

SA Additional Robustness Analyses

In this appendix, we investigate whether our results are robust to various variations to the demand side and the supply side of the model. We also estimate a parametric fixed cost function which allows for potential economies or diseconomies of scope in fixed costs. These robustness analyses as well as the robustness analyses in Appendix B.2 are summarized in Table SA.1.

Table SA.1: Summary of Robustness Analyses

Demand Specification		Section
(1)	add a random coefficient for the Apple dummy variable	SA.1
(2)	add a random coefficient for each carrier dummy variable	SA.1
(3)	add brand/year fixed effects	SA.1
(4)	add the age of a product and its square	SA.1
Supply Specification		
(5)	Apple and AT&T joint price setting when iPhone was exclusively on AT&T	SA.2
(6)	all smartphone firm/carrier pairs joint price setting	SA.2
(7)	nonlinear pricing with transfers to carriers	SA.2
(8)	nonlinear pricing with transfers to smartphone firms	SA.2
(9)	allow for economies or diseconomies of scope in fixed costs	SA.3
Merger Simulation Specification		
(10)	post-merger Samsung-LG brand effect = the average of the pre-merger Samsung and LG brand effects	B.1.1
(11)	two alternative fixed costs ranges	B.1.2
(12)	allow for adjusting old or all flagship products	B.1.3
(13)	allow random coefficients to be independent	B.1.4
(14)	Samsung-Motorola merger and LG-Motorola merger	B.2

Note: We repeat the estimation and all counterfactual simulations for (1)(2)(3)(4)(5).

We repeat the estimation and the counterfactual simulation in Section 5.1 for (6), (7) and (8). We will need additional assumptions to determine how a smartphone firm and a carrier split the profit in order to conduct merger simulations.

We repeat the merger simulation for (10)(11)(12)(13)(14). We also re-estimate the supply side for (13).

SA.1 Alternative Demand Specifications

On the demand side, one concern with our discrete choice model is that the assumption of independent idiosyncratic shocks may lead us to overestimate the effect of removing or adding a product on consumer surplus. One way we address this concern is that we report $\Delta(\text{consumer surplus})$ ignoring changes in Logit errors (see Section 5). In this section, we address this concern by conducting two robustness analyses where we add more random coefficients in order to allow for a greater correlation among the utilities that a consumer gets from different products.

In the first robustness analysis, we add a random coefficient for the Apple dummy variable and allow this random coefficient to be correlated with the quality random coefficient. The estima-

tion results in Table SA.2(a) indicate that the standard deviation of the Apple-dummy random coefficient is 2.625 and that this random coefficient is highly correlated with the quality random coefficient (the estimated correlation is 0.991). Unfortunately, both estimates are statistically insignificant. For the parameters common to both models, both the estimates and the statistical significance levels are robust. More importantly, the results from the counterfactual simulations, which allow us to address our research questions, are also robust (see Tables SA.2(b)-(d)). For example, we still find that removing a product reduces total surplus even considering the maximum possible saving in the fixed cost, that adding a product increases total surplus as long as the fixed cost is not much higher than its lower bound, and that a merger leads to a reduction in product offerings and eventually a decrease in total welfare.

In the second robustness analysis, we add four random coefficients, one for each carrier dummy variable. The estimation results in Table SA.3(a) show that the standard deviations of all carrier dummy variable coefficients, except that for T-Mobile, are small (compared to their corresponding means) and statistically insignificant. The estimates for the parameters common to the two models are robust. Moreover, all qualitative conclusions we draw from counterfactual simulation results also hold (see Tables SA.3(b)-(d)).

To further check the robustness of our results, we conduct two additional analyses where we replace the brand fixed effects by the brand/year fixed effects in one, and include the age of a product (i.e., how long a product has been in the market) and the square of it in the other. Our results are again robust (see Tables SA.4 and SA.5).

SA.2 Alternative Pricing Models

On the supply side, the pricing model of the baseline specification is a simple linear pricing model. In this section, we consider four alternative pricing models.

SA.2.1 Overview of Four Alternative Pricing Models

The simple linear pricing model in the baseline specification implies that there exists double marginalization as follows:

$$\mathbf{p} = (-\Gamma_c \circ \Delta_c)^{-1} \mathbf{s} + (-\Gamma_m \circ \Delta_m)^{-1} \mathbf{s} + \tilde{\mathbf{m}}\mathbf{c}, \quad (\text{SA.1})$$

where the operator \circ represents the element-wise multiplicity, and Γ_c is a matrix whose (i, j) element = 1 if products i and j are sold by the same carrier, and 0 otherwise. Analogously, Γ_m is a matrix whose (i, j) element = 1 if and only if products i and j are produced by the same smartphone firm. While Γ_c and Γ_m describe the “ownership,” the other two matrices, Δ_c and Δ_m , describe the price sensitivity of demand. Specifically, the (i, j) element of Δ_c and Δ_m are, respectively, $\frac{\partial s_j}{\partial p_i}$ and $\sum_k \frac{\partial s_j}{\partial p_k} \frac{\partial p_k^*}{\partial w_i}$.

Table SA.2: Robustness Analysis: Allowing an Apple Random Coefficient

(a) Estimation Results

	Parameter	Std. Error
Demand		
Quality coefficient		
battery talk time (hour)	0.052***	0.016
camera resolution (megapixel)	0.109***	0.046
chipset generation 2	0.444***	0.137
chipset generation 3	0.743***	0.180
chipset generation 4	1.145***	0.261
chipset generation 5	1.857***	0.385
screen size (inch)	1	
weight (gram)	-0.002*	0.002
Covariance of random coefficients		
std. dev., quality	0.214**	0.104
std. dev., Apple dummy	2.625	2.248
correlation	0.991	1.559
Price	-0.006	0.079
Apple	0.030	2.059
BlackBerry	1.149***	0.132
Samsung	0.337***	0.069
Flagship?	0.592***	0.069
Carrier/year and quarter dummies		Yes
Marginal Cost (\$)		
Exp(quality/10)	544.583***	2.908
Apple	-252.177***	0.150
BlackBerry	104.275***	0.510
Samsung	-20.101***	0.151
Carrier/year dummies		Yes

* indicates 90% level of significance. ** indicates 95% level of significance.
 *** indicates 99% level of significance.

