Summary of 4.7 to 7.7

1 L'Hopital's rule

LHopitals rule: If f and g are differentiable and (below a can be $\pm\infty$)
i)$f(a) = g(a) = 0$ for finite a,
Or ii)$\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = \pm\infty$,
Or iii)$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} g(x) = 0$ then

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

1.1 Dominance

We say that g dominates f as $x \to \infty$ if $\lim_{x \to \infty} f(x)/g(x) = 0$.

1.2 How to determine some bad limit?

There are several types of the limits that is "bad" which requires L'Hopital's rule to calculate: $0/0, \infty/\infty, \infty \cdot 0$. Although the first two cases we can use L'Hopital's rule to calculate, the others we cannot use it directly.
Read the book, and there are several things that we can consider.

- Consider taking log.

- Consider $1/f(x)$ so that we can transform ∞ to '1/0' or 0 to '1/∞'.

2 Improper integral

Formal definition of the improper integral I will let you read the book carefully, they are in the box. However, informally, there are two types of improper integral which we just interpret them as a limit.

- The first case is where we have the limit of the integration goes to infinity, i.e. $\lim_{b \to \infty} \int_a^b f(x)dx$.

- The integrand goes to infinity as $x \to a$.

The basic question that one want to know about the improper integral is basically is it well defined?

This turns to ask if an improper integral converges or not.

There are four ways people ususaly use to check this fact.

1. Check by definition, this means check the limit directly.

2. p-test.

	$p < 1$	$p = 1$	$p > 1$
Type I : $\displaystyle\int_a^\infty \frac{1}{x^p}\,dx$	diverges	$= \ln x \big|_a^\infty \Rightarrow$ diverges	converges
Type II : $\displaystyle\int_0^a \frac{1}{x^p}\,dx$	converges	$= \ln x \big|_0^a \Rightarrow$ diverges	diverges

3. Exponential decay test.

$$\int_0^\infty e^{-ax}\,dx$$

converges for $a > 0$.

4. Comparison test.
   If $f(x) \geq g(x) \geq 0$ on the interval $[a, \infty]$ then,

   - If $\int_a^\infty f(x)dx$ converges then so does $\int_a^\infty g(x)dx$.
   - If $\int_a^\infty g(x)dx$ diverges then so does $\int_a^\infty f(x)dx$.

5. Limit Comparison theorem.
   Limit Comparison Test. If $f(x)$ and $g(x)$ are both positive on the interval $[a, b)$ where b could be a real number or infinity. and

   $$\lim_{x \to b} \frac{f(x)}{g(x)} = C$$

   such that $0 < C < \infty$ then the improper integrals $\int_a^b f(x)dx$ and $\int_a^b g(x)dx$ are either both convergent or both divergent.