5,

Cross-Coupled Biaxial Computer Control for Manufacturing Systems

Yoram Koren

Goebel Chair Professor,
Department of Mechanical Engineering and Applied Mechanics,
The University of Michigan,
Ann Arbor, Mich. 48109

Biaxial control systems for generating predetermined paths under load disturbances, such as encountered in NC and CNC systems for machine tools, are conventionally designed such that the control of each axis is independent of the other. The present paper is concerned with providing cross-couplings for biaxial control systems, whereby an error in either axis affects the control loops of both axes. An algorithm for a cross-coupled control system is presented, and the performance of the cross-coupled system is mathematically analyzed and compared with the conventional CNC system having individual axis control. It is shown that cross-coupling between axes improves the contour accuracy while the velocity response of each axis is only slightly reduced. Although the proposed cross-coupled system requires additional hardware for implementation with an NC system, operation with a CNC-based system requires only software modifications to the system control program.

Introduction

Computerized numerical control (CNC) is attracting increasing attention for manufacturing. With CNC, a minicomputer is provided as part of the controller to perform the basic numerical control functions. The low prices and impressive capabilities of the current minicomputers and microprocessors are naturally contributing to the increasing use of CNC over a broad spectrum of manufacturing systems, including virtually all types of machine tools, laser-beam cutters, industrial robots, welders, EDM and ECM machines.

From a control point of view, the significant common requirement of all CNC systems is to generate coordinated movement of the separately driven axes-of-motion to trace a predetermined path of the "tool" relative to the workpiece. For example, consider a vertical spindle milling machine with the workpiece mounted on a table which can move within its own plane along the X-axis and Y-axis. The actual movement of the table along the predetermined path while the spindle is rotating and cutting produces the required part.

In existing biaxial control systems for machine tools, each axis-of-motion has a separate closed-loop control, so that the control loop of one axis receives no information regarding the other. Any load disturbances error in one of the axes is corrected only by its own loop, while the other loop carries on as before. This causes an error in the resultant path. Since biaxial control systems require both control and coordination of the motion along two axes, it should be possible to improve their accuracy by providing cross-coupling, whereby an error in either axis affects the control loops of both axes.

The first cross-coupling method was proposed by Sarachik and Ragazzini [1] and is shown in Fig. 1. This system has a "master-slave" structure (Y follows X): the storage device

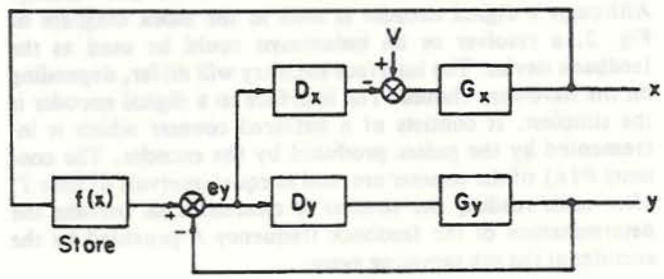

Fig. 1 Nonsymmetrical cross-coupled system

$f(x)$ provides the desired value of y as its output for an input x. The Y-axis is controlled by a closed loop and the X-axis by an open loop whose input is the Y-axis error, e_y. Elements D_y and D_x provide constraints on the path speed and e_y, respectively. The error in the Y-axis affects both the X and Y control loops, but an error signal in the X-axis is not generated.

While this nonsymmetrical cross-coupled system may be suitable for some open-contour operations, such as turning, a symmetrical system with equal loop gain in both axes is preferable in most practical NC systems. For a linear contour ($y = kx$) and negligible load disturbance, a symmetrical system provides a zero steady-state contour error, although there are time dependent errors in each axis [2]. The nonsymmetrical cross-coupled system, however, requires a substantial difference in the gain for each axis, resulting in a contour error which is dependent on the slope k of the contour [1]. Likewise for a circular contour, the symmetrical system generates a perfect circle with a small radial error, whereas a non-symmetrical system generates an elliptal shape [2].

A cross-coupled biaxial system with a symmetrical structure was developed by Koren and Ben-Uri [3]. Cross-coupling was obtained by the addition of two DDA integrators and digital comparator to a conventional biaxial control system.

Contributed by the Dynamic Systems and Control Division for publication in the JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL. Manuscript received by the Dynamic Systems and Control Division, January 1978; revised manuscript received September 1980.

Although it was experimentally shown that this cross-coupled biaxial system can operate with significantly less contour error than its conventional counterpart, the cost of the additional hardware components made the system economically impractical at that time. However, with the current trend away from conventional hardware-based NC to computer controlled software-based CNC [4] it is worthwhile to reconsider this cross-coupled concept. It should now be possible to obtain cross-coupling in CNC systems by modifying the computer control program without the need for additional hardware.

The present investigation was undertaken to determine the practical feasibility of using symmetric cross-coupled control in CNC systems. In the present paper, an algorithm is presented for applying the previous concept of symmetrical cross-coupled control to a CNC system. A mathematical analysis of the cross-coupled system is developed which provides a basis for evaluating the dynamic behavior of the system and the influence of load disturbances and axis mismatch on the contour errors.

