On Maximal Permissiveness in Partially-Observed Discrete Event Systems: Verification and Synthesis

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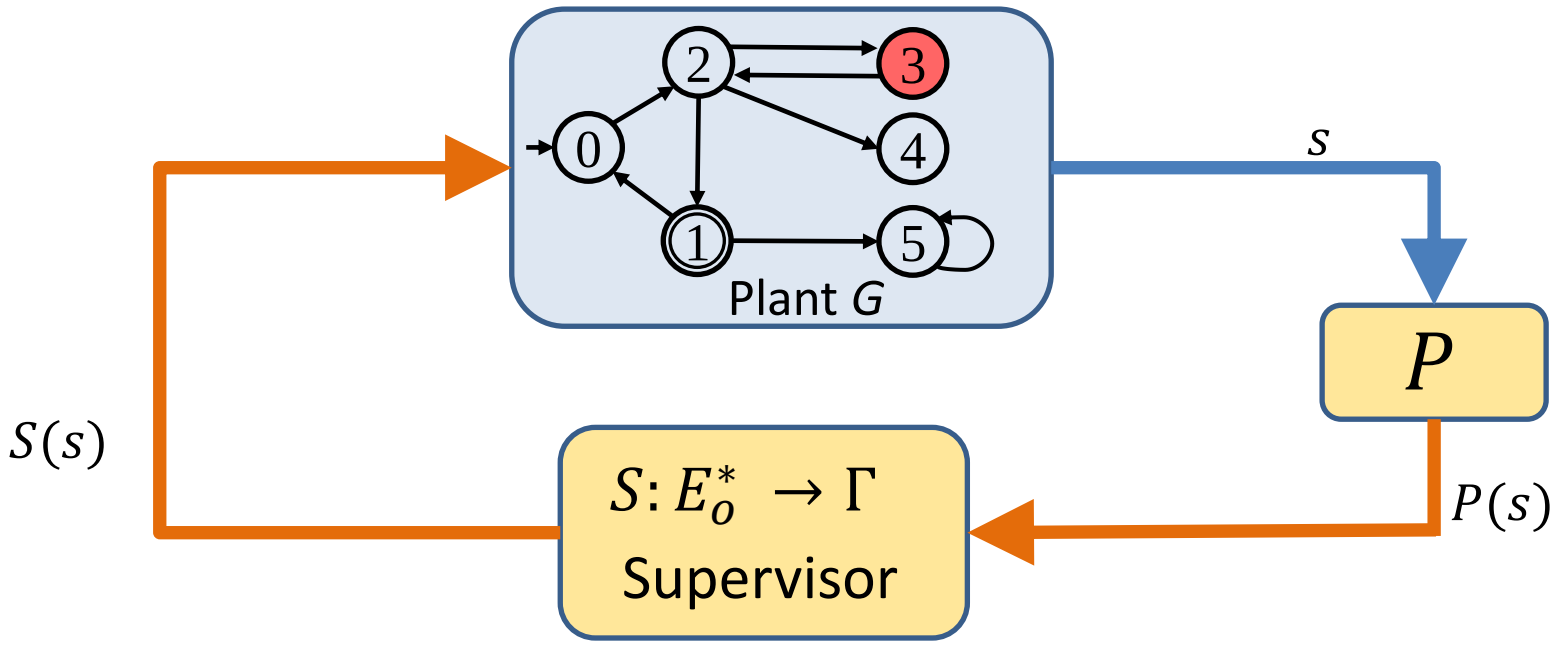
13th WODES, May 30-June 1, 2016, Xi'an, China

Introduction

Control Engineering Perspective



- $E = E_c \dot{\cup} E_{uc} = E_o \dot{\cup} E_{uo}$
- Supervisor: $S: E_o^* \rightarrow 2^{\mathrm{E}}$; Disable events in E_c based on its observations
- Closed-loop Behavior: $L(S/G)$

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- $G = (X, E, f, x_0)$ is a *deterministic* FSA
  - X is the finite set of states
  - E is the finite set of events
  - $f: X \times E \rightarrow X$ is the partial transition function
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- Safety specification automaton: $L(H) \subseteq L(G)$

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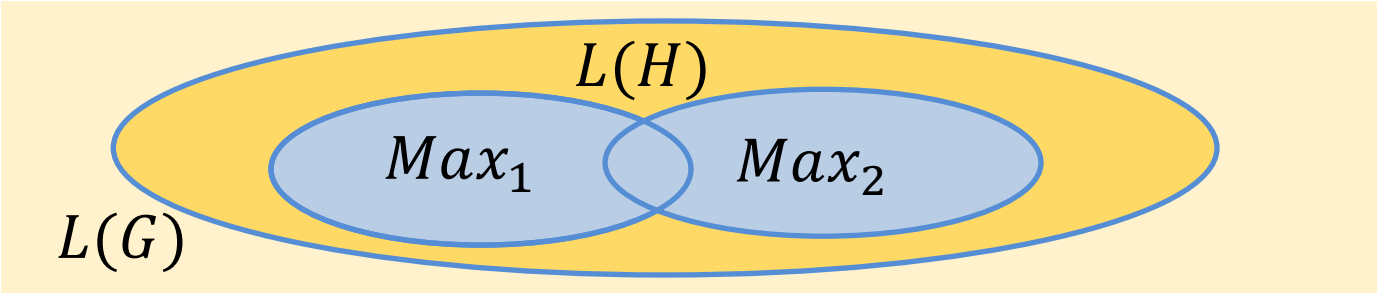

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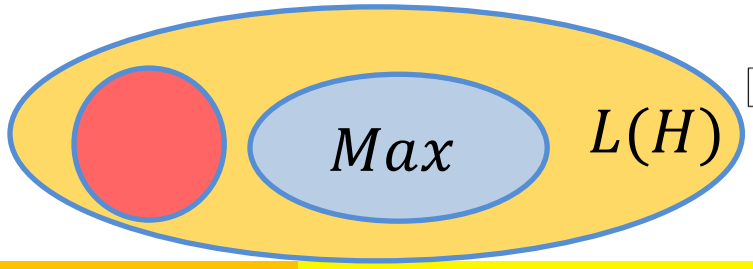

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- **Supervisor Verification Problem.**

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Motivation:

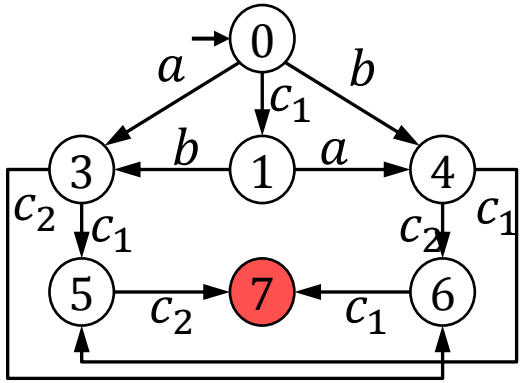
- Lower bound behavior L_r
- $\boldsymbol{L(S_R/G) = L_r{}^{\downarrow CO}}$, the infimal controllable and observable super-language
- Achieve both the lower bound and **permissiveness**

Bipartite Transition System

- **Information State**: a set of states, $I := 2^X$
- **BTS:** A bipartite transition system T w.r.t. G is a 7-tuple

$$T = (Q_Y^T, Q_Z^T, h_{YZ}^T, h_{ZY}^T, E, \Gamma, y_0)$$

where
- $Q_Y^T \subseteq I$ is the set of Y-states;
- $Q_Z^T \subseteq I \times \Gamma$ is the set of Z-states so that $z = (I(z), \Gamma(z))$;
- $h_{YZ}^T: Q_Y^T \times \Gamma \to Q_Z^T$ represents the unobservable reach;
- $h_{ZY}^T: Q_Z^T \times E \to Q_Y^T$ represents the observation transition;



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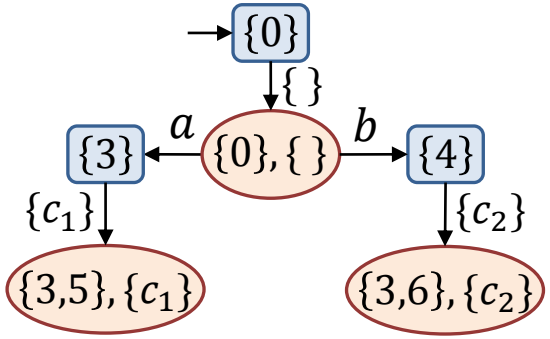

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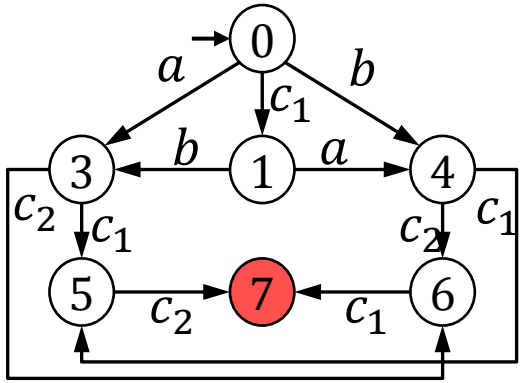

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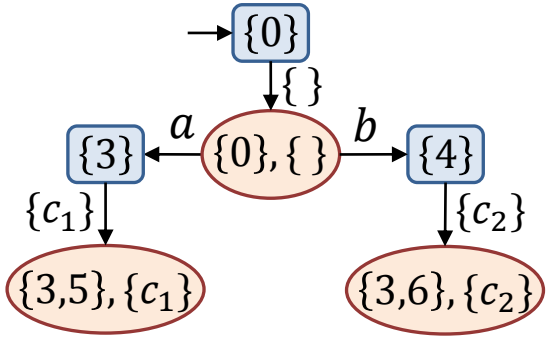

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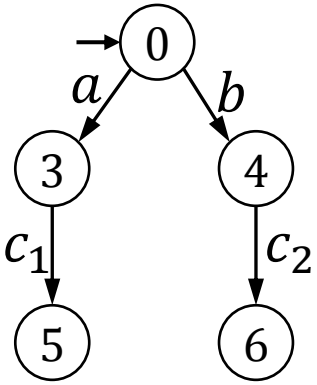


$\boldsymbol{L(S_R/G)}$

- **The given supervisor S_R can be realized as a BTS T_R**

All Inclusive Controller

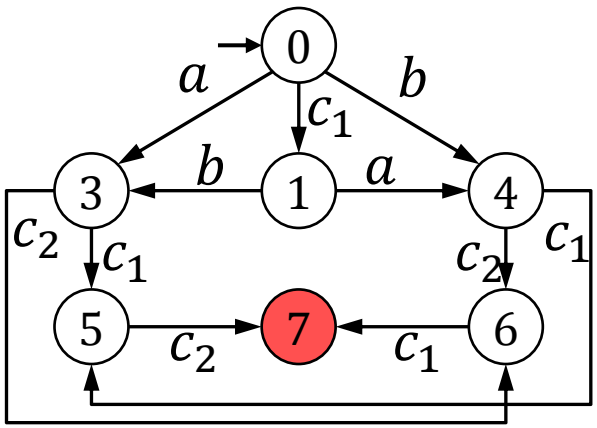
Definition. (AIC).

