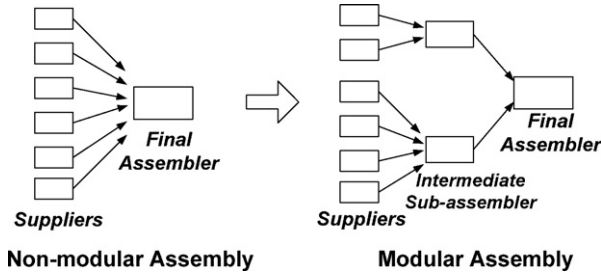


Fig. 2. Non-modular assembly to modular assembly supply chains.

assembly supply chain configuration. Section 4 concludes the paper.

## 2. Definitions and models of complexity

In this session, we define a unified measure and develop models of complexity for assembly systems and supply chains based on product variety. We use an example to illustrate our modeling techniques.

### 2.1. Mix and complexity

An example of a product family with its corresponding mixed-model assembly system and supply chain is illustrated in Fig. 3. The product has two components, A and B; each component has several variants (e.g.,  $A_1$  to  $A_3$ , and  $B_1$  to  $B_2$ ). The product structure can be represented by a product family architecture (PFA) diagram [10].

Fig. 3 illustrates all the possible variations of the customized products by combining the variants of each component. Here the maximal number of different end products is 6 (i.e.,  $3 \times 2$ ). Moreover, the product mix information is represented by a matrix  $\mathbf{P}$ , where  $p_{ij}$  is the demand (in %) of the  $j$ th variant of the  $i$ th feature. For instance, matrix  $\mathbf{P}$  for the product in Fig. 3 is the following:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & 0 \end{bmatrix}, \quad \text{where } \sum_j p_{ij} = 1, \quad \forall i \quad (1)$$

In the mixed-model assembly process, one variant of every component is selected and assembled sequentially along the flow of the assembly line. For example, as depicted in Fig. 3, if  $A_1$  is chosen for component A, and  $B_2$  for B, the final product will be  $A_1B_2$ .

In the supply chain, two suppliers provide components A and B to the downstream assembler. Each element of the supply chain can provide a number of variants to the downstream assembler or customer. The final assembler provides six variants to the customers,  $A_1B_1$ ,  $A_1B_2$ ,  $A_2B_1$ ,  $A_2B_2$ ,  $A_3B_1$ ,  $A_3B_2$ , which are the assembly combination from the variants provided by suppliers A and B.

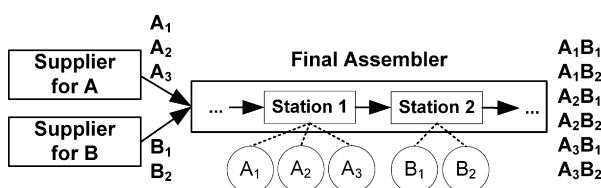


Fig. 3. An illustration of a mixed-model assembly line and supply chain.

A definition of product variety-induced complexity is given as follows.

**Definition.** Complexity is the average uncertainty in a random process  $i$  of handling product variety, which can be described by entropy function  $H_i$  in the following form:

$$H_i(p_{i1}, p_{i2}, \dots, p_{iM_i}) = -C \sum_{j=1}^{M_i} p_{ij} \log p_{ij} \quad (2)$$

where  $p_{ij}$  is the occurrence probability of a state  $j$  in the random process  $i$ ,  $j \in \{1, 2, \dots, M_i\}$ ,  $C$  is a constant depending on the base of the logarithm function chosen. If  $\log_2$  is selected,  $C = 1$  and the unit of complexity is bit.

### 2.2. Complexity in assembly system

Quite often, the mixed-model assembly process as shown in Fig. 3 is accomplished *manually*. Operators at every station must make correct choices among a number of alternatives according to customers' order. This process of selecting the right part is continued during the day. As variety increases, the operators face more uncertainty about making choices. This mechanism introduces complexity into the assembly task and in turn impacts assembly system performance. Therefore, complexity as defined in Eq. (2) characterizes operator's performance in making choices (thus *choice complexity*). Here  $p_{ij}$  in Eq. (2) refers to the probability of a choice taking the  $j$ th outcome in the  $i$ th choice.

#### 2.2.1. Station level complexity

At a station, in addition to the part choice mentioned above, the operator may perform other additional assembly activities in a sequential manner. Some examples of these choices are fixture choice, tool choice, assembly procedure choice, etc. All these choices contribute to the *operator choice complexity*. At the station, we number the sequential assembly activities (such as part, fixture, tool, and procedure choices) from 1 to  $K$ , and write  $C_p$  in Eq. (3) as the total complexity of Station  $p$ .

$$C_p = \sum_{k=1}^K H_p^k, \quad k = 1, 2, \dots, K \quad (3)$$

where  $H_p^k$  is the entropy computed from the variant mix ratio relevant to the  $k$ th activity at Station  $p$ .