Symmetrical Cross-Coupled Control System

With CNC systems of the sampled-data type, each axis is controlled independently in a loop closed by software within the computer [4,5,6]. A typical loop is shown in Fig. 2. The speed of each axis of the machine table is controlled by a d-c servomotor and its position is measured by a digital encoder which is able to transmit two sequences of pulses, one for each direction of rotation. Each pulse generated by the encoder represents an axial motion of one basic length-unit which might be on the order of 10 μm in a typical machine tool system. Therefore, the number of pulses represents position and the pulse frequency is proportional to the axis velocity. Although a digital encoder is used in the block diagram of Fig. 2, a resolver or an inductosyn could be used as the feedback device. The interface circuitry will differ, depending on the hardware chosen. The interface to a digital encoder is the simplest. It consists of a buffered counter which is incremented by the pulses produced by the encoder. The contents $P(n)$ of the counter are read at equal intervals of time T; after each reading the counter is cleared. This permits the determination of the feedback frequency F provided by the encoder at the nth sampling event:

$$F(n) = P(n)/T$$

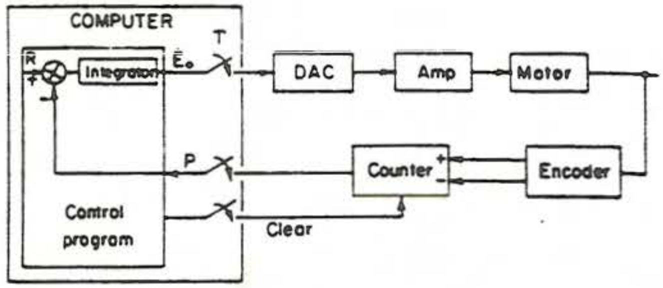

Fig. 2 Conventional CNC loop

The control program serves as an equalizer which compares two types of inputs: a reference number R proportional to the required speed of the axis; and the feedback signal F which is proportional to the actual speed of the axis. The difference between these two inputs is the speed error of the corresponding axis-of-motion. In order to obtain a zero speed error at the steady-state for a fixed reference number, R, the control program must integrate the speed error. This can be accomplished by the statement:

$$E_0(n) = E_0(n-1) + T[R(n) - F(n)] \tag{1}$$

E_0 is the axis position error which is converted by the DAC, amplified and fed to the d-c motor. Substituting for $F(n)$ in equation (1) yields the position error at the successive interval as:

$$E_0(n) = E_0(n-1) + TR(n) - P(n) \tag{2}$$

The proposed cross-coupled system employs the basic structure shown in Fig. 3. The control program is fed by two reference numbers, R_1 and R_2, proportional to the required speeds of axes 1 and 2, respectively. The corresponding error difference-equations are of the same type as in equation (1) for the conventional system:

$$E_i(n) = E_i(n-1) + T[R_i(n) - F_i(n)]; \; i = 1,2 \tag{3}$$

which yields the following statements for calculating the position errors in the successive intervals:

$$E_i(n) = E_i(n-1) + TR_i(n) - P_i(n); \; i = 1,2 \tag{4}$$

where $TR_i(n)$ is the nth required incremental position in the ith axis and $P_i(n)$ is the corresponding actual incremental position motion.

Nomenclature

Abbreviations

- CCS = cross-coupled system
- DAC = digital-to-analog converter
- DDA = digital differential analyzer
- A, A_i = gains in the CCS, defined by (24) or (59)
- D = characteristic polynomial of a matched CCS
- D_0 = characteristic polynomial of a conventional system
- D_u = characteristic polynomial of mismatched CCS
- E = magnitude proportional to the contour error in CCS
- E_0 = axis position error in a conventional system
- E_i, e_i = axis position errors in CCS
- ϵ = actual contour error
- F, f = feedback frequency, proportional to axis velocity
- G = weighted gain in CCS, defined in (26)
- H = polynomial defined by (29)
- K, K_i = partial loop gain, $K = K_m K_e$
- K_c = DAC gain in volt/bit
- K_e = encoder gain
- K_m = axial drive gain
- K_x, K_y = cross-coupling gains
- k = slope of straight line, $k = R_2/R_1$
- M, m = magnitude proportional to the load torque
- N = parameter defined by (53)
- p_i = $\exp(-T/\tau_i)$
- $P(n)$ = incremental position during the nth sampling period
- Q_i = polynomial defined in (61)
- R = axial-velocity reference
- s = Laplace-transform variable
- T = sampling time interval
- τ = time constant
- U, u = DAC input signal
- V, v = DAC output voltage
- W = weighting gain in CCS
- X, Y = symbols of the two axes-of-motion
- z = z-transform variable

NOTES: (1) Lower-case symbols are used for time variables, and upper-case ones for Laplace and z-transformed variables. (2) Indices 1 and 2 correspond to the X- and Y-axis, respectively.

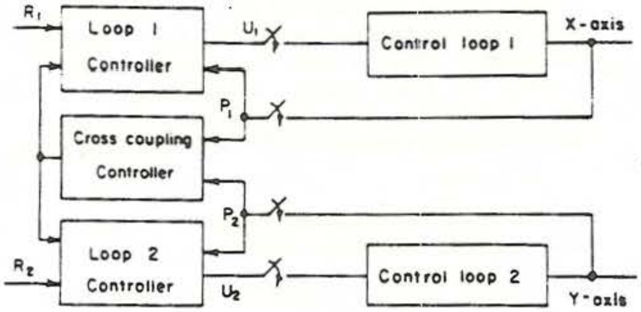

Fig. 3 Biaxial computer controlled system

However, the operation of the proposed symmetric cross-coupled system is based upon providing corrections which are proportional not only to the individual axial errors, but to the contour error as well. This is accomplished by combining the individual axis errors with a weighted contour error:

$$U_1(n) = E_1(n) - WE(n) \qquad (5a)$$

$$U_2(n) = E_2(n) + WE(n) \qquad (5b)$$

where U_1 and U_2 are the signals fed to the DAC's. The weighting factor W determines the cross-coupling effect upon the operation of the whole system.