The All Inclusive Controller

$$\mathcal{AIC}(G) = (Q_Y^{AIC}, Q_Z^{AIC}, h_{YZ}^{AIC}, h_{ZY}^{AIC}, E, \Gamma, y_0),$$

is defined as the *largest* BTS such

1. For any $y \in Q_Y^{AIC}$, there exists at least one control decision
2. For any $z \in Q_Z^{AIC}$, we have
   2.1. all feasible observable events are defined
   2.2. $I(z)$ only contains legal states



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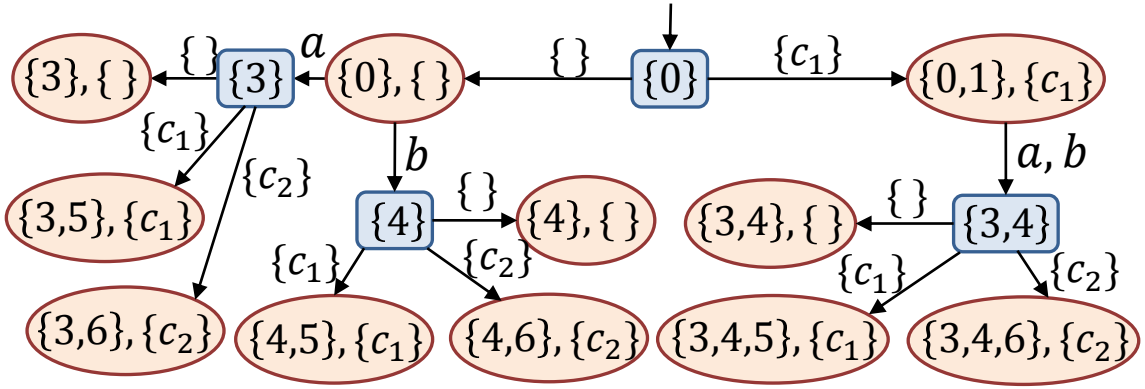

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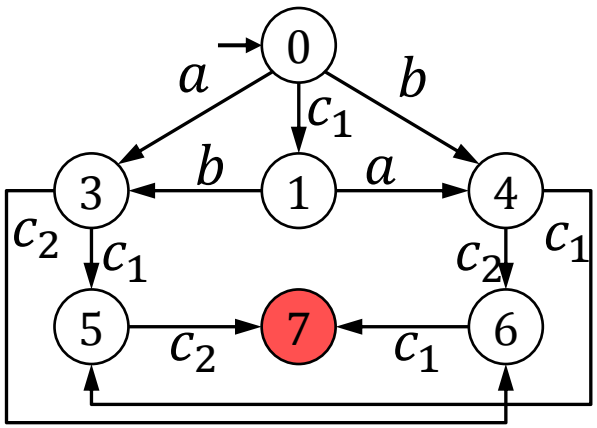

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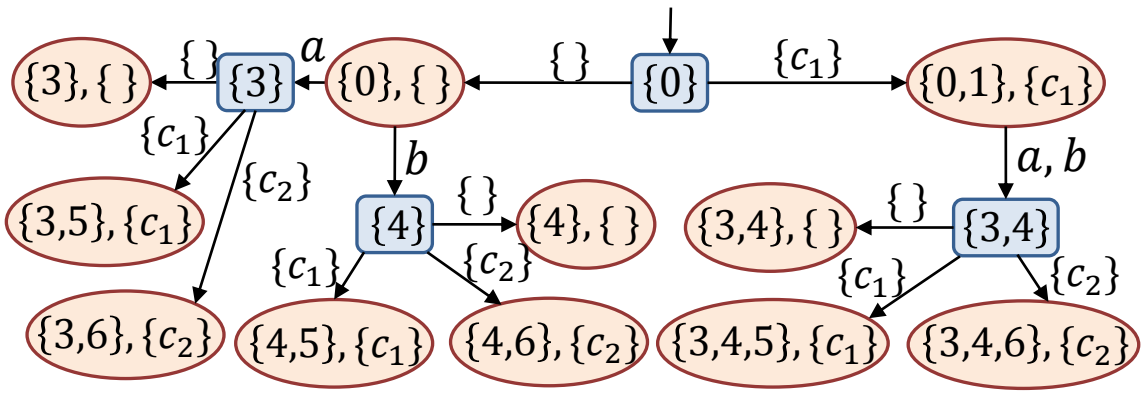

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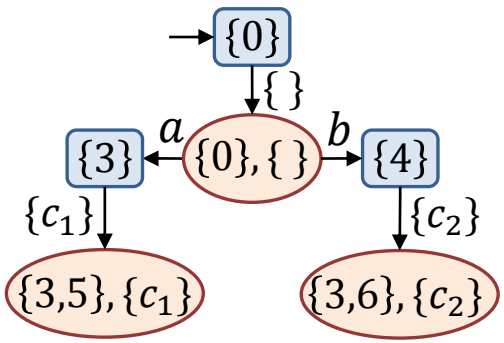
- **The AIC contains all safe supervisors**

Verification of Maximality: Basic Idea and Difficulties



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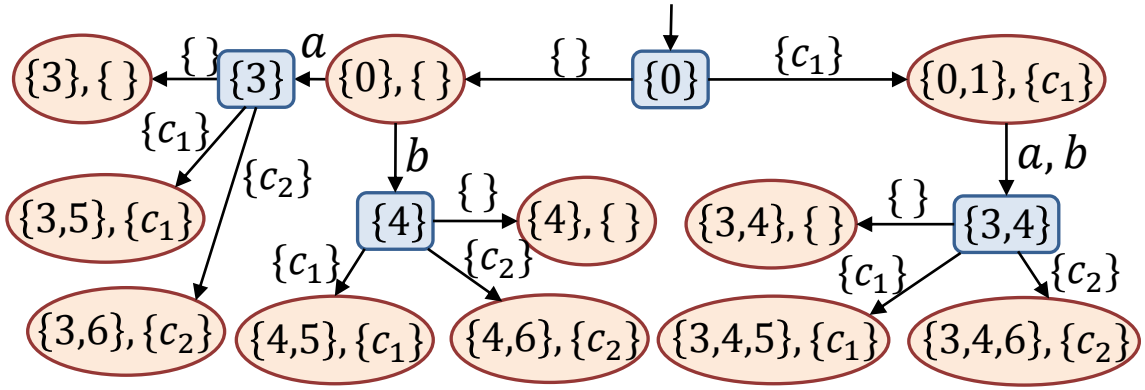

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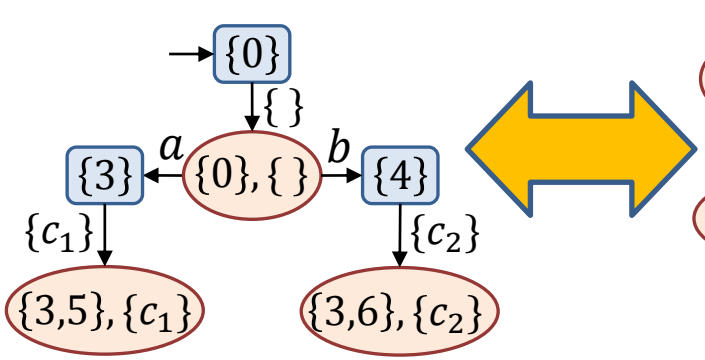

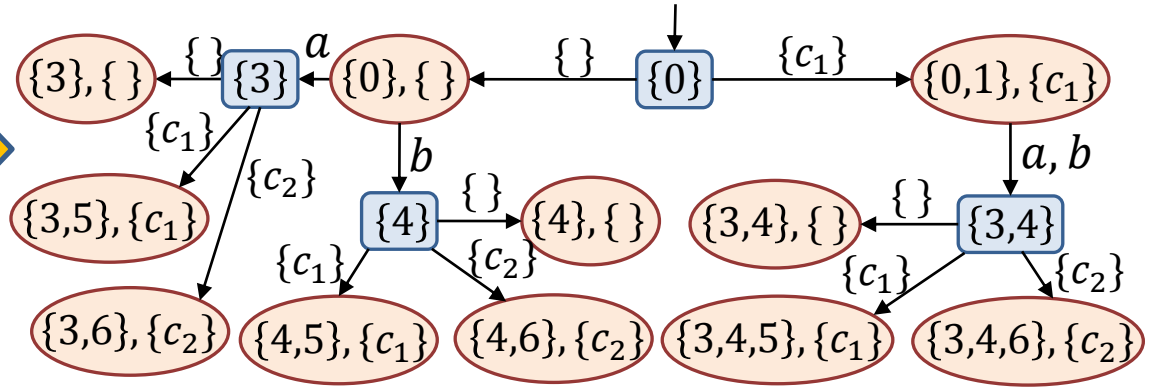

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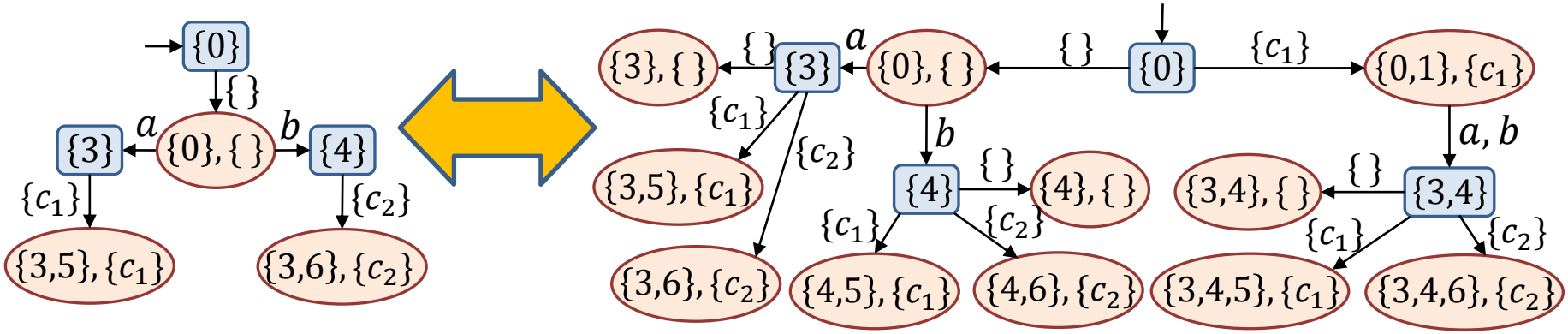

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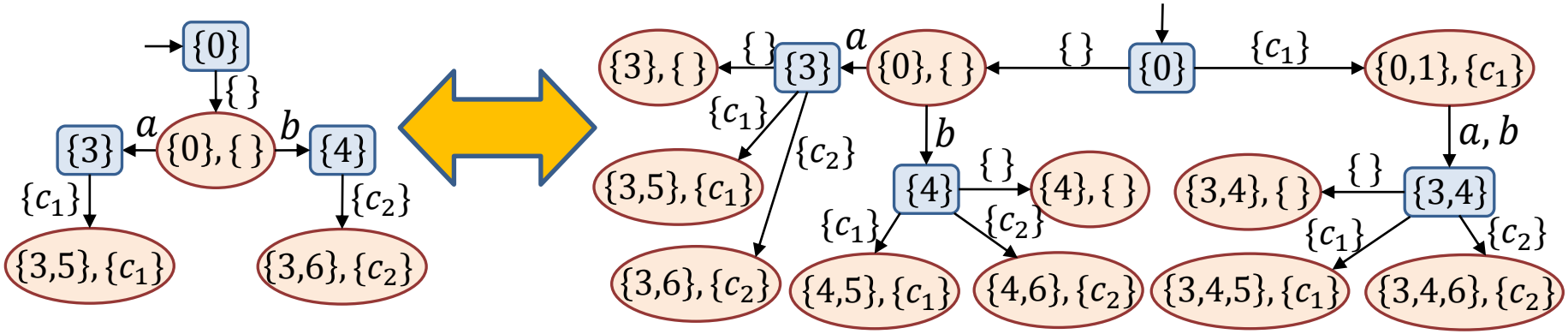