(b) Welfare Changes when a Product is Removed, March 2013 (million \$)

Removed product	Lowest-quality	Median	Highest
Δ (consumer surplus)	-1.35	-3.97	-13.19
Δ (carrier surplus)	-1.25	-2.42	-7.78
Δ (sum of smartphone firms' variable profits)	-0.81	-1.42	-3.05
Upper bound of savings in fixed costs	1.38	3.28	10.30

(c) Welfare Changes when a Product is Added, March 2013 (million \$)

	HTC	LG	Motorola	Samsung
Δ (consumer surplus)	4.03	4.07	4.19	4.90
Δ (carrier surplus)	2.41	2.44	2.51	3.03
Δ (sum of smartphone firms' variable profits)	1.46	1.44	1.38	2.33
Lower bound of added fixed costs	3.33	3.35	3.41	4.31

(d) The Effect of Samsung-LG Merger in March 2013

Variable	Pre-merger	Post-merger	Change
Number of products	70	63.20	-6.80
Variety	324.84	283.86	-40.98
Sales-weighted avg quality	6.84	6.83	-0.01
Sales-weighted avg price (\$)	89.19	92.94	3.74
Total sales	7,381,282	7,174,165	-207,117
Consumer surplus (million \$)	2664.69	2595.97	-68.71
Carrier profit (million \$)	1683.55	1639.39	-44.16
Smartphone firm profit (million \$)	1829.64	1864.89	35.24

Table SA.3: Robustness Analysis: Allowing Carrier Random Coefficients

(a) Estimation Results

	Parameter	Std. Error
Demand		
Quality coefficient		
battery talk time (hour)	0.067**	0.032
camera resolution (megapixel)	0.112***	0.043
chipset generation 2	0.456***	0.177
chipset generation 3	0.780***	0.229
chipset generation 4	1.097***	0.275
chipset generation 5	1.786***	0.373
screen size (inch)	1	
weight (gram)	-0.001	0.002
Std. dev. of random coefficients		
quality	0.349*	0.213
AT&T	0.018	23.410
Sprint	0.394	33.860
T-Mobile	4.241**	1.997
Verizon	0.394	33.860
Price	-0.008***	0.003
Apple	2.741***	0.192
BlackBerry	1.253***	0.175
Samsung	0.335***	0.076
Flagship?	0.587***	0.114
Carrier/year and quarter dummies		Yes
Marginal Cost (\$)		
Exp(quality/10)	459.944***	2.816
Apple	-47.073***	0.134
BlackBerry	87.343***	0.521
Samsung	-28.573***	0.148
Carrier/year dummies		Yes

* indicates 90% level of significance. ** indicates 95% level of significance.
 *** indicates 99% level of significance.

(b) Welfare Changes when a Product is Removed, March 2013 (million \$)

Removed product	Lowest-quality	Median	Highest
Δ (consumer surplus)	-1.38	-3.42	-11.29
Δ (carrier surplus)	-1.59	-1.72	-11.67
Δ (sum of smartphone firms' variable profits)	-0.09	-0.97	-2.49
Upper bound of savings in fixed costs	1.25	2.79	12.55

(c) Welfare Changes when a Product is Added, March 2013 (million \$)

	HTC	LG	Motorola	Samsung
Δ (consumer surplus)	3.68	3.74	3.91	4.62
Δ (carrier surplus)	1.57	1.60	1.68	2.09
Δ (sum of smartphone firms' variable profits)	1.01	0.99	0.89	1.80
Lower bound of added fixed costs	2.74	2.77	2.85	3.69

(d) The Effect of Samsung-LG Merger in March 2013

Variable	Pre-merger	Post-merger	Change
Number of products	70	45	-25
Variety	379.09	237.42	-141.67
Sales-weighted avg quality	8.46	8.53	0.06
Sales-weighted avg price (\$)	89.05	99.29	10.24
Total sales	8,017,672	7,785,397	-232,275
Consumer surplus (million \$)	2254.57	2183.44	-71.13
Carrier profit (million \$)	1616.98	1592.77	-24.21
Smartphone firm profit (million \$)	1349.27	1431.09	81.82

Table SA.4: Robustness Analysis: Allowing Brand/Year Fixed Effects

(a) Estimation Results

	Parameter	Std. Error
Demand		
Quality coefficient		
battery talk time (hours)	0.056***	0.015
camera resolution (megapixel)	0.092**	0.041
chipset generation 2	0.538***	0.146
chipset generation 3	0.840***	0.199
chipset generation 4	1.142***	0.276
chipset generation 5	1.811***	0.375
screen size (inch)	1	
weight (gram)	-0.003**	0.002
Covariance of random coefficients		
mean	0.530***	0.139
std. dev.	0.340***	0.075
Price	-0.003**	0.002
Flagship?	0.663***	0.063
Brand/year, carrier/year and quarter dummies		Yes
Marginal Cost (\$)		
Exp(quality/10)	463.048***	2.397
Apple	-73.115***	0.128
BlackBerry	89.889***	0.41
Samsung	-29.909***	0.131
Carrier/year dummies		Yes

* indicates 90% level of significance. ** indicates 95% level of significance.

*** indicates 99% level of significance.

(b) Welfare Changes when a Product is Removed, March 2013 (million \$)

Removed product	Lowest-quality	Median	Highest
Δ (consumer surplus)	-0.54	-3.25	-22.36
Δ (carrier surplus)	-0.38	-1.69	-17.41
Δ (sum of smartphone firms' variable profits)	-0.27	-1.34	-2.00
Upper bound of savings in fixed costs	0.49	2.79	20.52

(c) Welfare Changes when a Product is Added, March 2013 (million \$)

	HTC	LG	Motorola	Samsung
Δ (consumer surplus)	4.94	5.06	5.25	4.70
Δ (carrier surplus)	2.13	2.19	2.26	2.25
Δ (sum of smartphone firms' variable profits)	1.16	1.10	1.00	1.78
Lower bound of added fixed costs	3.68	3.73	3.82	3.82

(d) The Effect of Samsung-LG Merger in March 2013

Variable	Pre-merger	Post-merger	Change
Number of products	70	67.80	-2.20
Variety	453.04	443.55	-9.49
Sales-weighted avg quality	10.37	10.41	0.04
Sales-weighted avg price (\$)	160.67	165.28	4.60
Total sales	7,527,086	7,446,036	-81,050
Consumer surplus (million \$)	3752.69	3696.12	-56.57
Carrier profit (million \$)	2704.77	2674.91	-29.86
Smartphone firm profit (million \$)	2722.92	2753.62	30.70

Table SA.5: Robustness Analysis: Allowing Age in the Utility Function

(a) Estimation Results

	Parameter	Std. Error
Demand		
Quality coefficient		
battery talk time (hours)	0.046***	0.013
camera resolution (megapixel)	0.091***	0.036
chipset generation 2	0.240***	0.100
chipset generation 3	0.344***	0.131
chipset generation 4	0.461***	0.185
chipset generation 5	0.594**	0.260
screen size (inch)	1	
weight (gram)	0.0004	0.002
Covariance of random coefficients		
mean	0.802***	0.146
std. dev.	0.291***	0.093
Price	-0.238***	0.078
Apple	3.121***	0.113
BlackBerry	1.233***	0.117
Samsung	0.403***	0.070
Flagship?	0.911***	0.081
Age	-0.292***	0.037
Age ²	0.013***	0.003
Carrier/year and quarter dummies		Yes
Marginal Cost (\$)		
Exp(quality/10)	724.827***	2.988
Apple	-13.431***	0.108
BlackBerry	119.201***	0.458
Samsung	-18.228***	0.122
Carrier/year dummies		Yes

* indicates 90% level of significance. ** indicates 95% level of significance.
 *** indicates 99% level of significance.