The magnitude of E is proportional to the contour error (see Appendix A) and is calculated by comparing the feedback frequencies received from the two loops by the difference equation:

$$E(n) = E(n-1) + T[K_x F_1(n) - K_y F_2(n)] \qquad (6)$$

K_x and K_y are the cross-coupling gains, which are discussed below. In the control program, the statement corresponding to equation (6) is:

$$E(n) = E(n-1) + K_x P_1(n) - K_y P_2(n) \qquad (7)$$

As can be seen from equations (5), E is fed to loop 1 with a negative sign and to loop 2 with a positive sign. Considering the appropriate signs of P_1 and P_2 in equation (7), one sees that equations (5) and (7) provide negative feedback of the cross-coupling in both loops.

In the case of linear motion, the references R_1 and R_2 as well as K_x and K_y are constants. Since $R(n) = F(n)$ at steady-state, the frequency of the pulses applied by the encoder of loop 1 is (R_1/R_2) times that at the output of the second encoder. Therefore, in order to compare the frequencies, the condition:

$$K_x/K_y = R_2/R_1 \qquad (8)$$

must be satisfied. This provides a steady-state value of E and in turn steady-state performance of the whole system.

The main problem in the corresponding hardware circuit was realization of K_x and K_y with the highest possible accuracy for any possible inputs R_1 and R_2 [3]. This problem is obviated in the proposed software system, in which K_x and K_y are programmed coefficients. For a linear motion they may conveniently be chosen (see Appendix A):

$$K_x = R_2/(R_1 + R_2);\ K_y = R_1/(R_1 + R_2) \qquad (9)$$

so that $K_x + K_y = 1$.

In the case of a circular path with a radius A, the references R_1 and R_2 are $A\sin\omega t$ and $A\cos\omega t$, respectively, and the gains K_x and K_y can be chosen as $A\cos\omega t$ and $A\sin\omega t$, respectively. The sine and cosine values are continuously calculated by a circular interpolator which is included in the NC control program. Other non-linear contours are generated in NC systems by a combination of lines and circles.

In summary, the control program algorithm consists of equations (4), (5) and (7) with K_x and K_y satisfying equation (8). Satisfactory performance of the cross-coupled system depends on the values of K_x and K_y together with an appropriate choice of the weighting factor W.

Mathematical Analysis

Since the software cross-coupled control system is of the sampled-data type, the z-transform analysis can be applied. This technique is utilized to calculate the various errors in the cross-coupled system and to determine an appropriate value of the weighting factor W.

A block diagram for the system is shown in Fig. 4. The differential equation of the power amplifier and the motor is [7]:

$$\tau\dot{\omega} + \omega(t) = K_m[v(t) - m(t)] \qquad (10)$$

where ω is the motor speed, v is the DAC output voltage, and $m(t) = \alpha T_l(t)$; the product αT_l is denoted in Fig. 4 by M_i ($i = 1,2$). T_l is the load torque, and α is a constant which is directly proportional to the armature resistance (inductance neglected) and inversely proportional to the amplifier gain and motor torque constant. Most NC systems also contain an additional internal loop, consisting of the power amplifier, the motor and a tachogenerator mounted on the motor shaft as a second feedback device. This internal loop however, has a mathematical representation similar to the one in equation (10) [6].

The digital encoder provides K_e pulses per revolution of the motor shaft, so its output frequency $f(t)$ is:

$$f(t) = K_e\omega(t) \qquad (11)$$

Denoting the gain $K_m K_e$ of each individual loop by K_i ($i = 1,2$), the differential equations of the drive is obtained from equations (10) and (11):

$$\tau_i\dot{f}_i + f_i(t) = K_i v_i(t) - K_i m_i(t);\ i = 1,2 \qquad (12)$$

The corresponding Laplace-transform equation is:

$$(1 + s\tau_i)F_i(s) = K_i V_i(s) - K_i M_i(s);\ i = 1,2 \qquad (13)$$

The DAC holds the data from one sampling instant to the next and is actually a zero-order hold with a gain of K_c volt/bit. The corresponding transfer function is

$$\frac{V(s)}{U(s)} = \frac{K_c(1 - e^{-sT})}{s} \qquad (14)$$

Substitution of equation (14) into (13) yields:

$$F_i(s) = \frac{K_i K_c(1 - e^{-sT})U_i(s)}{s(1 + s\tau_i)} - \frac{K_i M_i(s)}{(1 + s\tau_i)};\ i = 1,2 \qquad (15)$$

The corresponding z-transform is

$$F_i(z) = K_i K_c\left(\frac{z-1}{z}\right)\left[\frac{z}{z-1} - \frac{z}{z-p_i}\right]U_i(z) - K_i\left(\frac{z/\tau_i}{z-p_i}\right)M_i(z) \qquad (16)$$

where

$$p_i = e^{-T/\tau_i} \qquad (17)$$

Equation (16) can be rewritten:

$$F_i(z) = K_i K_c\left(\frac{1-p_i}{z-p_i}\right)U_i(z) - K_i\left(\frac{z/\tau_i}{z-p_i}\right)M_i(z);\quad i = 1,2 \qquad (18)$$