As an example, in Fig. 3, we identify one assembly activity at Station 1, and two activities at Station 2. Specifically, we know from the process requirements that:

- (1) At Station 1, one of the three components,  $A_1$ ,  $A_2$ , or  $A_3$ , is chosen according to customer orders.
- (2) At Station 2, one of the two components,  $B_1$  or  $B_2$ , is chosen according to customer orders; also one of the three distinct tools is chosen according to the variant of component A installed at Station 1.

Therefore, the complexity values for the two stations are:

$$C_1 = H(P_{11}, P_{12}, P_{13}), \quad C_2 = H(P_{21}, P_{22}) + H(P_{11}, P_{12}, P_{13})$$

#### 2.2.2. System level complexity

Among the assembly activities, some activities are caused only by the feature variants at the current station, such as picking up a part, or making choices on tools for the selected part. The complexity associated with such assembly activity is defined as *feed complexity*. However, the choice of fixtures, tools, or assembly procedures at the current station may depend on the feature variant that has been added at an upstream station. This particular component of complexity is termed as *transfer complexity*. Hence, the total complexity at a station is simply the sum of the feed complexity at the station and the transfer complexity from all the

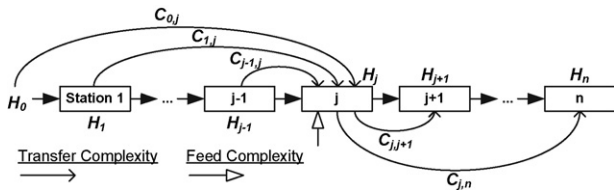


Fig. 4. Propagation of complexity at the system level in a multi-stage assembly system [11].

upstream ones, i.e., for Station  $q$ :

$$C_q = C_{qq} + \sum_{\forall p:p < q} C_{pq} \quad (4)$$

where  $C_{qq}$  (with two *identical* subscripts) is the feed complexity of Station  $q$ ,  $C_{pq}$  (with two *distinct* subscripts) is the complexity of Station  $q$  caused by variants added at an upstream Station  $p$ .

The propagation of the two types of complexity gives rise to the system level complexity model. Consider an assembly line with  $n$  workstations, see Fig. 4. Since the mix ratio in Eq. (1) is known, using Eq. (2), we can obtain the entropy  $H$  for the variants at each station according to their mix ratios. The propagation of complexity in a multi-stage system can be also analyzed by considering how the complexity of assembly operations (choices) at a station is influenced by the variety added at its upstream stations, as well as how variants added at the station impact the downstream stations, see Ref. [11] for details.

By the example in Fig. 3, we obtain feed and transfer complexity for the stations:

$$\begin{aligned} C_{11} &= H(P_{11}, P_{12}, P_{13}), \\ C_{12} &= H(P_{21}, P_{22}), \text{ and} \\ C_{22} &= H(P_{11}, P_{12}, P_{13}). \end{aligned}$$

where  $C_F = C_{11} + C_{22}$  is the total system feed complexity, and  $C_{12}$  is the transfer complexity from Station 1 to Station 2.

### 2.3. Complexity in assembly supply chains

In an assembly supply chain, each element can be assumed as a mixed-model assembly system providing a number of variants. There exists product variety-induced complexity in each and every element in the supply chain. The complexity of an assembly supply chain is determined by the following three factors: (i) the configuration of the assembly supply chain, including the number of elements in the supply chain and their relationships; (ii) the product variety in each element of the assembly supply chain; and (iii) the demand uncertainty of each element in the supply chain.

Assume there are  $N$  elements in the supply chain, denoted as 1, 2, ...,  $N$  and element  $i$  in the supply chain can provide  $n_i$  variants to its downstream element. Based on  $\mathbf{P}$  matrix in Eq. (1), we can obtain the mix ratio of product variants in each element of the supply chain. The complexity model is based on the supply–assembly relationship in the supply chain and the product variety information of downstream assembler is captured in each relationship. In order to capture the information of the suppliers in the first echelon, here we introduce a virtual supplier, denoted as 0, which is the supplier to all suppliers in the first echelon.

The complexity of an assembly supply chain can be obtained through the following steps [12]:

- **Step 1:** Adjacency matrix,  $\Phi = ((\phi_{ij}))_{(N+1) \times (N+1)}$ ,  $i, j = 0, 1, \dots, N$ , is used to capture the supply–assembly relationships and the configuration of an assembly supply chain, including the number of elements and their supply–assembly relationships. The number of columns and rows of matrix  $\Phi$  are equal to the number of elements in the supply chain. If element  $i$  is a supplier to element  $j$ ,  $\phi_{ij} = 1$ , otherwise  $\phi_{ij} = 0$ .
- **Step 2:** For each supply–assembly relationship where  $\phi_{ij} = 1$  in matrix  $\Phi$ , matrix  $\mathbf{Q}^{ij} = ((q_{uv}^{ij}))$ ,  $u = 1, \dots, n_i$ ,  $v = 1, \dots, n_j$ , is used

to capture the variety in supplier  $i$  and assembler  $j$  and the mix ratio of element  $j$ .