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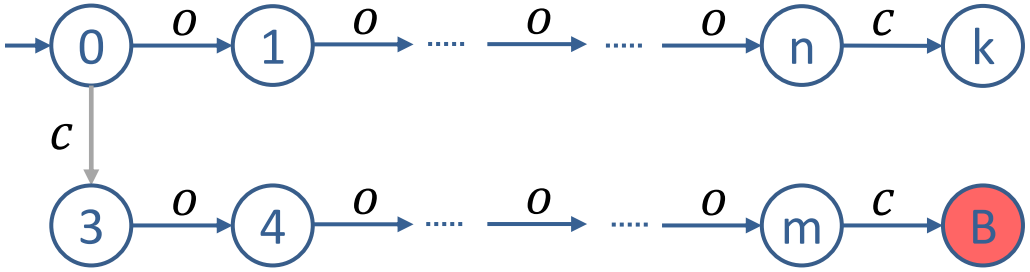

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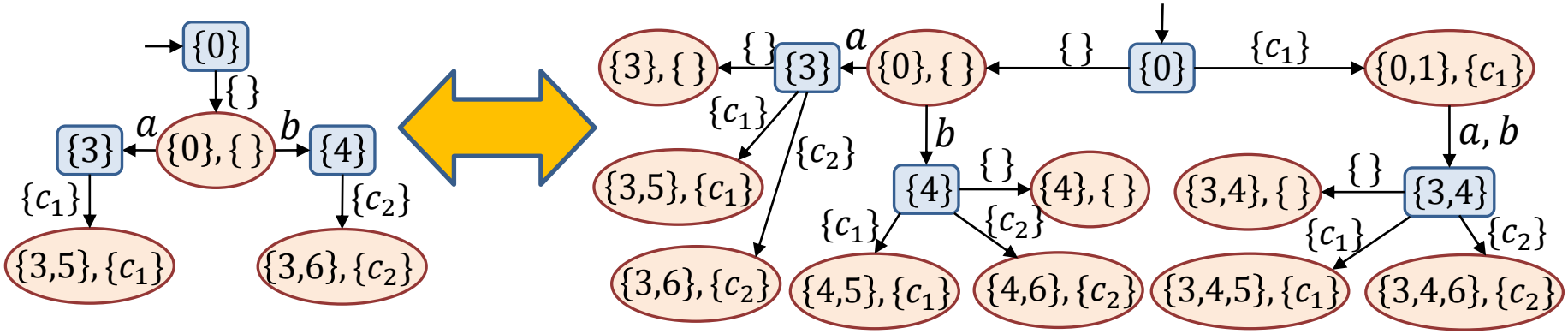

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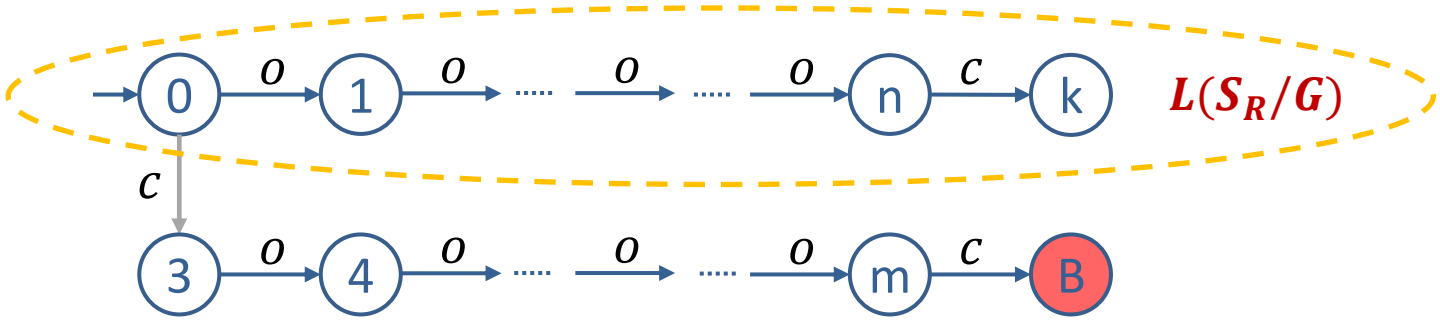

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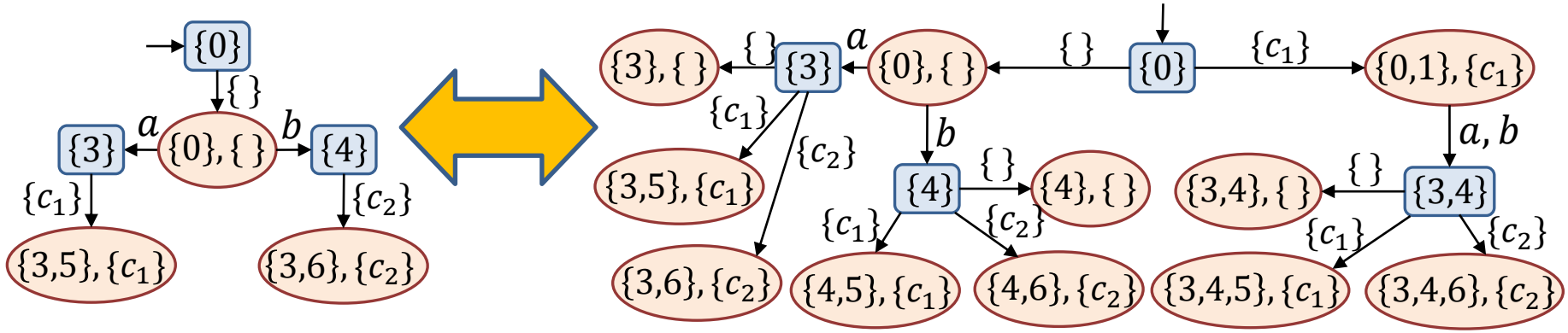

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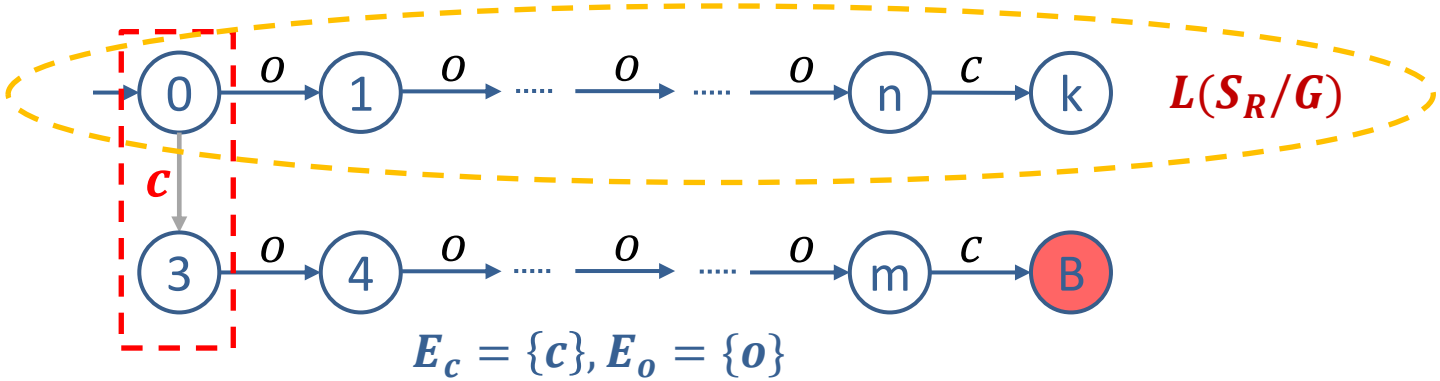

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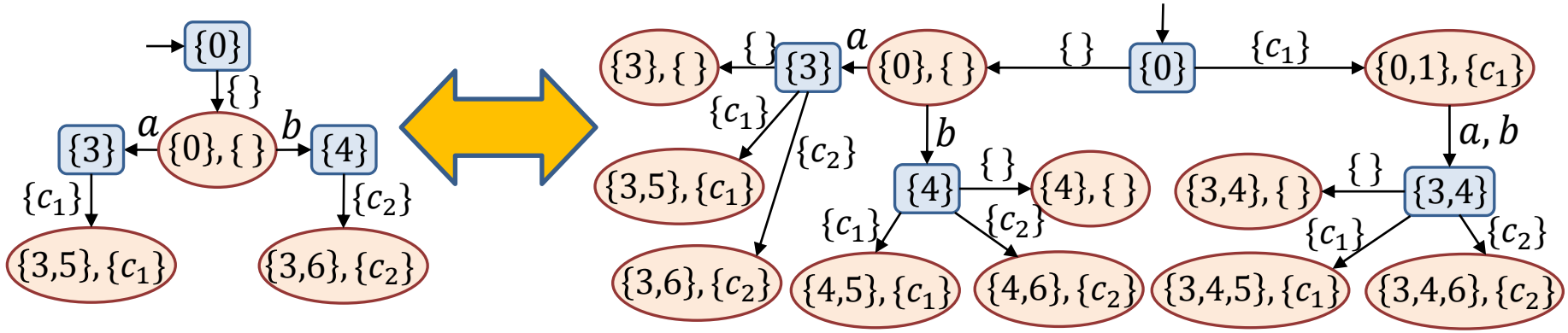

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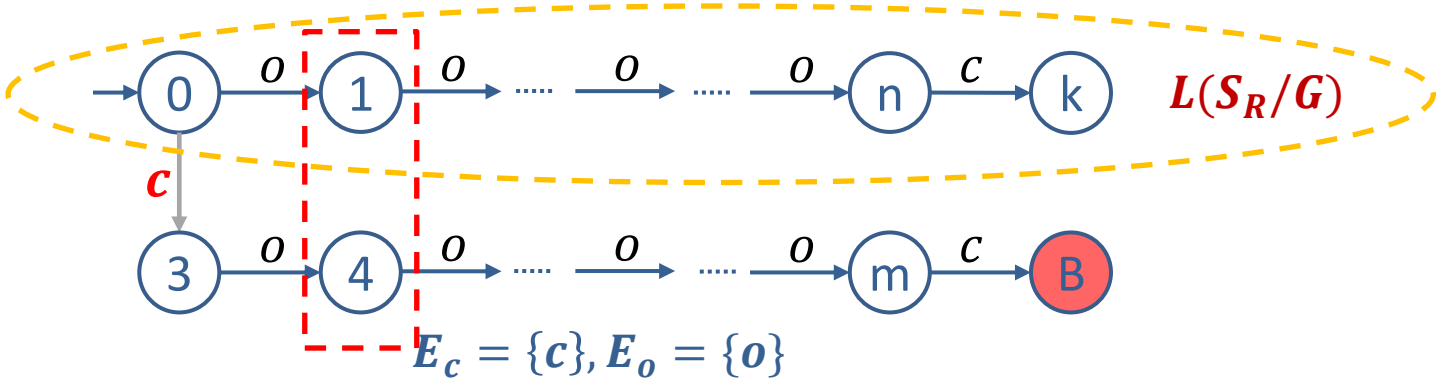