At the computer end the dominant difference-equations are obtained by combining equations (3), (5) and (6):

$$U_1(n) = U_1(n-1) + TR_1(n) - T(1 + WK_x)F_1(n) + TWK_y F_2(n) \qquad (19)$$

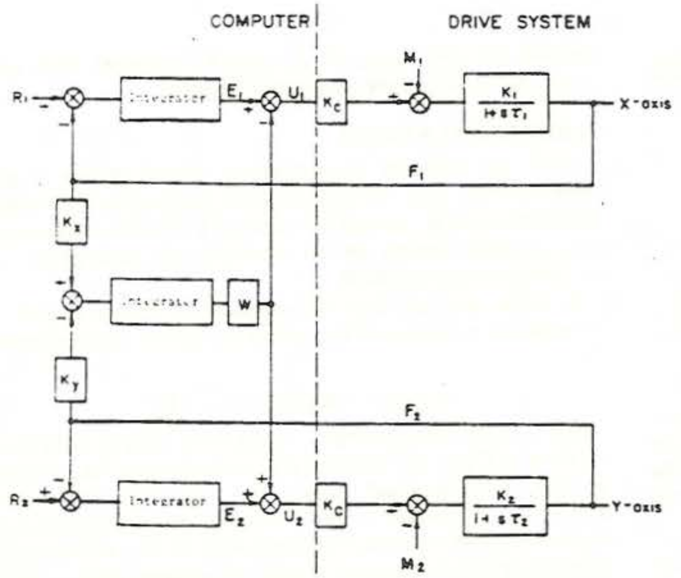

Fig. 4 Block diagram of the cross-coupled system

$$U_2(n) = U_2(n-1) + TR_2(n) - T(1+WK_y)F_2(n) + TWK_xF_1(n) \tag{20}$$

The corresponding z-transform equations are:

$$U_1(z) = [Tz/(z-1)][R_1(z) - (1+WK_x)F_1(z) + WK_yF_2(z)] \tag{21}$$

$$U_2(z) = [Tz/(z-1)][R_2(z) - (1+WK_y)F_2(z) + WK_xF_1(z)] \tag{22}$$

The whole system can thus be described by a set of four equations: equations (18), (21), and (22). The four state variables of this system are F_1, F_2, U_1, and U_2; the input variables are R_1, R_2, M_1, and M_2.

Assuming that the two axes of the control system are matched, namely $K = K_1 = K_2$ and $\tau = \tau_1 = \tau_2$, the solution for the encoder frequency $F_1(z)$ is:

$$\begin{aligned} F_1(z) = \{ & Az[(z-p)(z-1) + (1+WK_y)Az]R_1(z) \\ & + (Az)^2 WK_yR_2(z) - [z(z-1)K/\tau][(z-p)(z-1) \\ & + (1+WK_y)Az]M_1(z) - [z^2(z-1)A\,WK_yK/\tau]M_2(z)\}/D_0(z) \end{aligned} \tag{23}$$

where A is defined by

$$A = (1-p)TKK_c \tag{24}$$

and the characteristic polynomial of the system, in z-transform notation, is denoted by $D_0(z)$:

$$D_0(z) = (z-p)^2(z-1)^2 + (z-p)(z-1)Az(2+G) + (Az)^2(1+G) \tag{25}$$

The term G is defined as

$$G = W(K_x + K_y) \tag{26}$$

The characteristic polynomial, can be also written as the product of two terms:

$$D_0(z) = D(z)H(z) \tag{27}$$

where

$$D(z) = (z-p)(z-1) + Az \tag{28}$$

and

$$H(z) = (z-p)(z-1) + Az(1+G) \tag{29}$$

A symmetric equation to equation (23) is obtained for $F_2(z)$. The encoder frequencies are proportional to the speed of the motors, as seen from equation (11).

As a basis for comparing the conventional and cross-coupled system, consider the case of a linear contour path with fixed load disturbances on each axis. This approximates the situation which is often found in actual NC machining systems. For linear motions the references R_1 and R_2 are constants; at the start of machining motions the force disturbances are approximated by step functions, so that

$$R_i(z) = R_iz/(z-1);\ M_i(z) = M_iz/(z-1);\ i = 1,2 \tag{30}$$

To obtain the actual encoder frequency for these step inputs, equation (30) is substituted into equation (23), which yields:

$$\begin{aligned} F_1(z) = \{ & [(z-p)(z-1) + Az][Az^2/(z-1)]R_1 \\ & + [A^2z^3/(z-1)]WK_y(R_1+R_2) \\ & - (z^2K/\tau)[(z-p)(z-1) + (1+W)Az \\ & - W(1-K_y)AZ]M_1 - (z^2K/\tau)WK_yAzM_2\}/D_0(z) \end{aligned} \tag{31}$$

Substitution of K_x and K_y from equation (9) to (26) gives

$$G = W \tag{32}$$

and combining with equations (29) and (31), yields after some manipulation:

$$F_1(z) = \left[\frac{Az^2}{(z-1)D(z)}\right]R_1 - \frac{K}{\tau}\left[\frac{M_1}{D(z)} + L(z)\right]z^2 \tag{33}$$

where

$$L(z) = \left[\frac{M_2R_1 - M_1R_2}{R_1 + R_2}\right]\frac{WAz}{D_0(z)} \tag{34}$$

A similar symmetric equation also applies to $F_2(z)$.