(b) Welfare Changes when a Product is Removed, March 2013 (million \$)

Removed product	Lowest-quality	Median	Highest
Δ (consumer surplus)	-1.61	-1.88	-3.30
Δ (carrier surplus)	-1.48	-0.94	-2.88
Δ (sum of smartphone firms' variable profits)	-0.58	-0.65	-0.28
Upper bound of savings in fixed costs	1.55	1.54	2.91

(c) Welfare Changes when a Product is Added, March 2013 (million \$)

	HTC	LG	Motorola	Samsung
Δ (consumer surplus)	4.02	4.00	4.16	6.59
Δ (carrier surplus)	1.92	1.92	1.99	3.36
Δ (sum of smartphone firms' variable profits)	1.31	1.31	1.24	2.96
Lower bound of added fixed costs	3.20	3.19	3.26	5.65

(d) The Effect of Samsung-LG Merger in March 2013

Variable	Pre-merger	Post-merger	Change
Number of products	70	68.20	-1.80
Variety	310.41	295.64	-14.77
Sales-weighted avg quality	8.10	8.12	0.02
Sales-weighted avg price (\$)	68.34	70.15	1.81
Total sales	8,182,330	8,113,210	-69,120
Consumer surplus (million \$)	1361.15	1346.09	-15.07
Carrier profit (million \$)	1097.12	1088.13	-8.99
Smartphone firm profit (million \$)	1030.36	1037.55	7.19

As pointed out by Villas-Boas and Hellerstein (2006), it is possible that the pricing strategies of smartphone firms and/or carriers deviate from a linear pricing model. Villas-Boas and Hellerstein (2006) introduce two vectors Λ_c and Λ_m to capture such deviations so that the following equation describes the pricing behavior:

$$\mathbf{p} = \left[(-\bar{\Gamma}_c \circ \Delta_c)^{-1} \mathbf{s} \right] \circ \Lambda_c + \left[(-\bar{\Gamma}_m \circ \Delta_m)^{-1} \mathbf{s} \right] \circ \Lambda_m + \tilde{\mathbf{m}}\mathbf{c}, \quad (\text{SA.2})$$

where the “ownership” matrices $\bar{\Gamma}_c$ and $\bar{\Gamma}_m$ can also deviate from those in the simple linear pricing model (i.e., Γ_c and Γ_m).

The baseline model is a case where Λ_c and Λ_m are both constant-1 vectors and ($\bar{\Gamma}_c = \Gamma_c$, $\bar{\Gamma}_m = \Gamma_m$). With a slight abuse of notation, we refer to this case as ($\Lambda_c = 1$, $\Lambda_m = 1$, $\bar{\Gamma}_c = \Gamma_c$, $\bar{\Gamma}_m = \Gamma_m$). We now consider four alternative deviations from the baseline model:

Alternative Pricing Model 1. In this case, Apple and AT&T make the pricing decisions jointly during the time when they had an exclusive contract (i.e., AT&T was the sole seller for iPhones before February 2011). Specifically, we allow Apple and AT&T to set their pre-February 2011 iPhone prices jointly to maximize their joint profit from iPhones. At the same time, other carriers choose their retail prices to maximize their profits and AT&T chooses its retail prices for its non-iPhone products to maximize its profit from non-iPhone products.

In this model, $\Lambda_c = 1$, $\bar{\Gamma}_m = \Gamma_m$, and

$$\Lambda_m(j) = \begin{cases} 1 & \text{if } j \in \text{same non-Apple smartphone firm} \\ 0 & \text{otherwise} \end{cases} \quad (\text{SA.3})$$

$$\bar{\Gamma}_c(i, j) = \begin{cases} 1 & \text{if } i, j \in (\text{iPhones}) \text{ or } (\text{AT\&T and non-iPhones}) \text{ or } (\text{same non-AT\&T carrier}) \\ 0 & \text{otherwise.} \end{cases}$$

before February 2011 and they are the same as the baseline model afterwards.

Alternative Pricing Model 2. In this case, all smartphones and all carriers jointly set retail prices. Specifically, we consider each smartphone firm/carrier pair (m, c) to jointly solve the following maximization problem:

$$\max_{\mathbf{p}, j \in \mathcal{J}_{mc}} \underbrace{\sum_{j \in \mathcal{J}_{mc}} \Pi_j(\mathbf{p})}_{\text{pair } (m,c)\text{'s profit}} + \underbrace{\mu_m \sum_{j \in \mathcal{J}_m, j \notin \mathcal{J}_c} \tau \Pi_j(\mathbf{p})}_{\text{firm } m\text{'s profit from other products}} + \underbrace{\mu_c \sum_{j \in \mathcal{J}_c, j \notin \mathcal{J}_m} (1 - \tau) \Pi_j(\mathbf{p})}_{\text{carrier } c\text{'s profit from other products}}, \quad (\text{SA.4})$$

where $\Pi_j(\mathbf{p}) = (p_j - \tilde{\mathbf{m}}c_j) s_j(\mathbf{p})$ is the joint profit from selling product j , the parameter τ is the share of profit that goes to a smartphone firm (and thus $1 - \tau$ is the share for a carrier), and μ_m and μ_c are, respectively, the weights that the smartphone firm/carrier pair puts on the smartphone firm’s profit from selling other products and the carrier’s profit from selling other products. This

model is therefore equivalent to $\Lambda_c = 1, \Lambda_m = 0$, and

$$\bar{\Gamma}_c(i, j) = \begin{cases} 1 & \text{if } i, j \in \text{same smartphone firm/carrier,} \\ \mu_m \tau & \text{if } i, j \in \text{same smartphone firm, but different carriers,} \\ \mu_c (1 - \tau) & \text{if } i, j \in \text{same carrier, but different smartphone firms,} \\ 0 & \text{otherwise.} \end{cases} \quad (\text{SA.5})$$

Alternative Pricing Model 3. In this case, smartphone firms decide the retail prices directly. Smartphone firm m 's profit maximization problem is:

$$\max_{p_j, j \in \mathcal{J}_m} \sum_{j \in \mathcal{J}_m} (p_j - \tilde{m}c_j) s_j(\mathbf{p}). \quad (\text{SA.6})$$

The first-order condition is equivalent to (SA.2) where $(\Lambda_c = 0, \Lambda_m = 1, \bar{\Gamma}_c = \Gamma_c, \bar{\Gamma}_m = \Gamma_m)$.³⁰

Alternative Pricing Model 4. In this case, carriers choose the retail prices while facing a wholesale price that equals the marginal cost of each product. In other words, carrier c 's profit maximization problem is:

$$\max_{p_j, j \in \mathcal{J}_c} \sum_{j \in \mathcal{J}_c} (p_j - \tilde{m}c_j) s_j(\mathbf{p}). \quad (\text{SA.7})$$

The first-order condition is equivalent to (SA.2) where $(\Lambda_c = 1, \Lambda_m = 0, \bar{\Gamma}_c = \Gamma_c, \bar{\Gamma}_m = \Gamma_m)$.

