Math 116, Winter 2012

Quiz 5 Name: Key

For the first five problems, no work is required. Just state whether the series converges or diverges.

1. (*1 point*) : **Convergent or Divergent?**

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

Alt series test

Alternate ☑

$\frac{1}{n} \to 0$ ☑

$\frac{1}{n} > \frac{1}{n+1}$ ☑

2. (*1 point*) : **Convergent or Divergent?**

geometric series w/ ratio $\frac{1}{e} < 1$

$$\sum_{n=1}^{\infty} e^{-n} = \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \ldots = \frac{1/e}{1-\frac{1}{e}} = \frac{1}{e-1} \approx .582$$

value of Σ

3. (*1 point*) : **Convergent or Divergent?**

$$\sum_{n=1}^{\infty} \frac{(-1)^n\sqrt{n+1}}{\sqrt{n}}$$

$\lim_{n\to\infty} \frac{\sqrt{n+1}}{\sqrt{n}} = 1 \neq 0 \to$ DV by n^{th} term

4. (*1 point*) : **Convergent or ~~Divergent?~~**

$$\sum_{n=1}^{\infty} \frac{1}{n^n}$$

Ratio $\lim_{n\to\infty} \frac{1}{(n+1)^{n+1}} \div \frac{1}{n^n} = \lim_{n\to\infty} \underbrace{\frac{n^n}{(n+1)^n}}_{\frac{1}{e}} \cdot \frac{1}{n+1} = 0$

5. (*1 point*) : **Convergent or Divergent?**

$$\sum_{n=1}^{\infty} \frac{|\sin(n)|}{n^2} \leq \sum_{n=1}^{\infty} \frac{1}{n^2} = CV \qquad = \frac{\pi^2}{6} - 1$$

p-series

6. (*5 points*) : Is the following series divergent, conditionally convergent, or absolutely convergent? Prove your answer in detail, using complete sentences and perfect notation.

$$\sum_{n=1}^{\infty} \frac{\overbrace{3\cdot(-1)^n}^{=a_n}}{\sqrt{n+1}}$$

$|a_n| = \frac{3}{\sqrt{n+1}}$

$\lim_{n\to\infty} |a_n| = 0$

& $|a_n| > |a_{n+1}|$

& alternate signs

Σa_n CV by AST

$$\sum |a_n| = \sum \frac{3}{\sqrt{n+1}} \approx \sum \frac{3}{\sqrt{n}} = \sum 3n^{-1/2} = \text{Div}$$

p-series

LCT

$$\lim_{n\to\infty} \frac{3/\sqrt{n+1}}{3/\sqrt{n}} = \lim_{n\to\infty} \sqrt{\frac{n}{n+1}} = 1$$

because Σa_n CV & $\Sigma |a_n|$ Div, we have Σa_n Converges conditionally!

7) (5 pts) Find the Interval of Convergence of

$$\sum_{n=0}^{\infty} \frac{3^n x^n}{\sqrt{n^4+9}}$$

$$\lim_{n\to\infty} \frac{3^{n+1}}{\sqrt{(n+1)^4+9}} \div \frac{3^n}{\sqrt{n^4+9}} = \lim_{n\to\infty} \frac{3^{n+1}}{3^n} \frac{\sqrt{n^4+9}}{\sqrt{(n+1)^4+9}} =$$

RATIO TEST

$$= \lim_{n\to\infty} 3 \cdot \sqrt{\frac{n^4+9}{(n+1)^4+9}} = 3$$

Ratio of $\left|\frac{a_{n+1}}{a_n}\right| \to 3$ as $n\to\infty$, so $RoC = \frac{1}{3}$

END POINTS:

$$\sum_{n=0}^{\infty} \frac{3^n \cdot (\frac{1}{3})^n}{\sqrt{n^4+9}} = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n^4+9}} \leq \sum_{n=0}^{\infty} \frac{1}{\sqrt{n^4}} = \sum_{n=0}^{\infty} \frac{1}{n^2} = CV$$

p-series

$x = \frac{1}{3}$

$$\sum \frac{3^n(-\frac{1}{3})^n}{\sqrt{n^4+9}} = \sum \frac{(-1)^n}{\sqrt{n^4+9}}$$

converges b/c it converges absolutely

$x = -\frac{1}{3}$

$$IoC = \left[-\frac{1}{3}, \frac{1}{3}\right]$$