In the conventional system controlled by two separate loops, the weighting factor W is zero, which yields:

$$L(z) = 0 \tag{35}$$

This means that the effect of the cross-coupling on the individual axis response enters through the last term $L(z)$ in equation (33). This term does not affect the steady-state speed and has only an influence on the dynamic response of the individual axes.

The position errors for constant inputs in the cross-coupled system are obtained by substituting $F_1(z)$ and $F_2(z)$ into the z-transform versions of equations (3) and (6):

$$E_1(z) = \frac{T(z-p)z^2}{(z-1)D(z)}R_1 + \frac{K}{\tau}\left[\frac{M_1}{D(z)} + L(z)\right]\frac{Tz^3}{(z-1)} \tag{36}$$

$$E_2(z) = \frac{T(z-p)z^2}{(z-1)D(z)}R_2 + \frac{K}{\tau}\left[\frac{M_2}{D(z)} - L(z)\right]\frac{Tz^3}{(z-1)} \tag{37}$$

$$E(z) = \frac{K}{\tau}\left[\frac{M_2R_1 - M_1R_2}{(R_1+R_2)D(z)} - L(z)\right]\frac{Tz^3}{(z-1)} \tag{38}$$

Again, the effect of the cross-coupling on the individual axis errors, E_1 and E_2, can be obtained by an inverse-transform of the term containing $L(z)$ in equations (36) and (37).

Contour Error and Weighting Factor

The most important factor in the performance of the biaxial control system is the contour error, which is defined as the distance-difference between the required and actual path. For the cross-coupled system the steady-state error and its time response depend upon the value of the weighting factor W. Therefore, in order to choose an appropriate value of W, it is necessary to first determine the contour error.

Consider a linear path in a biaxial system as shown in Fig. 5. The required path is the straight line $y = kx$, but the asymmetry of the axis loads causes an error. At arbitrary time t_0 the actual path is followed to point 0, while the corresponding instructed point is $D(x^*, y^*)$; the resultant

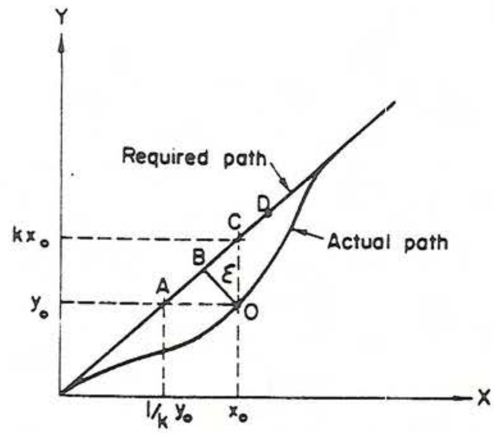

Fig. 5 Required and actual paths in the X-Y plane

error ϵ is the segment $\overline{OB}$ which is simply obtained from geometrical considerations (see Appendix B):

$$\overline{OB} = \epsilon = \frac{y_0 - kx_0}{1 + k^2} \tag{39}$$

The actual position $(x_0;y_0)$ at t_0 can be expressed in terms of the required position at the same time (x^*,y^*) and the position errors of the axes e_1 and e_2:

$$x_0 = x^* - e_1 \tag{40}$$

$$y_0 = y^* - e_2 \tag{41}$$

Substitution of equations (40) and (41) in (39) yields:

$$\epsilon = \frac{ke_1 - e_2}{1 + k^2} \tag{42}$$

Since $k = R_2/R_1$, the resultant contour error is:

$$\epsilon = \frac{R_2 e_1 - R_1 e_2}{\sqrt{R_1^2 + R_2^2}} \tag{43}$$

The z-transformed position errors are given by equations (36) and (37). Using the results in equation (43) the z-transformed contour error in the cross-coupling system can be written

$$\epsilon(z) = \frac{K}{\tau}\left[\frac{M_1 R_2 - M_2 R_1}{\sqrt{R_1^2 + R_2^2}}\right]\frac{Tz^3}{(z-1)H(z)} \tag{44}$$

By using the Final Value Theorem the steady-state error is obtained

$$\epsilon_{ss} = \frac{M_1 R_2 - M_2 R_1}{\tau(1-p)K_c(1+W)\sqrt{R_1^2 + R_2^2}} \tag{45}$$

The corresponding result for a conventional system controlled by two separate loops is obtained by substituting $D(z)$ for $H(z)$ into equation (44):

$$\epsilon_0(z) = \frac{K}{\tau}\left[\frac{M_1 R_2 - M_2 R_1}{\sqrt{R_1^2 + R_2^2}}\right]\frac{Tz^3}{(z-1)D(z)} \tag{46}$$

and the steady-state error is:

$$\epsilon_{ss0} = \frac{M_1 R_2 - M_2 R_1}{\tau(1-p)K_c\sqrt{R_1^2 + R_2^2}} \tag{47}$$

By comparing equations (45) and (47), it can be seen that the steady-state error is reduced by a factor of $(1 + W)$ with the cross-coupled system. This is achieved with almost negligible deterioration in the velocity response of each individual axis, since the term $L(z)$ in equation (33) has only a minor effect on F_1 as stated above.

