

## MATH 116 — PRACTICE FOR EXAM 2

NAME: SOLUTIONS

INSTRUCTOR: \_\_\_\_\_ SECTION NUMBER: \_\_\_\_\_

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1. This exam has 3 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a  $3'' \times 5''$  note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2013	3	2		6	
Fall 2014	2	4		12	
Winter 2012	3	2		13	
Total				31	

**Recommended time (based on points): 34 minutes**

2. [6 points] Let the sequence  $a_n$  be given by

$$a_1 = -1, \quad a_2 = \frac{\sqrt{2}}{3}, \quad a_3 = -\frac{\sqrt{3}}{5}, \quad a_4 = \frac{\sqrt{4}}{7}, \quad a_5 = -\frac{\sqrt{5}}{9}, \quad a_6 = \frac{\sqrt{6}}{11}$$

a. [1 point] Find  $a_7$ .

*Solution:*

$$a_7 = \frac{-\sqrt{7}}{13}.$$

b. [3 points] Write a formula for  $a_n$ .

*Solution:*

$$a_n = (-1)^n \frac{\sqrt{n}}{2n-1}.$$

c. [2 points] Does the sequence  $a_n$  converge? If so, find its limit.

*Solution:* Yes, it converges to 0.

4. [12 points] Consider the following sequences:

$$f_n = \frac{\pi^n}{e^n} \quad g_n = (-1)^n \sin(n) \quad h_n = \cos(e^{-n}) \quad i_n = \int_1^n \frac{1}{(x+3)^2} dx$$

For each sequence, circle **all** that apply. No justification is necessary.

a. [2 points] The sequence  $(f_n)$  is :

Bounded                       Increasing                      Decreasing

b. [2 points] The sequence  $(g_n)$  is :

Bounded                      Increasing                      Decreasing

c. [2 points] The sequence  $(h_n)$  is :

Bounded                       Increasing                      Decreasing

d. [2 points] The sequence  $(i_n)$  is :

Bounded                       Increasing                      Decreasing

e. [4 points] For each given sequence, if it converges, determine its limit and write that limit in the space provided. If the sequence diverges, write “diverges”. No justification is necessary.

$(f_n)$ : \_\_\_\_\_ **diverges** \_\_\_\_\_                       $(h_n)$ : \_\_\_\_\_ **1** \_\_\_\_\_

$(g_n)$ : \_\_\_\_\_ **diverges** \_\_\_\_\_                       $(i_n)$ : \_\_\_\_\_ **1/4** \_\_\_\_\_

2. [13 points] Determine if each of the following sequences is increasing, decreasing or neither, and whether it converges or diverges. Circle all the answers that apply. On parts **a-c**, if the sequence converges, find the limit. No justification is required.

- a. [3 points] For  $n \geq 1$ , let  $a_n = 3 + \frac{1}{n}$ .

*Solution:*

1. Increasing  **Decreasing**  Neither.
2. **Convergent** :  $\lim_{n \rightarrow \infty} a_n = 3$  Divergent

- b. [3 points] For  $n \geq 1$ , let  $a_n = \left(-\frac{\pi}{e}\right)^n$ .

*Solution:*

1. Increasing  Decreasing  **Neither.**
2. Convergent:  $\lim_{n \rightarrow \infty} a_n = \underline{\hspace{2cm}}$  **Divergent**

- c. [3 points] Let  $P(x)$  be the cumulative distribution function of a nonzero probability density function  $p(x)$ . Define  $a_n = P(n)$  for  $n \geq 1$ .

*Solution:*

1. **Increasing**  Decreasing  Neither.
2. **Convergent** :  $\lim_{n \rightarrow \infty} a_n = 1$  Divergent

- d. [2 points] For  $n \geq 1$ , let  $a_n = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + \frac{(-1)^n}{n!}$ .

*Solution:*

1. Increasing  Decreasing  **Neither.**
2. **Convergent**  (no need to compute the limit) Divergent

- e. [2 points] Let  $a_n = \int_2^n \frac{1}{\sqrt{x}-1} dx$ , for  $n \geq 2$ .

*Solution:*

1. **Increasing**  Decreasing  Neither.
2. Convergent (no need to compute the limit)  **Divergent**