SA.2.2 Results from the Four Alternative Pricing Models

In what follows, we present the results from these four robustness analyses. Specifically, we re-estimate the marginal cost parameters and the bounds on the fixed costs and repeat our counterfactual simulations. For simplicity of exposition, we suppress the subscript “t” and ignore the distinction between j and \tilde{j} .

Alternative Pricing Model 1

In this model, we allow Apple and AT&T to set the iPhone prices jointly to maximize their joint profit from iPhones during the time when they had an exclusive contract before February 2011. Our results in Table SA.6 show that our findings are robust to this alternative pricing model.

Alternative Pricing Model 2

We now consider the alternative supply-side model where a smartphone firm and a carrier jointly set the retail price of their products. Specifically, we consider two different choices of (μ_c, μ_m, τ) in equation (SA.4). In this model, how the two parties split the joint profit is determined by the

³⁰In this case, the (i, j) element of Δ_m is $\frac{\partial s_j}{\partial p_i}$.

Table SA.6: Robustness Analysis: Apple and AT&T Joint Price Setting before February 2011

(a) Estimation Results of Marginal Cost Parameters

	Parameter	Std. Error
Exp(quality/10)	460.828***	2.274
Apple	6.473***	0.107
BlackBerry	86.426***	0.393
Samsung	-17.546***	0.119
Carrier/year dummies	Yes	

*** indicates 99% level of significance.

(b) Welfare Changes when a Product is Removed, March 2013 (million \$)

Removed product	Lowest-quality	Median	Highest
Δ (consumer surplus)	-0.93	-3.31	-14.31
Δ (carrier surplus)	-0.86	-1.50	-12.32
Δ (sum of smartphone firms' variable profits)	-0.52	-1.26	-5.43
Upper bound of savings in fixed costs	0.96	2.74	15.92

(c) Welfare Changes when a Product is Added, March 2013 (million \$)

	HTC	LG	Motorola	Samsung
Δ (consumer surplus)	2.21	2.24	2.34	2.49
Δ (carrier surplus)	1.17	1.20	1.24	1.41
Δ (sum of smartphone firms' variable profits)	0.99	0.97	0.93	1.46
Lower bound of added fixed costs	1.92	1.94	1.99	2.33

(d) The Effect of Samsung-LG Merger in March 2013

Variable	Pre-merger	Post-merger	Change
Number of products	70	68.20	-1.80
Variety	360.25	343.14	-17.10
Sales-weighted avg quality	8.44	8.46	0.02
Sales-weighted avg price (\$)	121.75	124.50	2.75
Total sales	6,957,236	6,857,563	-99,674
Consumer surplus (million \$)	1674.59	1642.17	-32.42
Carrier profit (million \$)	1271.55	1252.55	-19.00
Smartphone firm profit (million \$)	1090.46	1105.40	14.94

parameter τ , i.e., the share of profit that goes to a smartphone firm. Table SA.7 presents the results, which are again robust.

Alternative Pricing Model 3

In this case, the per-unit profit for a carrier is zero and there should be a transfer from a smartphone firm to a carrier. Let T_m be the total transfer that a smartphone firm m pays, $T_{m,\setminus j}$ be the transfer when product j is removed from m 's product portfolio and $T_{m,\cup j}$ be the transfer when product j is added to m 's product portfolio. Then, the two inequalities (11) and (12) in Section 3, which capture the optimal conditions for m 's product choice in the baseline model, become:

$$E_{(\xi,\eta)}\pi_m(\mathbf{q}, \xi, \eta) - F_j - T_m \geq E_{(\xi \setminus \xi_j, \eta \setminus \eta_j)}\pi_m(\mathbf{q} \setminus q_j, \xi \setminus \xi_j, \eta \setminus \eta_j) - T_{m,\setminus j} \text{ for any } j \in \mathcal{J}_m \text{ (SA.8)}$$

$$E_{(\xi,\eta)}\pi_m(\mathbf{q}, \xi, \eta) - T_m \geq E_{(\xi \cup \xi_j, \eta \cup \eta_j)}\pi_m(\mathbf{q} \cup q_j, \xi \cup \xi_j, \eta \cup \eta_j) - F_j - T_{m,\cup j} \text{ for any } j \notin \mathcal{J}_m.$$

Table SA.7: Robustness Test: Smartphone Firm/Carrier Pairs Joint Price Setting

 (a) $(\mu_c = 0, \mu_m = 0)^a$

(a.1) Welfare Changes when a Product is Removed, March 2013 (million \$)

Removed product	Lowest-quality	Median	Highest
$\Delta(\text{consumer surplus})$	-1.03	-2.22	-13.88
$\Delta(\text{total producer surplus net of fixed costs})$	-0.66	-0.88	-7.10
Upper bound of savings in fixed costs	0.49	0.99	7.07

(a.2) Welfare Changes when a Product is Added, March 2013 (million \$)

	HTC	LG	Motorola	Samsung
$\Delta(\text{consumer surplus})$	1.78	1.79	1.80	2.31
$\Delta(\text{total producer surplus net of fixed costs})$	0.88	0.86	0.86	1.22
Lower bound of added fixed costs	0.82	0.82	0.84	1.02

^aIn this case, the value of τ is irrelevant.