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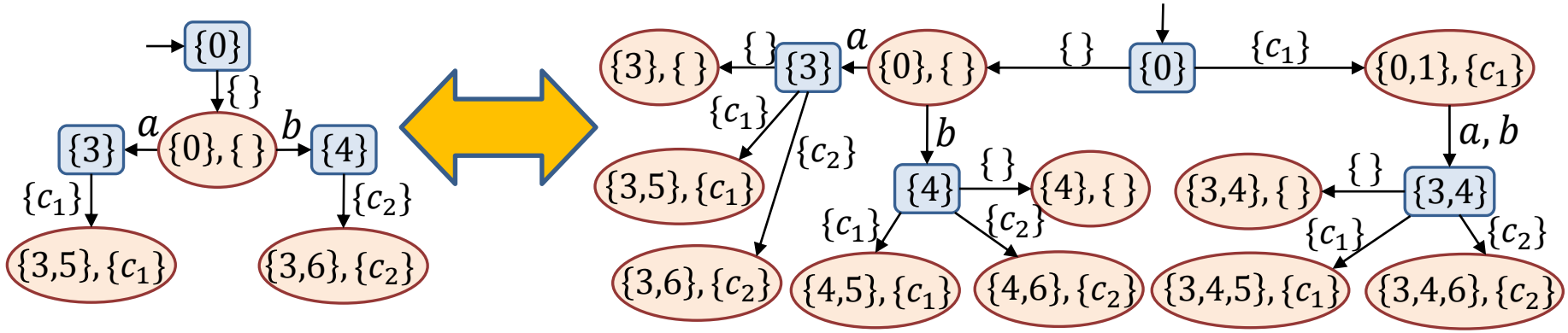

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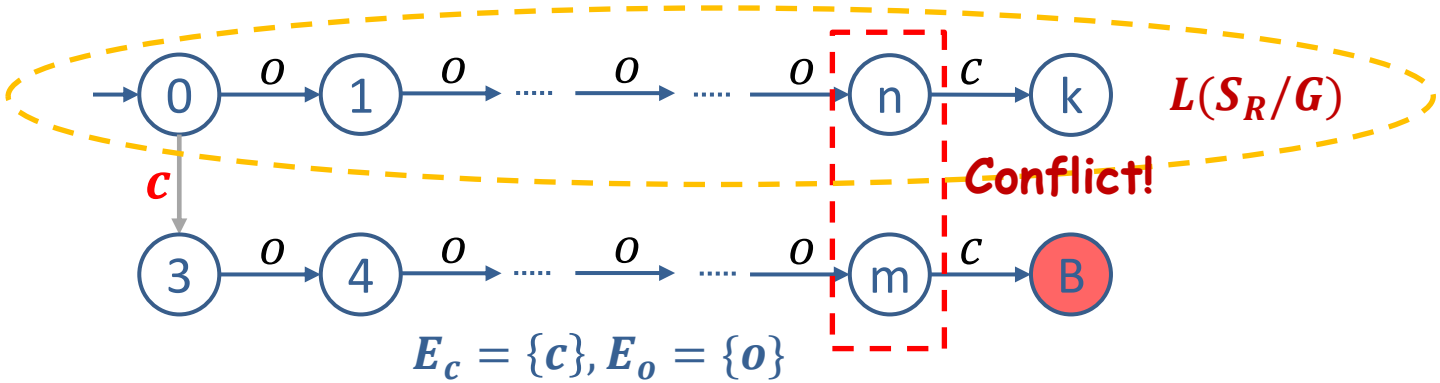

Control Simulation Relation

Definition. (Control Simulation Relation)

Let T_1 and T_2 between BTSs. A relation $\Phi = \Phi_Y \cup \Phi_Z \subseteq \left(Q_Y^{T_1} \times Q_Y^{T_2}\right) \times \left(Q_Z^{T_1} \times Q_Z^{T_2}\right)$ is said to be a **control** simulation relation from $\boldsymbol{T_1}$ **to** $\boldsymbol{T_2}$ if the following conditions hold:

1. $(y_0, y_0) \in \Phi_Y$;
2. For every $(y_1, y_2) \in \Phi_Y$, we have that: for any $y_1 \xrightarrow{\gamma_1} z_1$ in T_1, there exists $y_2 \xrightarrow{\gamma_2} z_2$ such that $(z_1, z_2) \in \Phi_Z$ and $\boldsymbol{\gamma_1 \subseteq \gamma_2}$.
3. For every $(z_1, z_2) \in \Phi_z$, we have that: for any $z_1 \xrightarrow{\sigma} y_1$ in T_1, there exists $z_2 \xrightarrow{\sigma} y_2$ such that $(y_1, y_2) \in \Phi_Y$.

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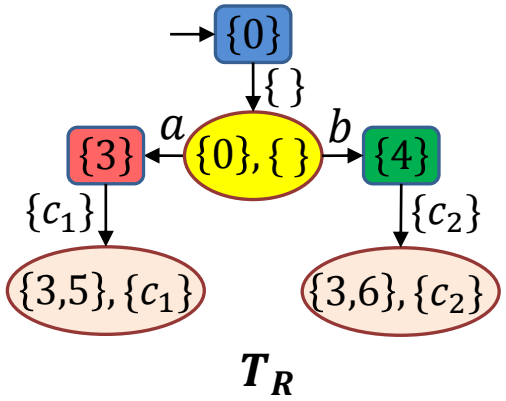
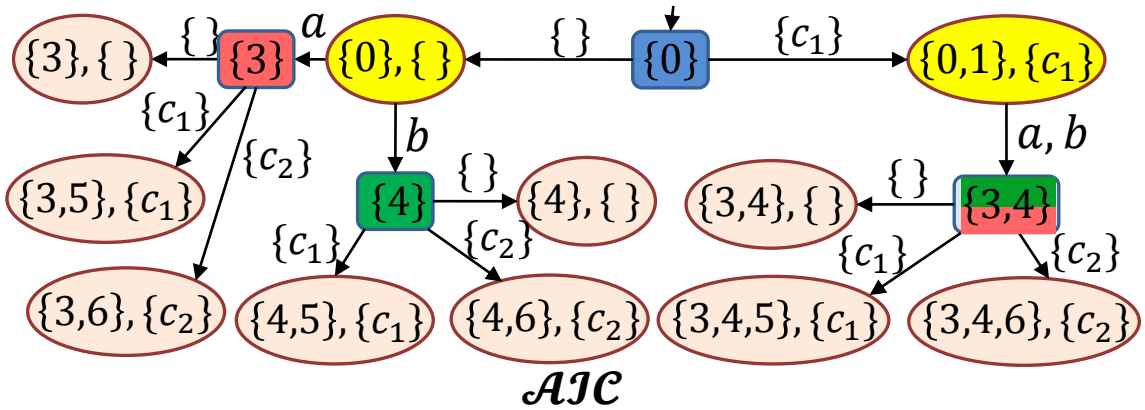
- There exists a unique maximal CSR $\Phi^*(T_1, T_2)$ from T_1 to T_2 if one exists
- $\Phi^*(T_1, T_2)$ can be computed by

$$\Phi^*(T_1, T_2) = \lim_{k \to \infty} F^k\left(\left(Q_Y^{T_1} \times Q_Y^{T_2}\right) \cup \left(Q_Z^{T_1} \times Q_Z^{T_2}\right)\right)$$

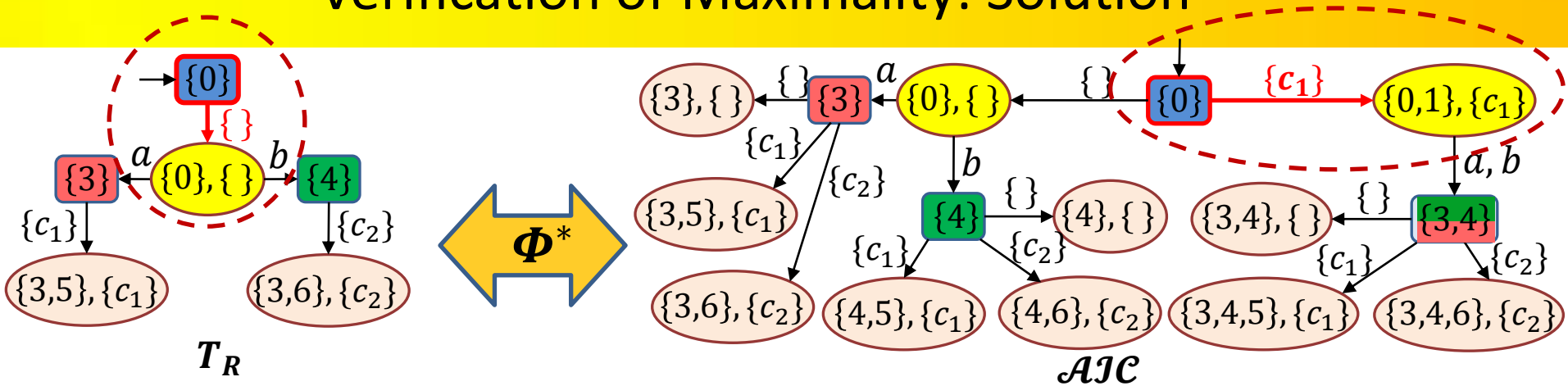
Verification of Maximality: Solution



Φ^*



Verification of Maximality: Solution



Replacement.

Let y be a Y-state in T_R and $c_{T_R}(y)$ be the control decision defined at y.

We say that control decision γ **replaces** $c_{T_R}(y)$ at y if

1. γ is defined at y in the AIC
2. $c_{T_R}(y) \subset \gamma$
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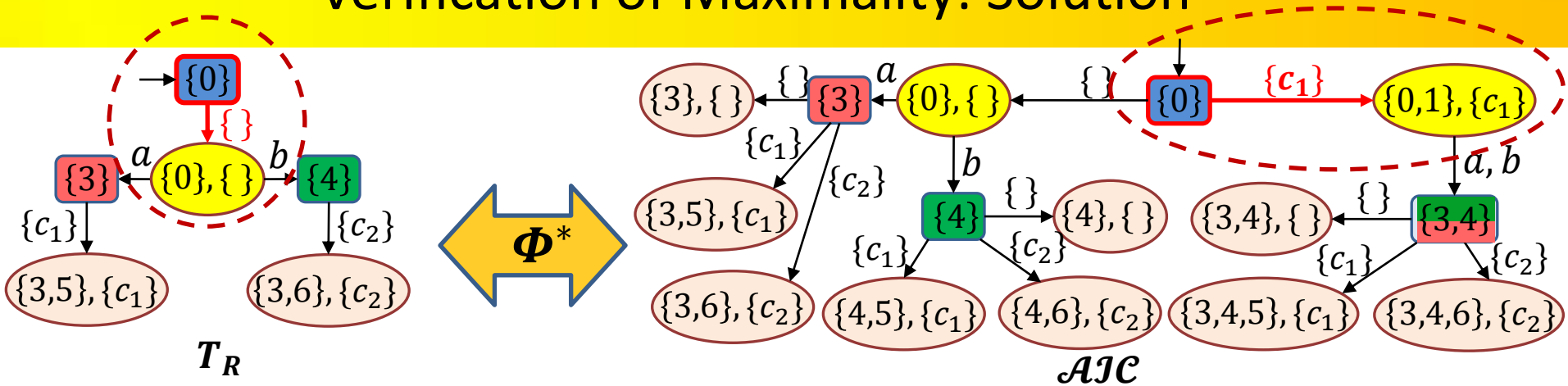

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Theorem.