Theoretically, this same reduction in contour error could be obtained with an uncoupled conventional system by simply increasing the individual gains K_c by a factor of $(1 + W)$, as seen from equation (47). However, this will reduce the damping of the individual axes and introduce undesirable oscillations into the actual machining path.

The advantage of the cross-coupled system can be readily appreciated for a single axis motions, which may be required for portions of the machining path. In this mode, $R_1 = M_1 = 0$ (or $R_2 = M_2 = 0$) leading to the condition $L(z) = 0$ in equation (33). That means that the velocity response of the individual axis in the cross-coupled system becomes identical to that of the conventional uncoupled system for single axis motion. Again a direct increase of the individual loop gains by $(1 + W)$ can cause undesirable velocity oscillations.

In order to decrease the steady-state contour error in the cross-coupled system, it is desirable that W will be as large as possible. However, there is an upper bound on W, which is prescribed by the system stability requirement.

The closed-loop system is stable if the characteristic equation $D_0(z) = 0$ possesses no zeros outside the unit circle in the z plane. In the general case, the determination of whether there is a zero outside this circle involves extensive effort, but since the polynomial in equation (25) can be written as a product of two quadratic polynomials, namely $D(z)H(z)$, the necessary and sufficient conditions for a stable system are [9]:

$$|D(0)| < 1;\ D(1) > 0;\ D(-1) > 0 \tag{48}$$

$$|H(0)| < 1;\ H(1) > 0;\ H(-1) > 0 \tag{49}$$

Since $D(z) = 0$ is the characteristic equation of a conventional system with two separate control loops, its corresponding stability conditions are given in (48), which lead to the following relations:

$$p < 1;\ A > 0;\ 2(1+p) > A$$

where p and A are defined in equations (17) and (24). Two of these conditions are satisfied for any positive K, but the third one places a bound on the open-loop gain

$$2(1+p)/(1-p) > TKK_c \tag{50}$$

In the cross-coupled system case the additional stability conditions (49) possess the extra constraint

$$2(1+p)/(1-p) > TKK_c(1+W) \tag{51}$$

which prescribes the following upper bound on W

$$N - 1 > W \tag{52}$$

where N is defined by

$$N = \frac{2(1+p)}{(1-p)TKK_c} \tag{53}$$

Typical values of N can be determined by using results from [4], which provides the optimal relationship among the conventional open-loop gain KK_c, the sampling period T and the time constant τ:

$$KK_c = \frac{1}{T + 2\tau} \tag{54}$$

The condition providing the stability constraint is [4]:

$$T/\tau < \pi \tag{55}$$

and, as a rule of thumb, $T = \tau$ usually results in acceptable response and an adequate stability margin. Substituting equation (54) and $T = \tau$ into equation (53) results in $N = 12$ as a typical value in CNC systems, and consequently $W < 11$ according to (52).

Nevertheless, smaller values of W should eventually be applied. Although the analyzed system has been approximated as 4th-order model, many practical systems are of higher order, and therefore setting of too large W may cause an unstable system. Therefore, in practice an adequate gain margin is required.