 (b) $(\mu_c = 0.5, \mu_m = 0.5, \tau = 0.5)$

(b.1) Welfare Changes when a Product is Removed, March 2013 (million \$)

Removed product	Lowest-quality	Median	Highest
$\Delta(\text{consumer surplus})$	-0.98	-2.31	-13.58
$\Delta(\text{total producer surplus net of fixed costs})$	-0.68	-0.88	-7.66
Upper bound of savings in fixed costs	0.50	1.03	7.46

(b.2) Welfare Changes when a Product is Added, March 2013 (million \$)

	HTC	LG	Motorola	Samsung
$\Delta(\text{consumer surplus})$	1.79	1.80	1.82	2.26
$\Delta(\text{total producer surplus net of fixed costs})$	0.88	0.88	0.86	1.24
Lower bound of added fixed costs	0.84	0.84	0.85	1.04

The two inequalities in (SA.8) imply that for any $j \in \mathcal{J}_m$,

$$\begin{aligned}
 F_j &\leq \left[E_{(\xi, \eta)} \pi_m(\mathbf{q}, \xi, \eta) - E_{(\xi \setminus \xi_j, \eta \setminus \eta_j)} \pi_m(\mathbf{q} \setminus q_j, \xi \setminus \xi_j, \eta \setminus \eta_j) \right] - [T_m - T_{m, \setminus j}] \quad (\text{SA.9}) \\
 &\triangleq \Delta \pi_{m, \setminus j} - [T_m - T_{m, \setminus j}] \triangleq \bar{F}_j,
 \end{aligned}$$

and for any $j \notin \mathcal{J}_m$,

$$\begin{aligned}
 F_j &\geq \left[E_{(\xi \cup \xi_j, \eta \cup \eta_j)} \pi_m(\mathbf{q} \cup q_j, \xi \cup \xi_j, \eta \cup \eta_j) - E_{(\xi, \eta)} \pi_m(\mathbf{q}, \xi, \eta) \right] - [T_{m, \cup j} - T_m] \quad (\text{SA.10}) \\
 &\triangleq \Delta \pi_{m, \cup j} - [T_{m, \cup j} - T_m] \triangleq \underline{F}_j.
 \end{aligned}$$

These transfers do not affect the equilibrium prices. Therefore, they do not affect consumer surplus or the sum of carriers' profits and smartphone firms' variable profits. They do, however, affect our estimates of the fixed cost bounds (see (SA.9) and (SA.10)). We argue that under a reasonable assumption on the transfers, we can obtain an overestimate of the bounds without modeling how the transfers are determined. Specifically, the assumption we need is: the total

transfer that a smartphone pays at least weakly increases with the number of its products, i.e.,

Assumption 1 $T_m - T_{m,\setminus j} \geq 0$ and $T_{m,\cup j} - T_m \geq 0$.

Under Assumption 1, we have $\bar{F}_j \leq \Delta\pi_{m,\setminus j}$ and $\underline{F}_j \leq \Delta\pi_{m,\cup j}$. We think this assumption is reasonable, in other words, we expect the upper bound (or the lower bound) to be smaller than $\Delta\pi_{m,\setminus j}$ (or $\Delta\pi_{m,\cup j}$). For example, if a carrier shares a portion (denoted by $\varphi \in (0,1)$) of the increase in a smartphone firm's variable profit when a product is added, i.e., $T_m - T_{m,\setminus j} = \varphi\Delta\pi_{m,\setminus j}$ and $T_{m,\cup j} - T_m = \varphi\Delta\pi_{m,\cup j}$, then $\bar{F}_j = (1-\varphi)\Delta\pi_{m,\setminus j} < \Delta\pi_{m,\setminus j}$ and $\underline{F}_j = (1-\varphi)\Delta\pi_{m,\cup j} < \Delta\pi_{m,\cup j}$.

In Table SA.8 where we present the simulation results when a product is removed or added, we report these overestimated bounds: $\Delta\pi_{m,\setminus j}$ and $\Delta\pi_{m,\cup j}$. Table SA.8 shows that even with such an overestimation, our results are robust: removing a product leads to a decrease in total welfare even considering the (over-estimated) maximum possible saving in the fixed cost while adding a product leads to an increases in the total welfare as long as the fixed cost of the added product is not much higher than its (over-estimated) lower bound. In sum, our results on welfare changes when a product is added or removed are robust to this change to the supply side of the model.³¹

Table SA.8: Robustness Test, $\Lambda_c = 0, \Lambda_m = 1, \bar{\Gamma}_m = \Gamma_m$

(a) Welfare Changes when a Product is Removed, March 2013 (million \$)

Removed product	Lowest-quality	Median	Highest
$\Delta(\text{consumer surplus})$	-1.25	-2.70	-17.49
$\Delta(\text{total producer surplus net of fixed costs})^a$	-0.69	-0.82	-6.30
$\Delta\pi_{m,\setminus j}$	1.17	2.30	16.04

^aThe sum of carriers' profits and smartphone firms' variable profits.

(b) Welfare Changes when a Product is Added, March 2013 (million \$)

	HTC	LG	Motorola	Samsung
$\Delta(\text{consumer surplus})$	2.21	2.22	2.30	2.66
$\Delta(\text{total producer surplus net of fixed costs})$	0.91	0.90	0.86	1.49
$\Delta\pi_{m,\cup j}$	1.98	1.98	2.02	2.55

Alternative Pricing Model 4

In this case, the transfer should be from a carrier to a smartphone instead. Let T_m be the total transfer that a smartphone firm m receives, and $T_{m,\setminus j}$ and $T_{m,\cup j}$ be that when j is removed from or when j is added to m 's product portfolio. Then, the two inequalities (11) and (12) become:

$$T_m - F_j \geq T_{m,\setminus j} \iff F_j \leq T_m - T_{m,\setminus j} \text{ for any } j \in \mathcal{J}_m \quad (\text{SA.11})$$

$$T_m \geq T_{m,\cup j} - F_j \iff F_j \geq T_{m,\cup j} - T_m \text{ for any } j \notin \mathcal{J}_m. \quad (\text{SA.12})$$

³¹We do not conduct robustness analyses regarding the merger simulations because doing so requires us to make assumptions on how large the transfer from each smartphone firm to each carrier is and how a merger affects the transfers between smartphone firms and carriers.

We again make an assumption on the transfers. Specifically, let the changes in the (pre-transfer) profit of j 's carrier be:

$$\begin{aligned}\Delta\pi_{c,\setminus j} &= E_{(\boldsymbol{\xi},\boldsymbol{\eta})}\pi_c(\mathbf{q},\boldsymbol{\xi},\boldsymbol{\eta}) - E_{(\boldsymbol{\xi}\setminus\xi_j,\boldsymbol{\eta}\setminus\eta_j)}\pi_c(\mathbf{q}\setminus q_j,\boldsymbol{\xi}\setminus\xi_j,\boldsymbol{\eta}\setminus\eta_j), \\ \Delta\pi_{c,\cup j} &= E_{(\boldsymbol{\xi}\cup\xi_j,\boldsymbol{\eta}\cup\eta_j)}\pi_c(\mathbf{q}\cup q_j,\boldsymbol{\xi}\cup\xi_j,\boldsymbol{\eta}\cup\eta_j) - E_{(\boldsymbol{\xi},\boldsymbol{\eta})}\pi_c(\mathbf{q},\boldsymbol{\xi},\boldsymbol{\eta}).\end{aligned}\tag{SA.13}$$

We assume that the increase in the amount of transfer that the smartphone firm receives is not larger than the increase in the carrier's (pre-transfer) profit, i.e.,

Assumption 2 $T_m - T_{m,\setminus j} \leq \Delta\pi_{c,\setminus j}$ and $T_{m,\cup j} - T_m \leq \Delta\pi_{c,\cup j}$.