Supervisor S_R is maximal iff no control decision in T_R can be replaced

Synthesis of Larger Supervisor: Basic Idea and Difficulties

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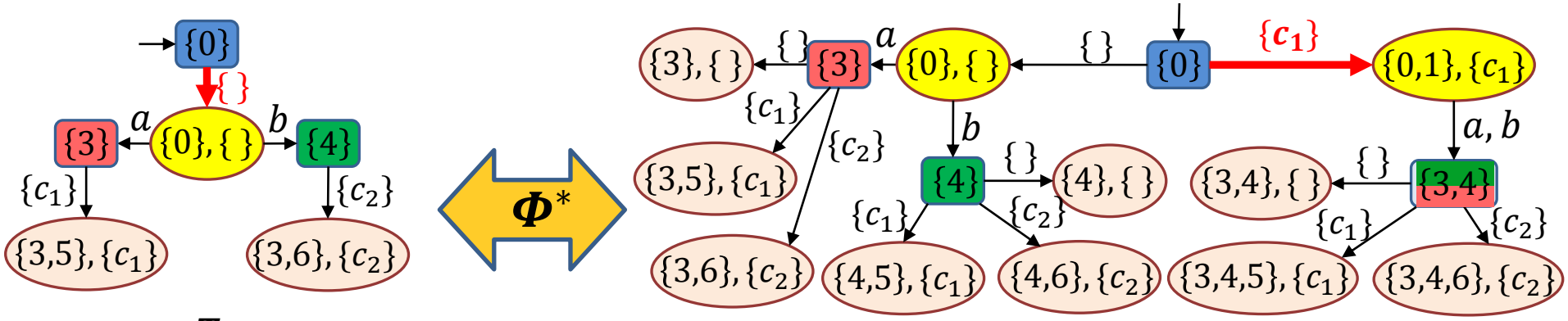

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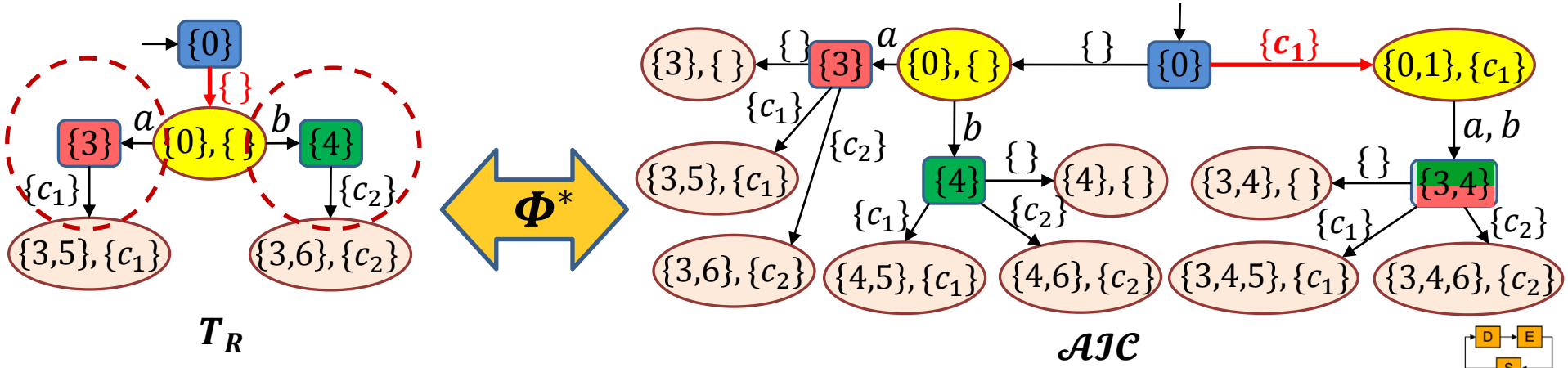

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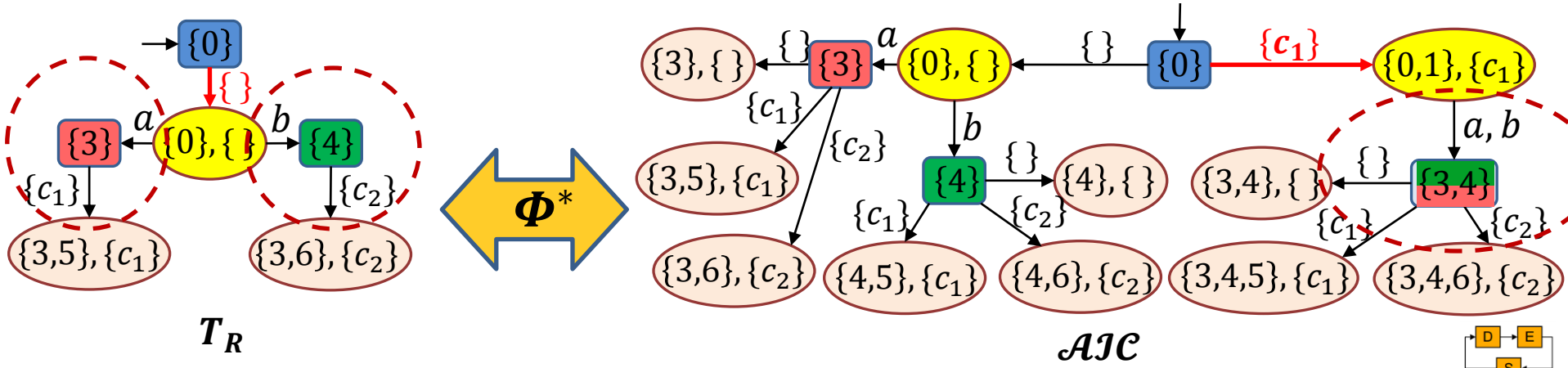

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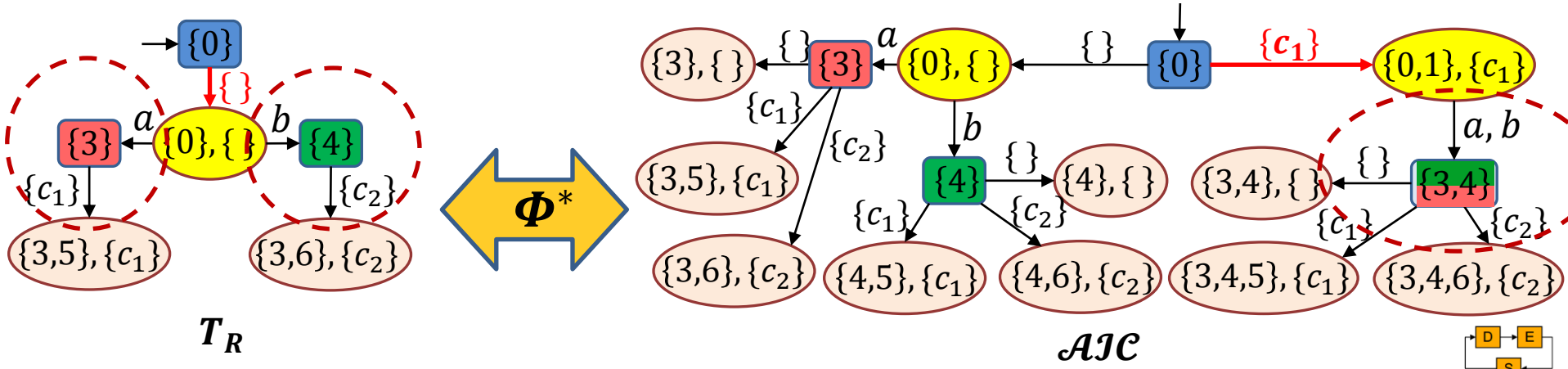

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- **Construct a new BTS: replace the original control decision in T_R by a larger one**
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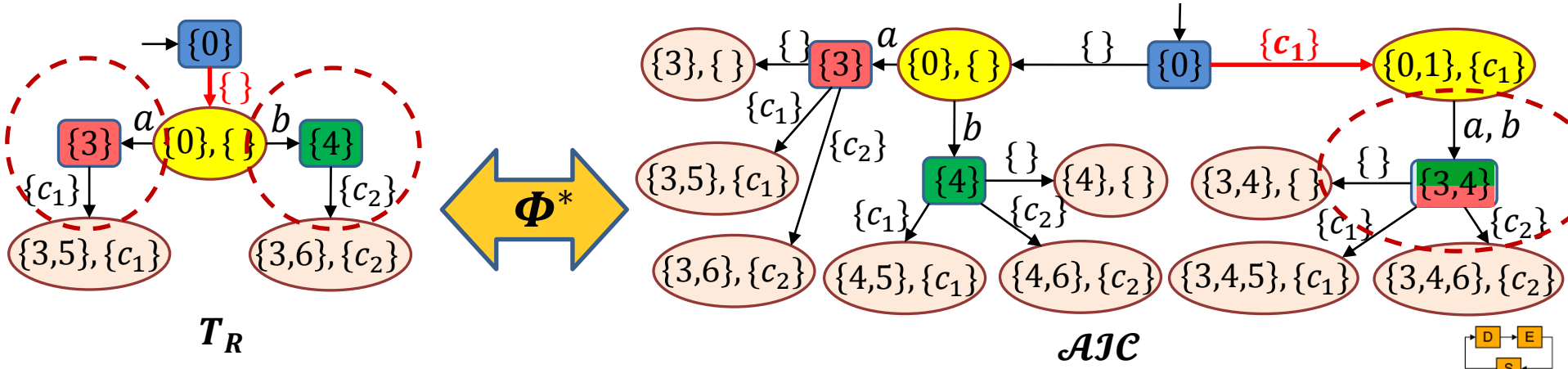
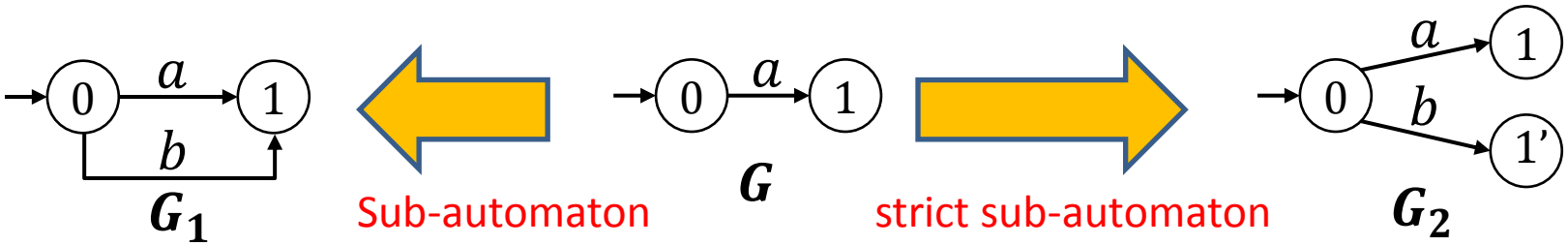
- **2^X may not be sufficient for the synthesis problem!**



The Role of Strict Sub-automaton

- The information merge phenomenon **will not occur** under the assumption that R is a **strict sub-automaton** of G, where $L(R) = L(S_R/G)$.