The term $TKK_c(1 + W)$ appearing in equation (51) is a gain

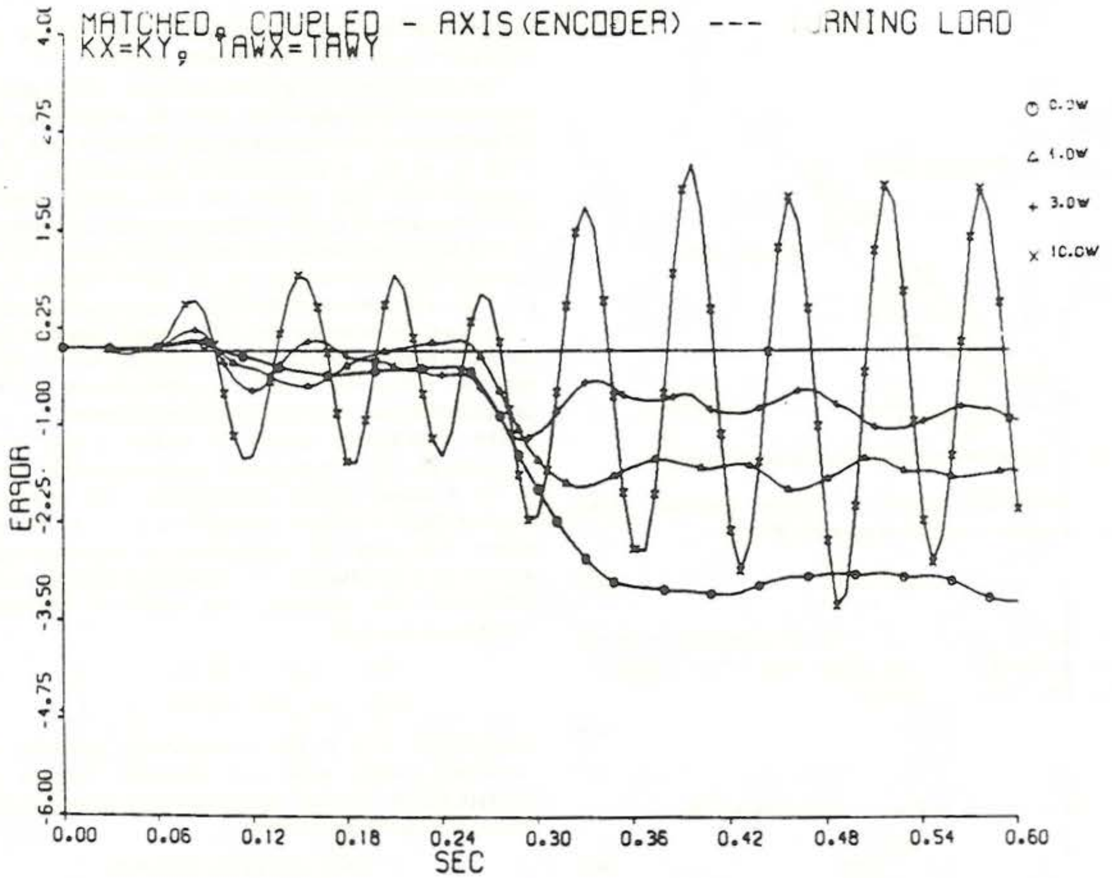

Fig. 6 Simulation results of contour errors due to machining load in conventional system ($W = 0$) and matched CCS ($W = 1,3,10$)

of the biaxial cross-coupled system. A rule of thumb is to allow a gain margin between 1/4 to 1/2 for good transient response. Consequently, the recommended values of W are

$$N/4 - 1 \leq W \leq N/2 - 1 \tag{56}$$

which for $N = 12$ provides the interval $[2 \leq W \leq 5]$ as the best compromise between the desire to reduce the contour error and to obtain acceptable transient response, and to provide an adequate "cushion" of stability margin for differences between the model and the actual system.

Typical behavior is illustrated in Fig. 6, where contour error time responses are given for the conventional system ($W = 0$) and for three different values of W with the cross-coupled system. In this simulation the following data is applied: $KK_c = 20\ \mathrm{s}^{-1}$, $T = \tau = 20$ m s, which, according to equation (53), yields $N = 10$ and consequently W is bounded by $W < 9$ but should be selected within the range $1.5 < W < 4$ for optimal results. For certain velocity and loading conditions the steady-state error in the conventional system is 3 units (e.g. 0.03 mm). Setting $W = 1$ in the corresponding cross-coupled system reduces the contour error to 1.5 units and slightly reduces the system damping as is shown in Fig. 6. Increasing the gain to $W = 3$ results in a steady-state error which is smaller than 1 unit. A further increase in W, to $W = 10$, causes an unstable system. In this case $W = 3$ is suggested as the best selection of the weighting gain in the CCS.

Axis Mismatch

Another factor which affects the contour accuracy is the axis mismatch, which means that the respective loop gains and time constants are not identical. Assuming that the system is not externally loaded, so that $M_1(z) = M_2(z) = 0$. The solution for the component $F_1(z)$ in this case is:

$$F_1(z) = \frac{[(z-1)(z-p_2) + zA_2(1 + WK_y)]zA_1}{D_u(z)} R_1(z) + \frac{z^2 A_1 A_2 W K_y}{D_u(z)} R_2(z) \tag{57}$$

where $D_u(z)$ is the characteristic polynomial of the mismatched system, defined by

$$D_u(z) = (z-p_1)(z-p_2)(z-1)^2 + [A_1(z-p_2)(1 + WK_x) + A_2(z-p_1)(1 + WK_y)](z-1)z + z^2 A_1 A_2 (1 + W) \tag{58}$$

and the gains A_i are:

$$A_i = (1 - p_i) T K_i K_{ci}; \quad i = 1,2 \tag{59}$$

Assuming a linear motion (step inputs), and combining equation (30) with equation (57), we obtain:

$$F_1(z) = Q_1(z) R_1 / D_u(z) \tag{60}$$

where

$$Q_1(z) = [(z-1)(z-p_2) + zA_2(1 + W)] A_1 z^2 / (z-1) \tag{61}$$

Similar symmetric equations hold for $F_2(z)$ and $Q_2(z)$, The position error in each loop is:

$$E_i(z) = \frac{Tz}{(z-1)} \left[\frac{z}{z-1} - \frac{Q_i(z)}{D_u(z)} \right] R_i; \quad i = 1,2 \tag{62}$$

Substitution of these errors into equation (43) yields:

$$\epsilon(z) = \frac{R_1 R_2 [(z-p_1)A_2 - (z-p_2)A_1] T z^3}{\sqrt{R_1^2 + R_2^2}\,(z-1) D_u(z)} \tag{63}$$

By applying the definitions of A_i for $i = 1,2$, and defining an average gain K as $K = \sqrt{K_1 K_2}$, equation (63) results in a steady-state contour error of