In Table SA.9, which presents the simulation results in this robustness analysis, we report $\Delta\pi_{c,\setminus j}$ in Table SA.9(a) and $\Delta\pi_{c,\cup j}$ in Table SA.9(b). Under Assumption 2, the bounds of the fixed cost reported in Table SA.9 are again over estimated. Therefore, from Table SA.9, we draw a similar robustness conclusion as in the case of $(\Lambda_c = 0, \Lambda_m = 1, \bar{\Gamma}_m = \Gamma_m)$.

Table SA.9: Robustness Test, $\Lambda_c = 1, \Lambda_m = 0, \bar{\Gamma}_c = \Gamma_c$,

(a) Welfare Changes when a Product is Removed, March 2013 (million \$)				
Removed product	Lowest-quality	Median	Highest	
$\Delta(\text{consumer surplus})$	-0.93	-2.86	-12.06	
$\Delta(\text{total producer surplus net of fixed costs})$	-0.87	-1.07	-11.13	
$\Delta\pi_{c,\setminus j}$	1.05	2.49	14.90	

(b) Welfare Changes when a Product is Added, March 2013 (million \$)				
	HTC	LG	Motorola	Samsung
$\Delta(\text{consumer surplus})$	1.96	1.96	1.96	2.48
$\Delta(\text{total producer surplus net of fixed costs})$	0.96	0.96	0.96	1.30
$\Delta\pi_{c,\cup j}$	1.83	1.83	1.83	2.34

In summary, for the case of $(\Lambda_c = 0, \Lambda_m = 1, \bar{\Gamma}_m = \Gamma_m)$ and $(\Lambda_c = 1, \Lambda_m = 0, \bar{\Gamma}_c = \Gamma_c)$, we argue that under reasonable assumptions on the transfers (between a smartphone firm and a carrier), we can obtain an overestimate of the fixed-cost upper bound and lower bound. The results in Tables SA.8 and SA.9, where we report these overestimated bounds, show that removing a product leads to a decrease in total welfare even considering the (over-estimated) maximum possible saving in the fixed cost and adding a product leads to an increase in total welfare as long as the fixed cost of the added product is not much higher than its (over-estimated) lower bound.

SA.3 Parametric Fixed Cost Function

So far we assume that the total fixed cost of a firm is the sum of the fixed cost for each product (i.e., there are no economies or diseconomies of scope in fixed costs). Under this assumption, we find that a merger leads to a reduction in product offerings. Is this finding robust to this assumption?

Intuitively, if there are diseconomies of scope in fixed costs, the merged firm's per-product fixed cost may increase after the merger, leading to a further reduction in product offerings. If, however, there are economies of scope, the merged firm's per-product fixed cost decreases after the merger, which may lead to an increase in product offerings.

To address this concern, we now take a parametric approach and specify a function of the fixed cost allowing for economies or diseconomies of scope as follows:

$$FC_{jt} = \phi_1 q_j + \phi_2 \log(n_{m(j)t}) + \varphi_{m(j)t}, \quad (\text{SA.14})$$

where q_j is product j 's quality index, $n_{m(j)t}$ is the number of products that the smartphone firm $m(j)$ has in period t , and $\varphi_{m(j)t}$ represents the brand/time fixed effects.³² Note that a negative estimate of the coefficient ϕ_2 indicates economies of scope in fixed costs; and conversely, a positive estimate indicates diseconomies of scope.

Note that the purpose of this exercise is to address the concern that potential economies of scope in fixed costs may lead to an increase in the number of products after a merger, which would be the opposite of our baseline results. Therefore, our goal in this section is to obtain a (conservative) estimate of the lower bound for ϕ_2 . If our conservative estimate of the lower bound is positive (or negative but of small magnitude), then we can conclude that there are diseconomies of scope (or small economies of scope) in fixed costs.

To obtain the estimate of the lower bound for ϕ_2 , we consider the following three types of deviations:

(1) Dropping a product j

Nash equilibrium implies that dropping a product does not increase the expected profit of a firm. Let $\Pi_{mt}(\mathcal{J}_{mt}) = E_{(\boldsymbol{\xi}_t, \boldsymbol{\eta}_t)} \pi_{mt}(\mathbf{q}_t, \boldsymbol{\xi}_t, \boldsymbol{\eta}_t)$ be the expected profit that a smartphone firm m gets from its observed product portfolio \mathcal{J}_{mt} and $\Pi_{mt}(\mathcal{J}_{mt} \setminus j) = E_{(\boldsymbol{\xi}_t \setminus \xi_{jt}, \boldsymbol{\eta}_t \setminus \eta_{jt})} \pi_{mt}(\mathbf{q}_t \setminus q_j, \boldsymbol{\xi}_t \setminus \xi_{jt}, \boldsymbol{\eta}_t \setminus \eta_{jt})$ be that when it drops product j . Then, for any $j \in \mathcal{J}_{mt}$,

$$[\Pi_{mt}(\mathcal{J}_{mt}) - (\phi_1 q_j + \varphi_{mt}) - \phi_2 n_{mt} \log(n_{mt})] - [\Pi_{mt}(\mathcal{J}_{mt} \setminus j) - \phi_2 (n_{mt} - 1) \log(n_{mt} - 1)] + v_{jt} \geq 0,$$

where v_{jt} is added to the inequality to represent an expectation error that is uncorrelated with product choices (e.g., Holmes (2011) and Pakes, Porter, Ho and Ishii (2015)). Then,

$$\begin{aligned} & \phi_1 q_j + \varphi_{mt} + \phi_2 [n_{mt} \log(n_{mt}) - (n_{mt} - 1) \log(n_{mt} - 1)] \\ & \leq \Pi_{mt}(\mathcal{J}_{mt}) - \Pi_{mt}(\mathcal{J}_{mt} \setminus j) + v_{jt}. \end{aligned} \quad (\text{SA.15})$$

(2) Replacing product j by a high-quality product j'

³²For notational simplicity, we use j instead of \tilde{j} to represent a product, i.e., we ignore the distinction between j and \tilde{j} as explained in Section 3.2.2.