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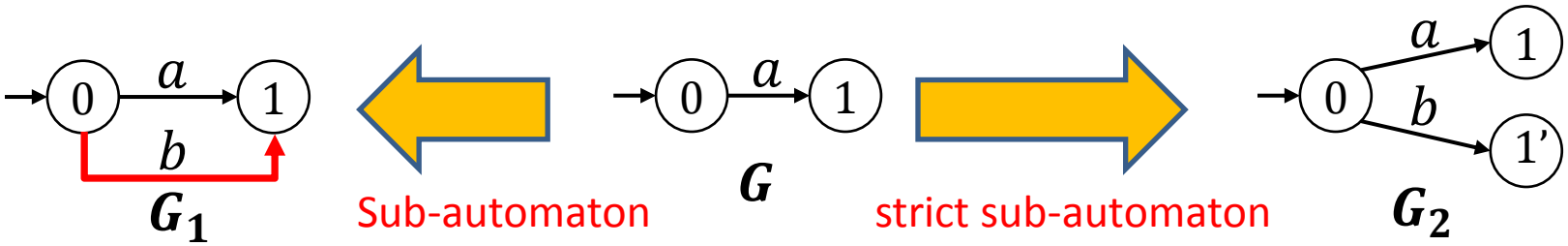

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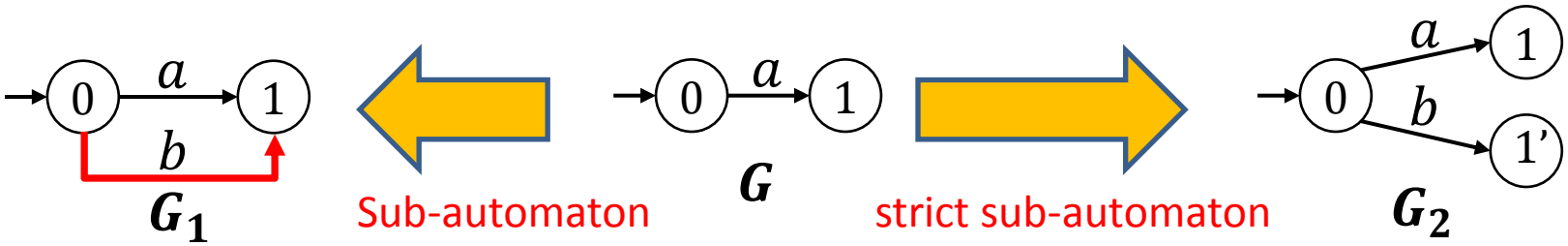
- We can always obtain strict sub-automaton [Cho & Marcus, 1989]



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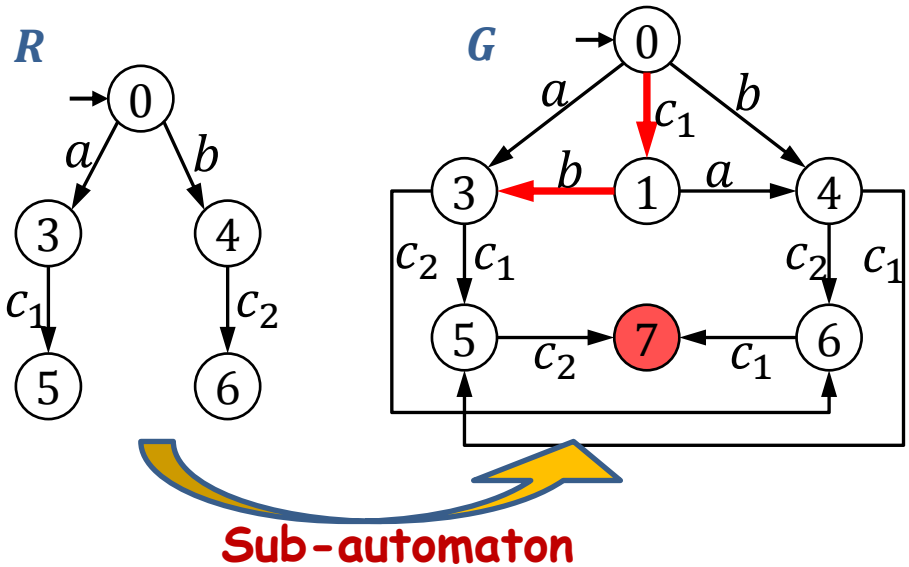

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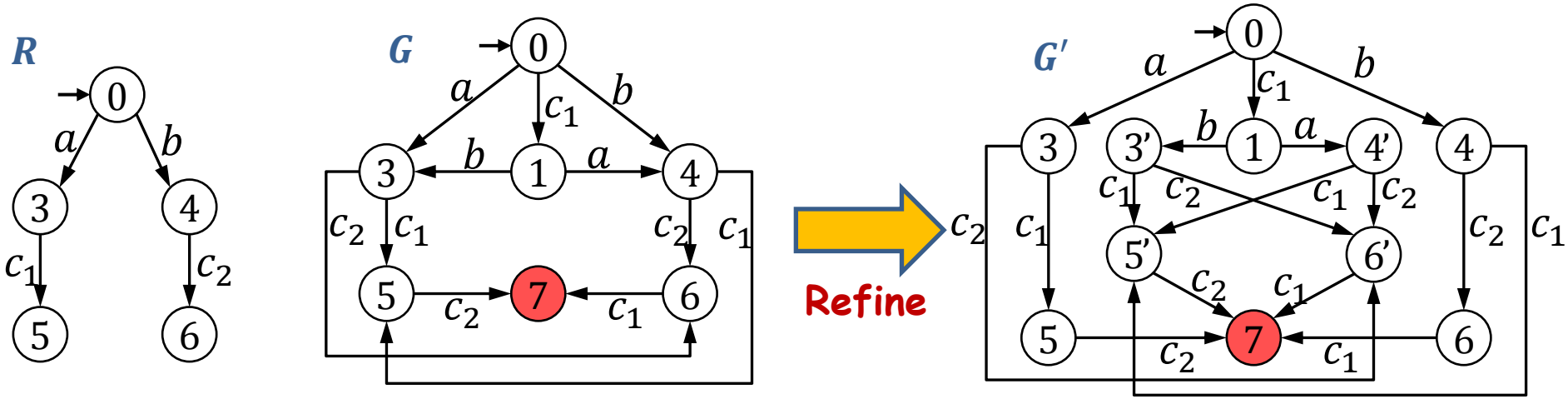

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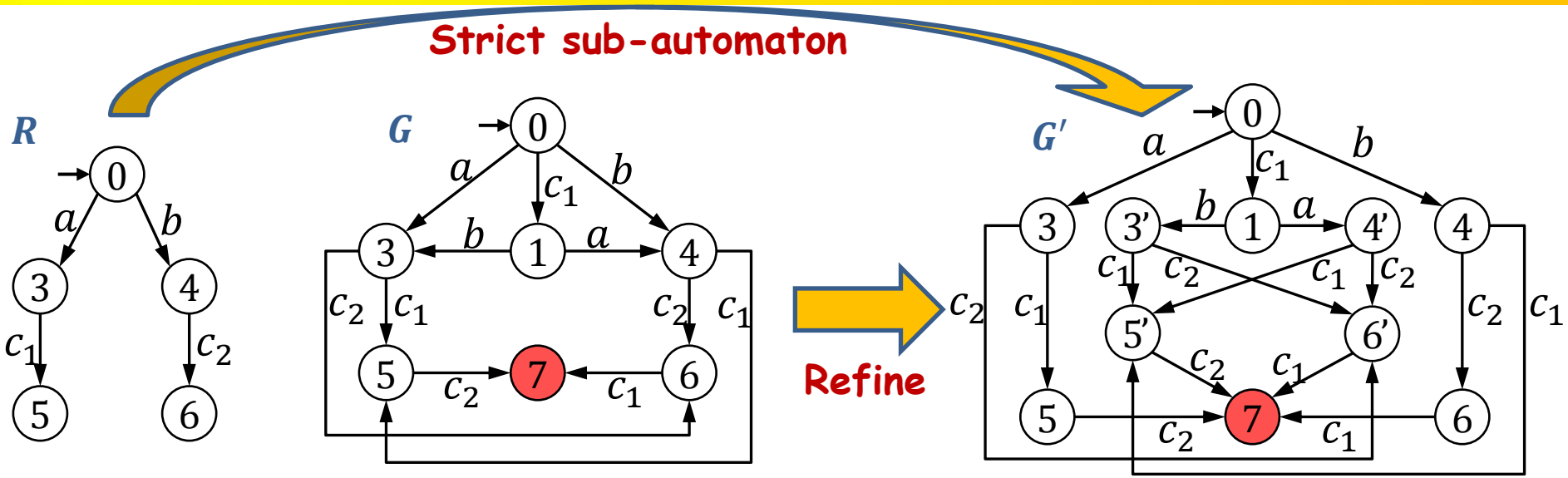

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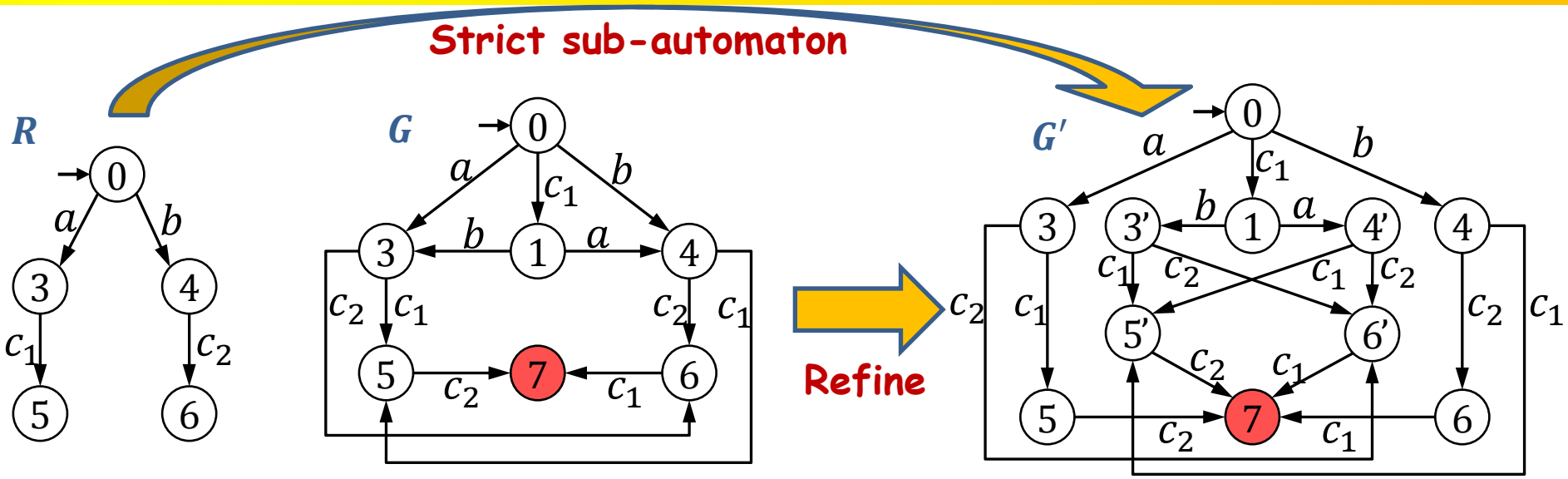

