Math 116 — Practice for Exam 2

Generated March 8, 2021

Name: ______________________________

Instructor: ______________________ Section Number: __________

1. This exam has 6 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2012	2	8		12	
Winter 2016	2	9		12	
Winter 2013	2	4		13	
Winter 2012	2	7	rod	9	
Winter 2018	2	8		7	
Winter 2013	2	6		11	
Total				64	

Recommended time (based on points): 58 minutes

8. [12 points] Determine if the following integrals converge or diverge. Justify your answer. If you use the comparison test, be sure to show all your work.

a. [3 points] $\displaystyle\int_1^\infty \frac{1}{x+e^x}dx.$

b. [4 points] $\displaystyle\int_1^e \frac{1}{x(\ln x)^2}dx.$

c. [5 points] $\displaystyle\int_{2\pi}^\infty \frac{x\cos^2 x+1}{x^3}dx.$

9. [12 points]

a. [6 points] Show that the following integral diverges. Give full evidence supporting your answer, showing all your work and indicating any theorems about improper integrals you use.

$$\int_1^{\infty} \frac{\cos(\frac{1}{t})}{\sqrt{t}}\,dt$$

b. [6 points] Find the limit

$$\lim_{x\to\infty} \frac{\int_1^x \frac{\cos(\frac{1}{t})}{\sqrt{t}}\,dt}{\sqrt{x}}$$

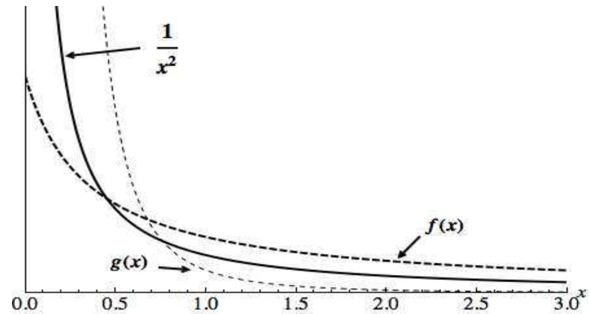
4. [13 points]

a. [8 points] Consider the functions $f(x)$ and $g(x)$ where

$$\frac{1}{x^2} \le g(x) \quad \text{for} \quad 0 < x < \frac{1}{2}.$$

$$g(x) \le \frac{1}{x^2} \quad \text{for} \quad 1 < x$$

$$\frac{1}{x^2} \le f(x) \quad \text{for} \quad 1 < x.$$



Using the information about $f(x)$ and $g(x)$ provided above, determine which of the following integrals is convergent or divergent. Circle your answers. If there is not enough information given to determine the convergence or divergence of the integral circle NI.

i) $\int_1^\infty f(x)dx$ CONVERGENT DIVERGENT NI

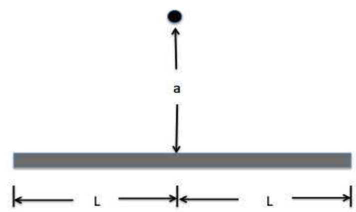
ii) $\int_1^\infty g(x)dx$ CONVERGENT DIVERGENT NI

iii) $\int_0^1 f(x)dx$ CONVERGENT DIVERGENT NI

iv) $\int_0^1 g(x)dx$ CONVERGENT DIVERGENT NI

b. [5 points] Does $\int_e^\infty \frac{1}{x(\ln x)^2}dx$ converge or diverge? If the integral converges, compute its value. Show all your work. Use u substitution.

7. [9 points] If a particle of mass m is positioned at a perpendicular distance a from the center of a rod of length $2L$ and constant mass density δ as shown below



The force of gravitational attraction F between the rod and the particle is given by

$$F = Gm\delta a \int_{-L}^{L} \frac{1}{(a^2 + x^2)^{\frac{3}{2}}} dx.$$

a. [5 points] Does the force of gravitational attraction F approach infinity as the length of the rod goes to infinity? Justify your answer using the comparison test.

b. [4 points] Consider the integral

$$I = \int_{1}^{\infty} \frac{1}{(a^2 + x^2)^p} dx$$

i. Give a power function that if integrated over $[1, \infty)$ will have the same convergence or divergence behavior as I.

ii. For what values of p is I convergent or divergent?

8. [7 points] Consider the integral

$$\int_1^\infty \frac{e^{rx}}{x}\,dx,$$

where r is a constant.

a. [3 points] Show that this integral converges for $r < 0$. **Show all work and indicate any convergence tests used.**

b. [4 points] Show that the integral diverges for $r \geq 0$. **Show all work and indicate any convergence tests used.**

6. [11 points]

a. [8 points] Use the **comparison method** to determine the convergence or divergence of the following improper integrals. Justify your answers. Make sure to properly cite any results of convergence or divergence of integrals that you use.

i) $\displaystyle\int_1^{\infty} \frac{3+\sin(4x)}{\sqrt[3]{x}} dx.$

ii) $\displaystyle\int_4^{\infty} \frac{1}{\sqrt{x}+x^2} dx.$

b. [3 points] For which values of p does the following integral converges?

$$\int_2^{\infty} \frac{x^2-1}{x^p+4x^2+2} dx.$$

No justification is required.