Such a deviation gives us the following inequality:

$$\begin{aligned} & [\Pi_{mt}(\mathcal{J}_{mt}) - (\phi_1 q_j + \varphi_{mt}) - \phi_2 n_{mt} \log(n_{mt})] \\ & - [\Pi_{mt}(\mathcal{J}_{mt} \setminus j \cup j') - (\phi_1 q_{j'} + \varphi_{mt}) - \phi_2 n_{mt} \log(n_{mt})] + v_{jj't} \geq 0, \end{aligned}$$

implying

$$\phi_1 (q_{j'} - q_j) \geq \Pi_{mt}(\mathcal{J}_{mt} \setminus j \cup j') - \Pi_{mt}(\mathcal{J}_{mt}) - v_{jj't}. \quad (\text{SA.16})$$

(3) Replacing product j by two products k_1 and k_2 such that $q_{k_1} + q_{k_2} = q_j$

Similarly, we have

$$\begin{aligned} & [\Pi_{mt}(\mathcal{J}_{mt}) - (\phi_1 q_j + \varphi_{mt}) - \phi_2 n_{mt} \log(n_{mt})] \\ & - [\Pi_{mt}(\mathcal{J}_{mt} \setminus j \cup k_1 \cup k_2) - (\phi_1 q_{k_1} + \varphi_{mt}) - (\phi_1 q_{k_2} + \varphi_{mt}) - \phi_2 (n_{mt} + 1) \log(n_{mt} + 1)] + v_{jk_1 k_2 t} \geq 0, \end{aligned}$$

implying

$$\begin{aligned} & \varphi_{mt} + \phi_2 [(n_{mt} + 1) \log(n_{mt} + 1) - n_{mt} \log(n_{mt})] \\ & \geq \Pi_{mt}(\mathcal{J}_{mt} \setminus j \cup k_1 \cup k_2) - \Pi_{mt}(\mathcal{J}_{mt}) - v_{jk_1 k_2 t}. \end{aligned} \quad (\text{SA.17})$$

To obtain a conservative lower bound for ϕ_2 , we take the difference of (SA.17) and (SA.15) and obtain

$$\begin{aligned} & \phi_2 [(n_{mt} + 1) \log(n_{mt} + 1) - 2n_{mt} \log(n_{mt}) + (n_{mt} - 1) \log(n_{mt} - 1)] \\ & \geq [\Pi_{mt}(\mathcal{J}_{mt} \setminus j \cup k_1 \cup k_2) - \Pi_{mt}(\mathcal{J}_{mt})] - [\Pi_{mt}(\mathcal{J}_{mt}) - \Pi_{mt}(\mathcal{J}_{mt} \setminus j)] + \phi_1 q_j - v_{jt} - v_{jk_1 k_2 t} \\ & \geq [\Pi_{mt}(\mathcal{J}_{mt} \setminus j \cup k_1 \cup k_2) - \Pi_{mt}(\mathcal{J}_{mt})] - [\Pi_{mt}(\mathcal{J}_{mt}) - \Pi_{mt}(\mathcal{J}_{mt} \setminus j)] \\ & \quad + \frac{\Pi_{mt}(\mathcal{J}_{mt} \setminus j \cup j') - \Pi_{mt}(\mathcal{J}_{mt})}{q_{j'} - q_j} q_j - v_{jt} - v_{jk_1 k_2 t} - \frac{q_j}{q_{j'} - q_j} v_{jj't}, \end{aligned} \quad (\text{SA.18})$$

where the second inequality is obtained by plugging (SA.16) into the first inequality. This inequality eventually gives us

$$\begin{aligned} \phi_2 & \geq \frac{[\Pi_{mt}(\mathcal{J}_{mt} \setminus j \cup k_1 \cup k_2) - \Pi_{mt}(\mathcal{J}_{mt})] - [\Pi_{mt}(\mathcal{J}_{mt}) - \Pi_{mt}(\mathcal{J}_{mt} \setminus j)] + \frac{\Pi_{mt}(\mathcal{J}_{mt} \setminus j \cup j') - \Pi_{mt}(\mathcal{J}_{mt})}{q_{j'} - q_j} q_j}{[(n_{mt} + 1) \log(n_{mt} + 1) - 2n_{mt} \log(n_{mt}) + (n_{mt} - 1) \log(n_{mt} - 1)]} \\ & \quad - \frac{-v_{jt} - v_{jk_1 k_2 t} - \frac{q_j}{q_{j'} - q_j} v_{jj't}}{[(n_{mt} + 1) \log(n_{mt} + 1) - 2n_{mt} \log(n_{mt}) + (n_{mt} - 1) \log(n_{mt} - 1)]}. \\ & \triangleq \Phi_{jt} + \epsilon_{jt}, \end{aligned} \quad (\text{SA.19})$$

We denote the first line in (SA.19) by Φ_{jt} , and with a slight abuse of notation, the second line by

ϵ_{jt} .³³ As mentioned, we assume that ϵ_{jt} is uncorrelated with product choices. Then, we have

$$\phi_2 \geq E\Phi_{jt}. \tag{SA.20}$$

In estimation, we set the quality difference $q_{j'} - q_j$ in (SA.16) to be 0.05, and $q_{k_1} = 0.4q_j$ and $q_{k_2} = 0.6q_j$ in (SA.17).³⁴ According to our estimate, the lower bound of the estimated set for ϕ_2 is $\frac{1}{\#\mathcal{J}} \sum_{jt \in \mathcal{J}} \Phi_{jt} = 0.016$. Following Imbens and Manski (2004), the lower bound of the 95% confidence interval for ϕ_2 is $\frac{1}{\#\mathcal{J}} \sum_{jt \in \mathcal{J}} \Phi_{jt} - \frac{\sqrt{\widehat{\text{var}}(\Phi_{jt})}}{\sqrt{\#\mathcal{J}}} c_{0.05} = 0.011$, where $\widehat{\text{var}}(\Phi_{jt})$ is an estimator of the variance of Φ_{jt} and $c_{0.05}$ represents the critical value. These results suggest that there are some diseconomies of scope in fixed costs. For example, $\phi_2 = 0.016$ means that, for the merged firm Samsung-LG, when it drops a product, the fixed cost for each of its remaining products decreases by 1.5 million dollars. Therefore, if anything, we underestimate the decrease in product offerings in our baseline results.

SB Plot of Estimated Demand and Marginal Cost Shocks

In this section, we plot the estimated demand shocks $\hat{\xi}_{jt}$ and marginal cost shocks $\hat{\eta}_{jt}$ for three groups of observations separately: (1) “newly added”: jt s.t. $j \in \mathcal{J}_t$ but $j \notin \mathcal{J}_{t'}, t' < t$; (2) “discontinued”: jt s.t. $j \in \mathcal{J}_t$ but $j \notin \mathcal{J}_{t'}, t' > t$; and (3) “others”: all other jt . Figure SB.1 shows that these three groups do not seem to be very different.