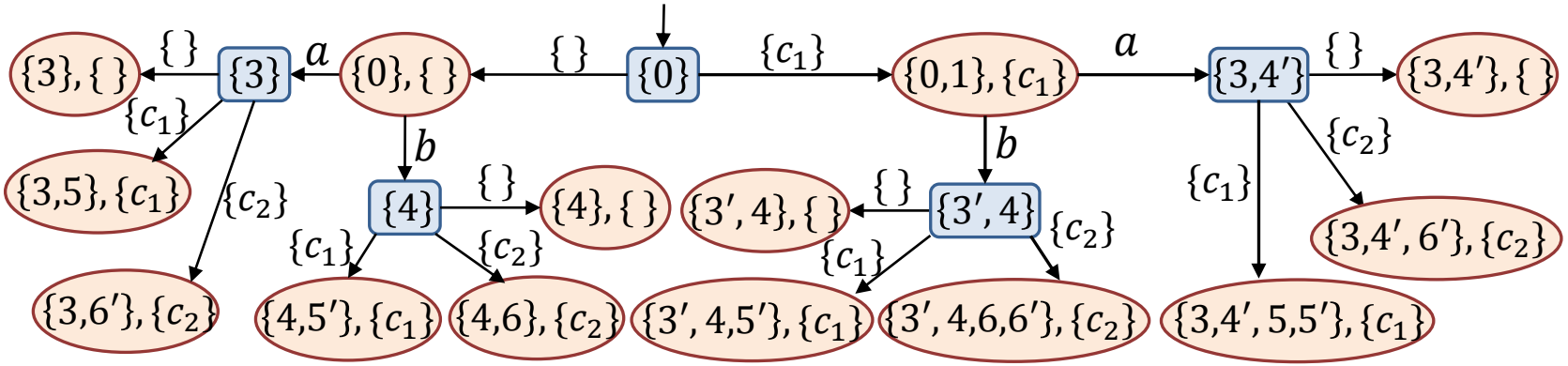

AIC for the refined system

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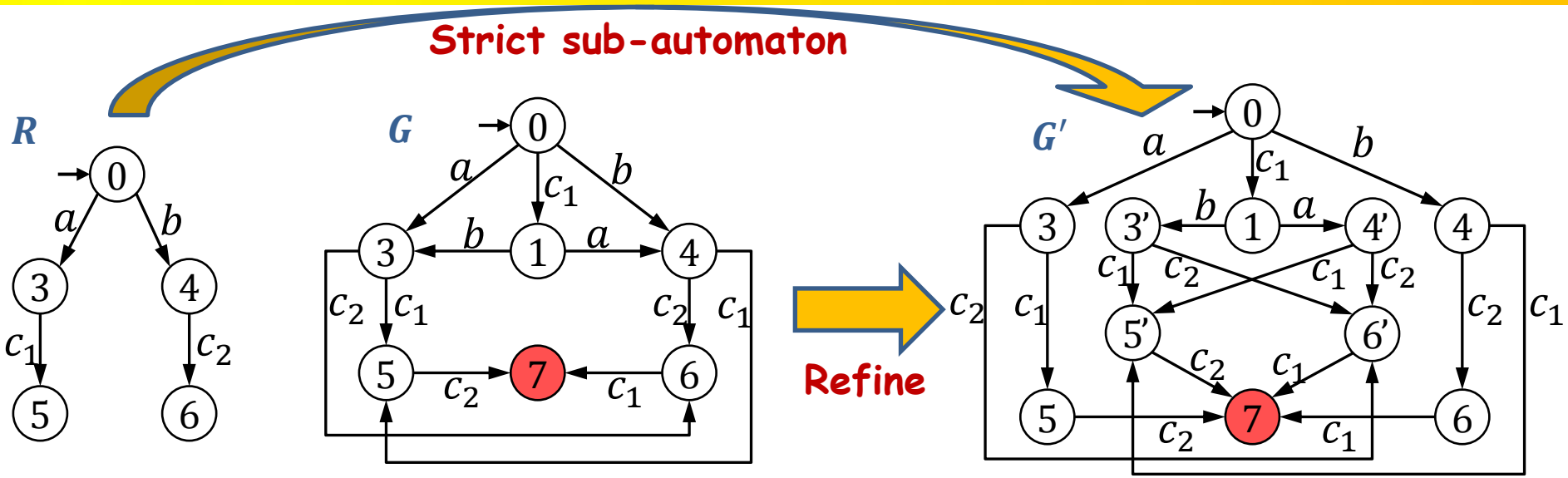

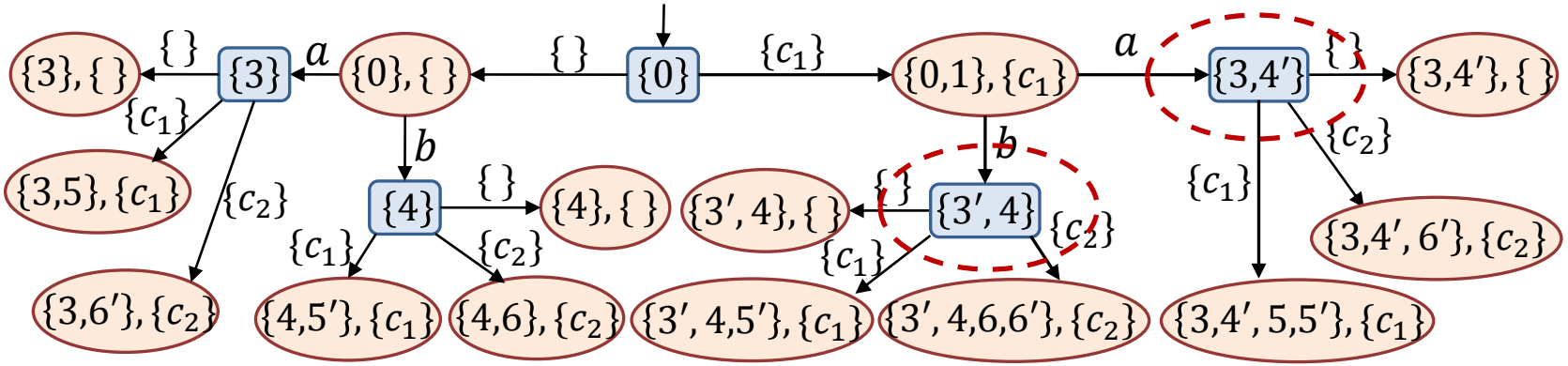

AIC for the refined system

Synthesis Algorithm

Synthesis Steps:

1. Construct BTS T_R and $\mathcal{AIC}(G)$ (make sure R is a strict sub-automaton of G)
2. Compute the maximal CSR Φ^* from T_R to $\mathcal{AIC}(G)$
3. Find a Y-state y in T_R such that its control decision can be replaced by γ
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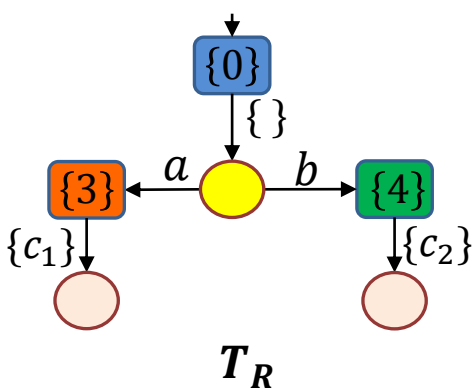

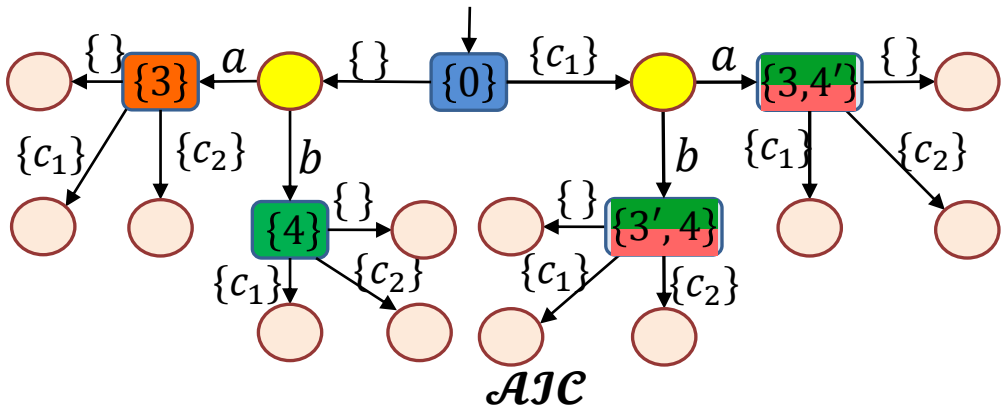

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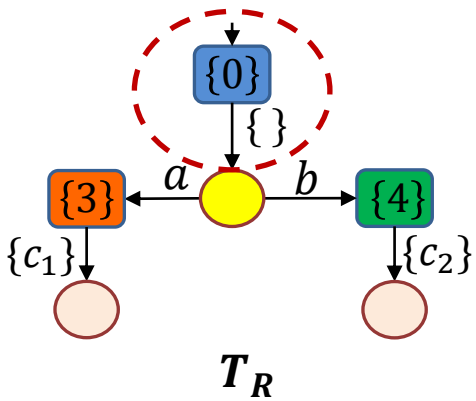

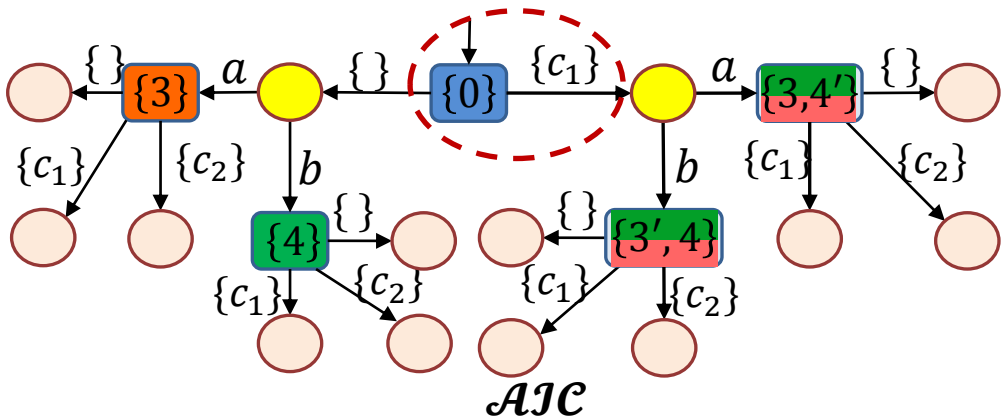

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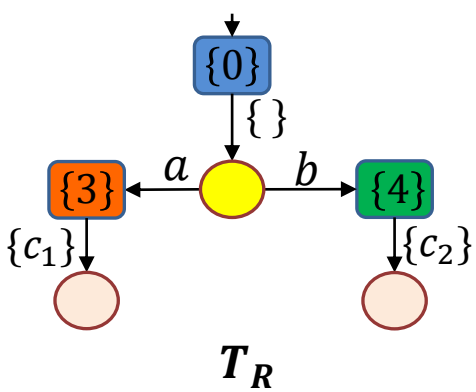

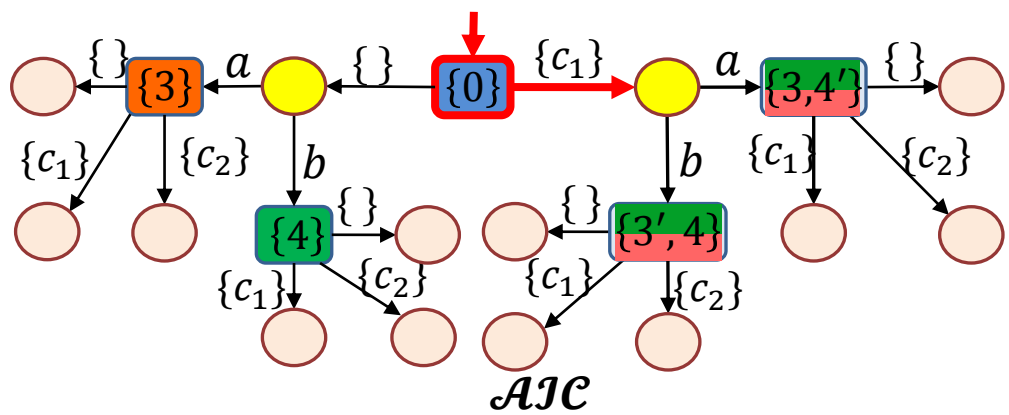

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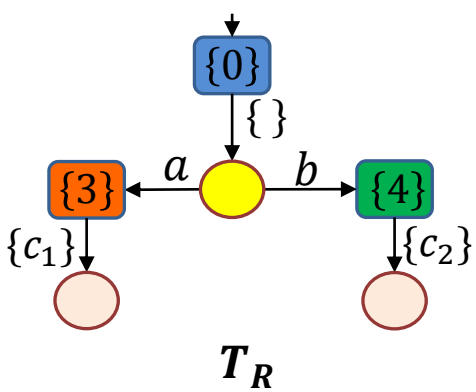