- **Step 3:** Each matrix  $\mathbf{Q}^{ij} = ((q_{uv}^{ij}))$ , is normalized:

$$\bar{q}_{uv}^{ij} = \frac{q_{uv}^{ij}}{K}, \quad \text{where } K = \sum_i \sum_j \sum_u \sum_v q_{uv}^{ij} \quad (5)$$

- **Step 4:** The complexity definition in Eq. (2) is used to obtain the complexity of an assembly supply chain, with state probability  $\bar{q}_{uv}^{ij}$ . The complexity of any supply–assembly relationship, where  $\phi_{ij} = 1$ , is defined in the following form:

$$C_{ij} = - \sum_u \sum_v \bar{q}_{uv}^{ij} \log_2 \bar{q}_{uv}^{ij} \quad (6)$$

Then the complexity of an assembly supply chain is obtained by summing the complexity values of all supply–assembly relationships.

$$C_{sc} = \sum_i \sum_j C_{ij} \quad (7)$$

When only the feed complexity is considered, the supply chain complexity becomes an extension of the complexity of the system with the supply chain structure incorporated. It can be shown that

$$C_{sc} = f(C_s) + g(S_{\text{structure}}) \quad (8)$$

where  $f(C_s)$  is a function of the assembly system complexity and  $g(S_{\text{structure}})$  is a function related to supply chain configuration.

For the example of Fig. 3, the complexity of that assembly supply chain is

$$\begin{aligned} C_{sc} &= [H(p_{11}, p_{12}, p_{13}) + H(p_{21}, p_{22})] + \log_2 4 \\ &= f(C_s) + \log_2 4. \end{aligned}$$

where the last term is the value of  $g(\text{Structure})$  in this specific example, and the number “4” indicates the total number of supplier–assembler links in the supply chain, including two links from the virtual supplier to the first echelon suppliers A and B. It is easy to verify that the complexity of an assembly supply chains increases with the number of elements in the supply chain, product variety of each element in the supply chain and uncertainty from products demand distribution of each element in the supply chain.

### 3. Complexity mitigation

Once the measure and models of complexity are developed, they can be applied to the design of assembly systems and supply chains to mitigate complexity. Several potential applications are described below.

#### 3.1. Assembly sequence planning to minimize complexity

Assembly sequence planning is an important task in assembly system design. Since the assembly sequence determines the directions in which complexity flows, see Fig. 5, proper assembly sequence planning can reduce complexity.

Generally, suppose we have a product with  $n$  assembly tasks, and the tasks are to be carried out sequentially in an order subject to precedence constraints. By applying the complexity model, we assume that the transfer complexity can be found between every two assembly tasks. Since only one of the two transfer complexity

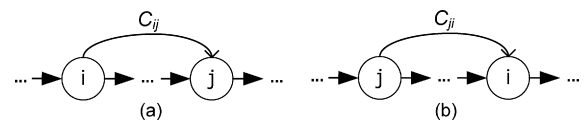


Fig. 5. Differences in transfer complexity values for different assembly sequences,  $C_{ij} \neq C_{ji}$  [13].

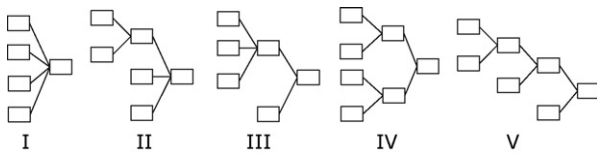


Fig. 6. Possible supply chain network with four original suppliers.

values in Fig. 5 is effective (because only the upstream task/station has influence on the downstream ones) for one particular assembly sequence, an optimization problem can be formulated to minimize the system complexity by finding an optimal assembly sequence while satisfying the precedence constraints. Please refer to Ref. [13] for details.

### 3.2. Optimal assembly supply chain configuration

Now we move from assembly system design to assembly supply chain design. As discussed in Section 1, modular assembly supply chain is a way to handle variety for mass customization at the enterprise level. The complexity model developed in Section 2 is a good means to studying supply chain complexity caused by product variety. It can be used to find the optimal assembly supply chain configuration. The procedure to find the optimal assembly supply chain can be divided into the following three main steps:

- (1) Generate all possible supply chain configurations.
- (2) Calculate complexity for each possible configuration.
- (3) Compare the results and obtain the optimal supply chain configuration.

