Math 116 — Practice for Exam 1

Generated March 8, 2021

Name: SOLUTIONS

Instructor: ______________________ Section Number: __________

1. This exam has 3 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2019	2	6	bucket	8	
Fall 2017	1	9	chain	8	
Fall 2015	1	9	trash bag	10	
Total				26	

Recommended time (based on points): 23 minutes

6. [8 points] Derivative Girl lifts a bucket of water at a constant velocity from the ground up to a platform 50 meters above the ground. The bucket and water start at a total mass of 20 kg, but while it is being lifted, a total of 3 kg of water drips out at a steady rate through a hole in the bottom of the bucket.

For this problem, you may assume that acceleration due to gravity is $g = 9.8$ m/s^2.

a. [2 points] Give an expression giving the mass of the bucket and water when the bucket is h meters above ground. Include units.

Answer: Mass of water = $20 - \frac{3}{50}h$ **kg**

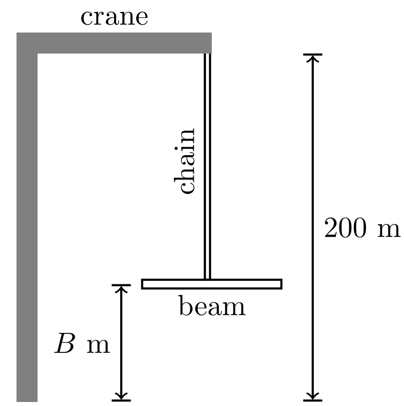
b. [3 points] Suppose Δh is small. Write an expression (not involving integrals) that approximates the work required to lift the bucket from a height of h meters above the ground to a height of $h + \Delta h$ meters above the ground. Include units.

Answer: Work $\approx$ $9.8(20 - \frac{3}{50}h)\Delta h$ **joules**

c. [3 points] Write, but do not evaluate, an integral that gives the work required to lift the bucket from the ground to the platform. Include units.

Answer: $\displaystyle\int_0^{50} 9.8(20 - \frac{3}{50}h)\, dh$ **joules**

9. [8 points]
During the construction of a skyscraper, a 200 meter tall crane lifts a steel beam from the ground to a height of 175 meters. The steel beam has a mass of 50 kilograms. The crane has a chain that is also made of steel, and the chain has a mass of 15 kilograms per meter. The total length of the chain is 200 meters, but as the beam is lifted, the crane no longer needs to lift any of the chain that has already been "reeled in", i.e. has already reached the top of the crane.



For this problem, you may assume the acceleration due to gravity is $g = 9.8$ m/s^2 .

a. Write an expression in terms of B that gives the total mass, in kilograms, of the steel beam together with the chain that has not yet been reeled in at the moment that the steel beam is B meters above the ground.

Solution: If the steel beam is B meters above the ground, that means that a length of B meters of chain has been reeled in, so $(200-B)$ meters of chain remains. The mass of this remaining chain is (15 kilograms per meter) $\cdot$ ($200 - B$ meters) $= 3000 - 15B$ kilograms. We add to this the mass of the steel beam to find a total mass of

$$\text{Mass} = 50 + 3000 - 15B \text{ kilograms.}$$

b. Assuming ΔB is very small but positive, write an expression in terms of B that approximates the work done by the crane in lifting the steel beam up ΔB meters starting from a height of B meters above the ground. Assume that the weight of the chain being lifted is constant over this very short distance. Include units.

Solution: The force due to gravity is the weight. At the moment the steel beam is B meters above the ground, the weight of the beam together with the chain that has not yet been reeled in is

$$\text{Weight} = (\text{mass})(g) = (3050 - 15B \text{ kilograms})(9.8 \text{ m/s}^2) = 29890 - 147B \text{ Newtons.}$$

The work to lift the steel beam over the small distance ΔB is then approximately

$$(\text{Force}) \cdot \Delta B = (29890 - 147B)\Delta B \text{ Joules.}$$

c. Write, but do **not** evaluate, an expression involving one or more integrals that gives the total work that must be done by the crane in order to lift the steel beam from the ground to a height of 175 meters. Include units.

Solution: Using our answer from part **b.** above, summing over the entire path of the beam, and taking the limit as ΔB approaches 0, we find that the total work is

$$\text{Work} = \int_0^{175} g(3050 - 15B)\, dB = \int_0^{175} (29890 - 147B)\, dB \text{ Joules.}$$

9. [10 points] Tracy Johnson is taking the trash out of her apartment on State Street. In order to throw the trash bag into the dumpster, she lifts the bag at a constant speed from the ground up to a height of 2 meters. The bag of trash weighs 5 kg initially, but an unfortunate hole causes the bag to leak trash at a constant rate. Recall that the gravitational constant is $g = 9.8$ m/s^2.

If the bag weighs 3 kg when it is one meter from the ground, compute the work required for Tracy to lift the bag of trash into the dumpster. Evaluate **by hand** any integrals you compute.

Solution: The trash bag is leaking at a constant rate, so the weight of the bag is a linear function of the height above the ground. Using the information, the bag has a mass of $5 - 2h$ kg at a height of h meters. Since the force on the bag at height h is $(5 - 2h)g$, the total work is $\int_0^2 (5 - 2h)g\, dh = 6g = 58.8$ joules.