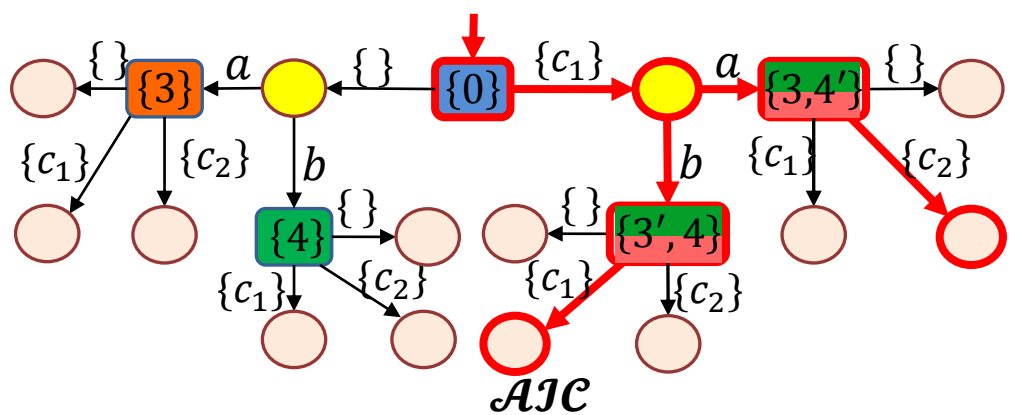

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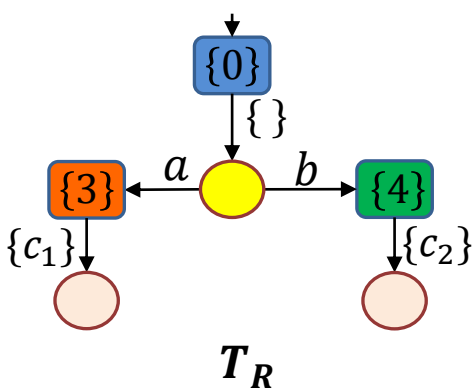
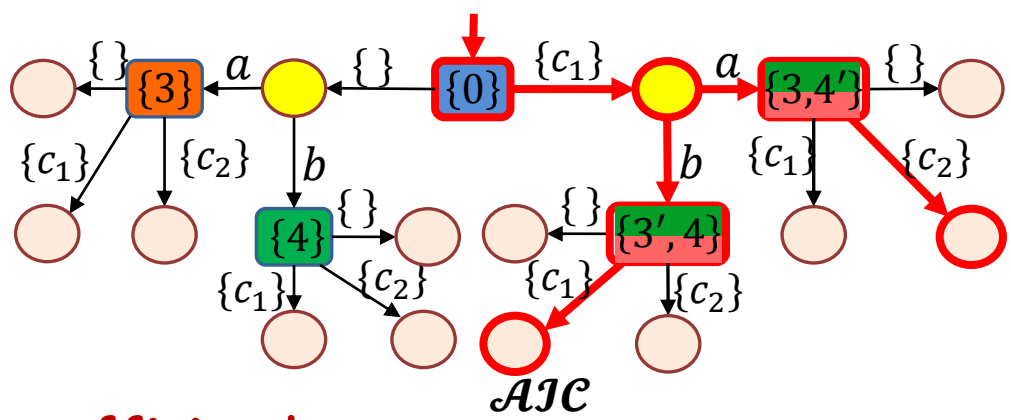
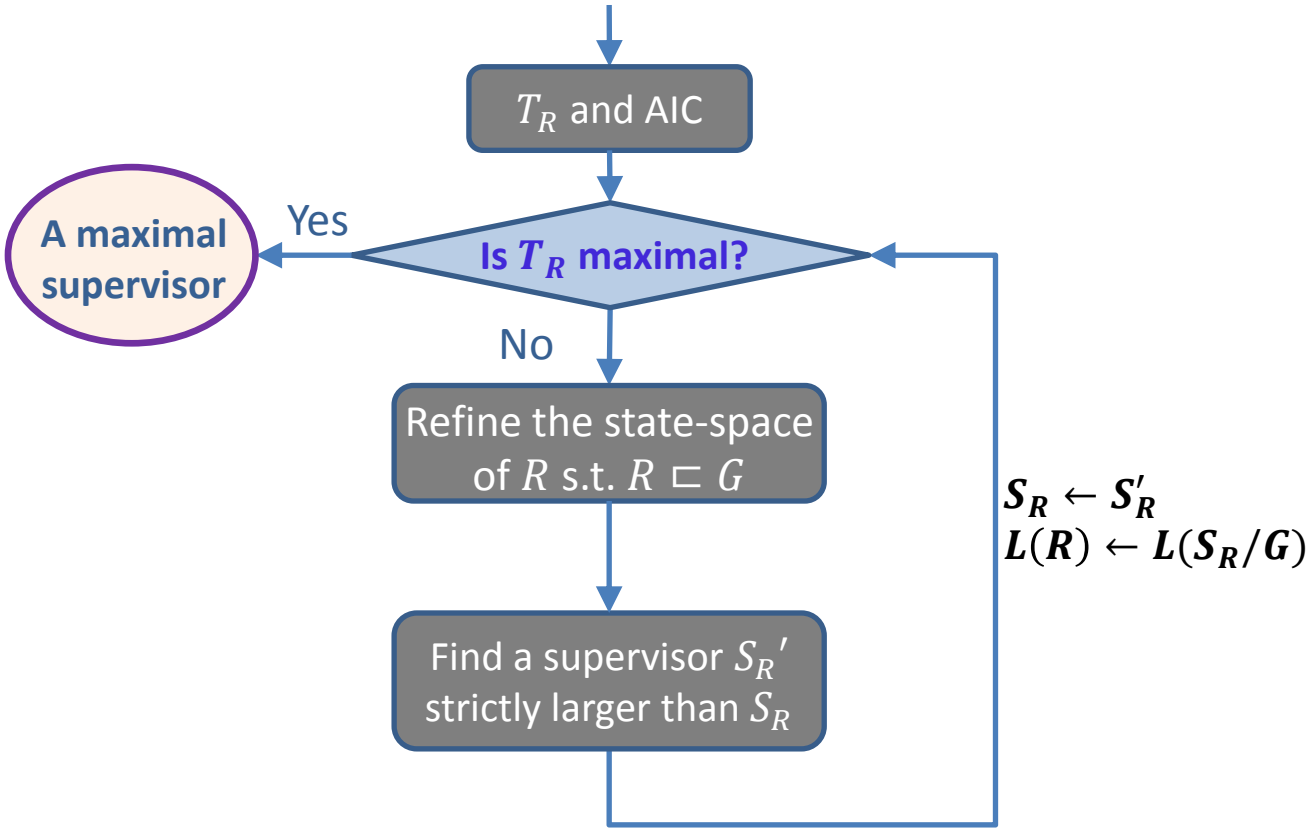




- 2^X is sufficient!

Achieve More Permissiveness



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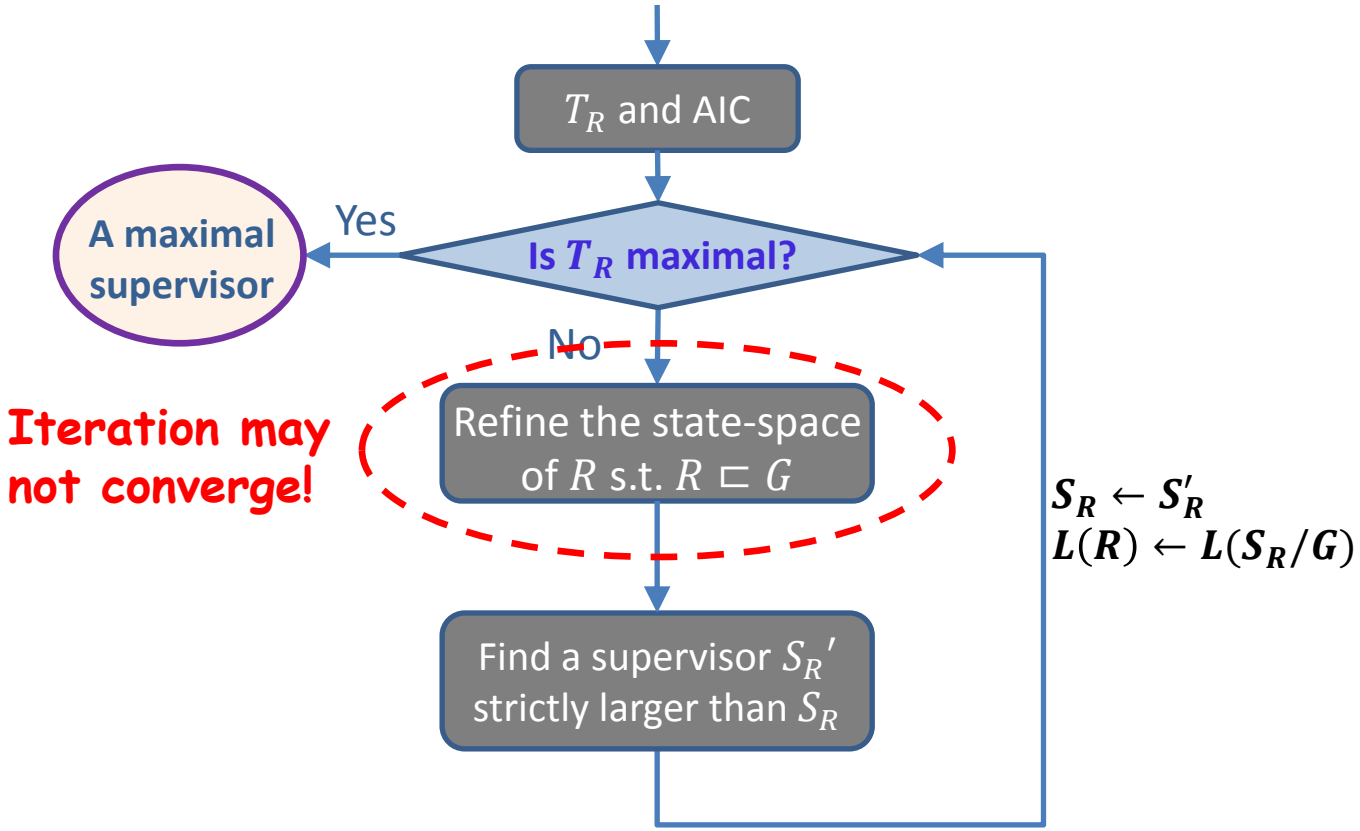

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On Going Work

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