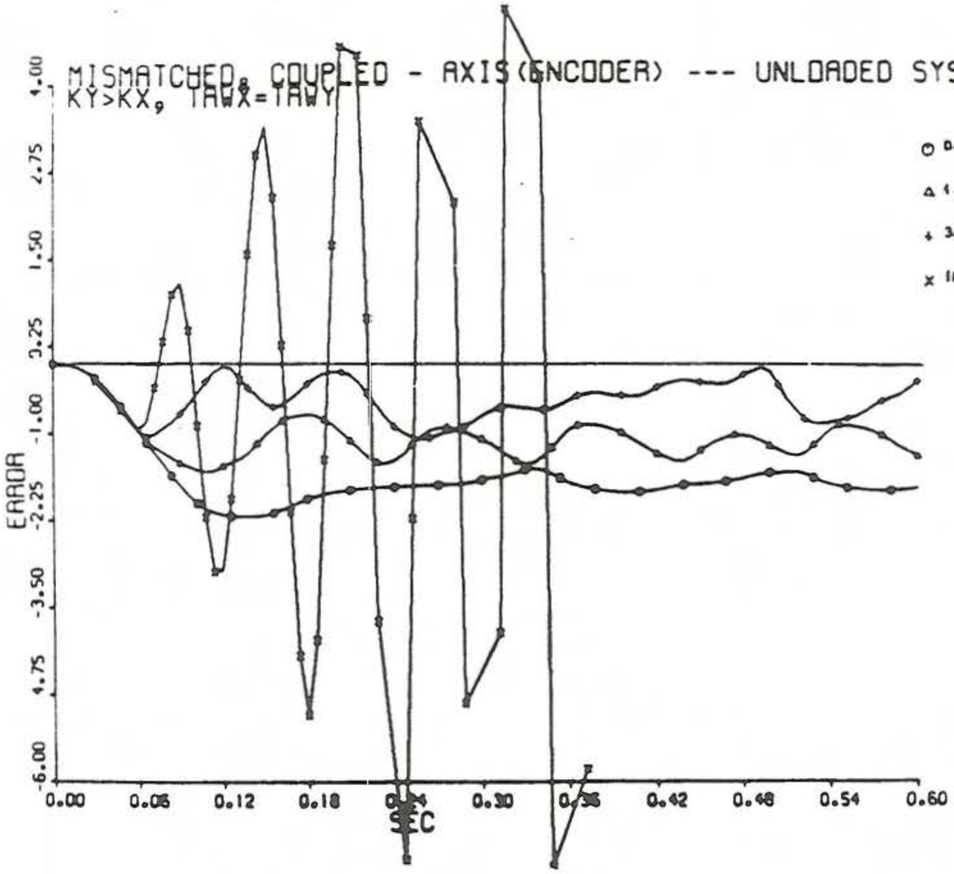

Fig. 7 Simulation results of contour errors in a mismatched conventional system (W = 0) and CCS (W = 1,3,10)

$$\epsilon_{ss} = \frac{R_1 R_2}{\sqrt{R_1^2 + R_2^2}} \frac{\Delta K/K}{KK_c(1+W)} \tag{64}$$

where $\Delta K = K_2 - K_1$. The corresponding results in a conventional system controlled by two separate loops is:

$$\epsilon_{ss0} = \frac{R_1 R_2}{\sqrt{R_1^2 + R_2^2}} \frac{\Delta K/K}{KK_c} \tag{65}$$

As might be expected, the steady-state contour error in the cross-coupled system is reduced by a factor of $(1 + W)$, with a minor effect on the performance of the individual loop; W is selected within the range given by condition (56).

In order to find the magnitude of typical errors let us assume that a gain of $KK_c = 20\ s^{-1}$ is applied and that a small difference of about 2 percent exists between the two loop gains. For a typical maximum contour velocity of 25 mm/s (60 ipm), and equal velocities in both axes, the steady-state contour error in a conventional system, as obtained from equation (65), is 18 μm (7 10^{-4} in.). Setting $W = 3$ in the corresponding cross-coupled system reduces the contour error to 4.5 μm. The simulation results are illustrated in Fig. 7. It is seen that increasing W to the value of 10.0 causes an unstable system; decreasing to $W = 1.0$ gives a larger steady-state error. Again selecting $W = 3$ seems to be the best compromise between the desire to reduce the contour error and stability considerations which rule out higher values of W.

In order to examine the combined effect of axis mismatch and external loading, constant axial loads were added to the previous simulation and the results are presented in Fig. 8. It is readily seen that while the steady-state contour error in a conventional system ($W = 0$) is about 50 μm, a proper choice of W, (i.e., W = 3) in the cross-coupled system, reduces this error to 10 μm, providing a substantial improvement in contour accuracy.

Conclusions

A symmetrical cross-coupled system for computer control of a biaxial system is analyzed and compared with its conventional counterpart. An improvement over the conventional system is obtained by considering the whole system as a single unit, rather than in terms of its individual loops. The influence of load disturbances and axis mismatch is reduced in the proposed cross-coupled system, while the velocity response along the generated path is not deteriorated. A cross-coupled software system requires no extra investment in hardware; all that is needed are a few additional statements in the control program with a view to improving the accuracy of the resultant path $y = f(x)$.

The cross-coupled structure is suitable for systems where the time response of each individual controlled variable takes on a smaller role relative to the intervariable dependence $y = f(x)$, such as in the case of machining process with manufacturing systems. The proposed method lends itself to multiaxial generalization and may be a first step towards improved control strategies to CNC of manufacturing systems.

Acknowledgment

The author would like to thank Professor S. Malkin for his helpful suggestions, and Dr. O. Masory for the simulation results.

References

1 Sarachik, P., and Ragazzini, J. R., "A Two Dimensional Feedback Control System," *Trans. AIEE*, Vol. 76, Part II, May 1957, pp. 55–61.

2 Poo, A., Bollinger, G., and Younkin, G. W., "Dynamic Errors in Type 1 Contouring Systems," *IEEE Trans. Ind. Appl.*, Vol. 1A-8, No. 4, July–Aug. 1972.

3 Koren, Y., and Ben-Uri, J., "Digital Control of Multi-Axial Motion