SC $\Delta(\text{Consumer Surplus})$ without Changes in Logit Errors

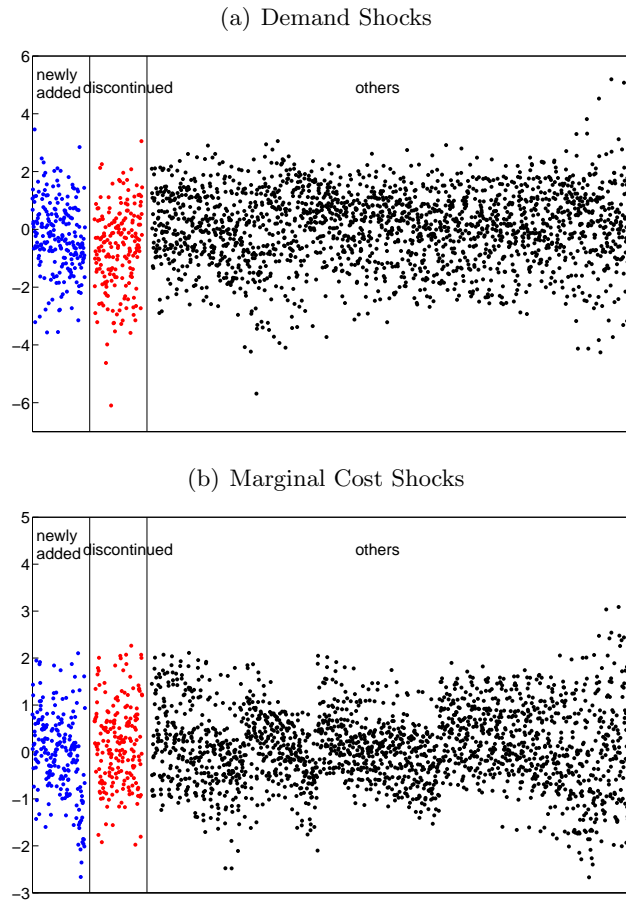
One concern with our finding that removing (or adding) a product leads a decrease (or an increase) in total welfare is that we may overestimate the consumer surplus changes because when we remove (or add) a product, we remove (or add) the Logit error term corresponding to this product, which is independent of other Logit error terms. To address this concern, in this section, we recalculate $\Delta(\text{consumer surplus})$ without removing or adding a Logit error term. Specifically, when product j is added to a set of existing products \mathcal{J}_t , we assign the Logit error of an existing product $k \in \mathcal{J}_t$ to the added product j so that there is no added Logit error term. We choose product k to be the closest to j 's quality among all existing products of j 's manufacturer.³⁵ We take a similar approach for the case of removing a product. Note that when product j is removed from the set \mathcal{J}_t , the decrease in consumer surplus is essentially the increase in consumer surplus when product j is added to the set $\mathcal{J}_t \setminus j$.

³³As will be explained later, in the estimation, we choose $q_{j'}$, q_{k_1} and q_{k_2} as a constant function of q_j . As the result, the second summand in (SA.19) is only jt specific.

³⁴The results are robust to other choices of $q_{j'} - q_j$ and (q_{k_1}, q_{k_2}) .

³⁵We could also set the Logit error of the removed product to be zero. However, since our estimates are based on a model with Logit errors, doing so means that we cannot match the market share data. Our approach allows us to get rid of the effect of adding an independent Logit error term while being close to our data.

Figure SB.1: Plot of the Estimated Demand Shocks



With this alternative measure of consumer surplus, Tables 7 (removing a product) and 8 (adding a product) become:

Table SC.1: Welfare Changes When a Product Is Removed, March 2013 (Million \$)

Removed product	Lowest-quality	Median	Highest
$\Delta(\text{consumer surplus without changes in Logit errors})$	-0.45	-2.33	-10.25
$\Delta(\text{carrier surplus})$	-1.01	-1.48	-11.18
$\Delta(\text{sum of smartphone firms' variable profits})$	-0.56	-1.14	-4.41
Upper bound of savings in fixed costs	1.11	2.65	14.20
Rows 1+2+3+4	-0.91	-2.30	-11.64

The changes in consumer surplus are indeed smaller than what are reported in Tables 7 and 8. However, the sum of the four rows in Table SC.1 is still negative, and the ratio of the first three rows to the last row in Table SC.2 varies between 1.95 and 2.01, close to 2.3 in the baseline.

Table SC.2: Welfare Changes When a Product Is Added, March 2013 (Million \$)

	HTC	LG	Motorola	Samsung
Δ (consumer surplus without changes in Logit errors)	1.70	1.76	1.72	1.96
Δ (carrier surplus)	1.20	1.21	1.26	1.52
Δ (sum of smartphone firms' variable profits)	0.92	0.91	0.86	1.44
Lower bound of added fixed costs	1.92	1.93	1.97	2.45
(Rows 1+2+3)/(Row 4)	1.99	2.01	1.95	2.01

SD Monte Carlo Test of the Heuristic Algorithm

In this section, we conduct Monte Carlo simulations to evaluate the performance of the heuristic algorithm explained in Section 5. To this end, we study product-choice problems where the number of potential products is small enough for us to find the optimal product portfolio without using the algorithm. We evaluate the performance of the algorithm by comparing the optimal product portfolio determined by the algorithm to the true optimal product portfolio.

To construct these Monte Carlo simulations, we first randomly draw K products from Samsung's products in March 2013. For each of these K products, we compute the variable profit if this product were the only product in the market. We then draw a K -by-1 vector of fixed costs uniformly from an interval between 0 and the maximum of the K variable profits.³⁶ Given these fixed-cost draws, we compute the firm profit (variable profit less the fixed cost) corresponding to each of the 2^K possible product portfolios to find the most profitable one. We also use the heuristic algorithm to search for the profit-maximizing portfolio and record the outcome obtained from using each of the 2^K product portfolios as the starting point for the algorithm. We conduct such a simulation 200×500 times, where 200 is the number of draws for the K potential products and 500 is the number of draws for the K fixed costs. Finally, we compute the failure rate (i.e., the number of simulations where the heuristic algorithm fails to find the true optimal product portfolio/100,000), separately for every starting point.

We repeat the above Monte Carlo simulations for the numbers of potential products $K = 3, \dots, 10$. In Figure SD.1, for each of these Monte Carlo studies where K varies between 3 and 10, we plot the maximum failure rate across all 2^K starting points. Figure SD.1 shows that, as the number of potential products (K) increases, so does the maximum failure rate. However, it is smaller than 1.06% even for $K = 10$. This result indicates that the heuristic algorithm works well at least for a relatively small optimal product-choice problem.

³⁶We do not use the bounds obtained in the estimation results section (Section 4.2) for this exercise because K in this exercise is much smaller than the number of products in the data. As a result, the change in variable profit from adding or removing a product is larger than that in Section 4.2. If we were to use the bounds reported there, we would find, in this exercise, that it is always optimal to have all K products in the market.

Figure SD.1: Failure Rate of the Heuristic Algorithm