Among these steps, the first step is the most challenging. For example, given four original suppliers, there are five possible supply chain network configurations, shown in Fig. 6. For each of these possible networks, there are many possible supply chain configurations because the locations of each original supplier can be different from one configuration to another. For example, the network IV has three different possible supply chain configurations, as seen in Fig. 7.

The method of Webbink and Hu [14] for assembly system configuration can be modified for generating supply chain configurations. Wang et al. [12] developed a modified algorithm to generate all possible assembly supply chain configurations. After all the possible configurations are generated, the complexity of each configuration can be calculated by Eq. (7) and then the optimal configuration can be found by picking up the configuration with the smallest complexity value.

Based on the above algorithm, it can be shown that as the product variety increases, the optimal assembly supply chain configuration moves from non-modular assembly to modular assembly.

## 4. Conclusions

This paper introduces a unified measure of product variety-induced manufacturing complexity for assembly systems and supply chains. Models are developed to characterize the propaga-

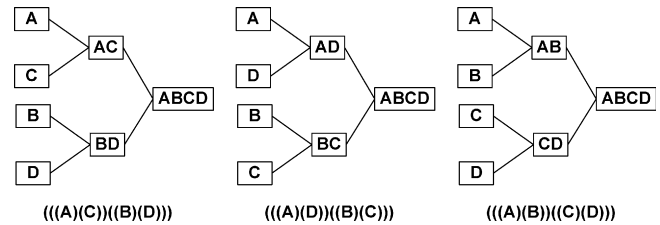


Fig. 7. Possible supply chain configurations for network IV.

tion of complexity in multi-stage mixed-model assembly systems and multi-echelon assembly supply chains. Relationship is established between assembly system complexity and supply chain complexity. These models of complexity can be used to configure assembly systems and supply chains to ensure robust performance by mitigating complexity.

## Acknowledgment

The authors gratefully acknowledge the financial support from the Engineering Research Center for Reconfigurable Manufacturing Systems of the National Science Foundation under Award No. EEC-9529125, and the General Motors Collaborative Research Laboratory in Advanced Vehicle Manufacturing, both at The University of Michigan.

## References

- [1] Pine BJ (1993) *Mass Customization: The New Frontier in Business Competition*. Harvard Business School Press, Boston.
- [2] <http://www.bmwgroup.com/e/nav/index.html>.
- [3] MacDuffie JP, Sethuraman K, Fisher ML (1996) Product Variety and Manufacturing Performance: Evidence from the International Automotive Assembly Plant Study. *Management Science* 42(3):350–369.
- [4] Fisher ML, Ittner CD (1999) The Impact of Product Variety on Automobile Assembly Operations: Empirical Evidence and Simulation. *Management Science* 45(6):771–786.
- [5] Deshmukh AV, Talavage JJ, Barash MM (1998) Complexity in Manufacturing Systems. Part 1. Analysis of Static Complexity. *IIE Transactions* 30(7):645–655.
- [6] Fujimoto H, Ahmed A, Iida Y, Hanai M (2003) Assembly Process Design for Managing Manufacturing Complexities because of Product Varieties. *International Journal of Flexible Manufacturing Systems* 15(4):283–307.
- [7] ElMaragha HA, Kuzgunkayaa O, Urbanic RJ (2005) Manufacturing Systems Configuration Complexity. *Annals of the CIRP* 54:445–448.
- [8] Frizelle G, Woodcock E (1995) Measuring Complexity as an Aid to Developing Operational Strategy. *International Journal of Operations & Production Management* 15:26–39.
- [9] Sivadasan S, Efstathiou J, Frizelle G, Shirazi R (2002) An information-Theoretic Methodology for Measuring the Operational Complexity of Supplier–Customer Systems. *International Journal of Operations & Production Management* 22(1):80–102.
- [10] Tseng MM, Jiao J (1996) Design for Mass Customisation. *Annals of the CIRP* 45:153–156.
- [11] Zhu X, Hu SJ, Koren Y, Marin SP (2006) Modeling of Manufacturing Complexity in Mixed-Model Assembly Lines. *Proceedings of ASME International Conference on Manufacturing Science and Engineering*.
- [12] Wang H, Zhu X, Hu SJ, Koren Y, Complexity Analysis of Assembly Supply Chain Configurations. Working Paper.
- [13] Zhu X, Hu SJ, Koren Y, Marin SP, Huang N (2007) Sequence Planning to Minimize Complexity in Assembly Mixed-Model Lines. *IEEE International Symposium on Assembly and Manufacturing*.
- [14] Webbink RF, Hu SJ (2005) Automated Generation of Assembly System–Design Solutions. *IEEE Transactions on Automation Science and Engineering* 2(1):32–39.