## Math 116 - Quiz 4

Name: \_\_\_\_\_

Answer the following questions showing all work. 5 points each. For questions 1-4, consider the graph given by the parametric equations  $x(t) = 2e^t - 4$  and  $y(t) = \frac{1}{2}e^{2t} - t$ .

1. Precisely one of the points  $(2e^5 - 4, \frac{1}{2}e^5 - 5)$  and  $(2e - 4, \frac{e^2}{2} - 1)$  is on the graph. Which one is?

2. Find the equation of the tangent line to the graph at the point you found in 1.

3. Is the graph concave up or concave down at the point you found in 1?

4. Find length along the graph from  $(4, \frac{1}{2})$  to  $(8e^2 - 4, \frac{1}{2}e^4 - 1)$ .

5. Suppose  $r = f(\theta)$  satisfies the differential equation  $(\frac{dr}{d\theta})^2 + r^2 = \theta^2$ . Compute the arc length of f from  $\theta = 0$  to  $\theta = 2\pi$ .

6. In honor of St. Patricks Day, you decide to pick a shamrock. Unfortunately, you can't find any three leaf clovers, which makes you sad. Your friend offers to cheer you up by conjuring up one in the form of the polar equation  $r = 4\cos(3\theta)$ . With that in mind, compute the area of a single leaf.

7. Also, write down (but do not evaluate) a definite integral to compute the perimiter of one leaf.

8. Consider the region,  $\Omega$ , in the first quadrant bounded by the x-axis, the y-axis, the line x + y = 10, and the circle  $x^2 + y^2 = 4$ . Give inequalities on r and  $\theta$  that describe  $\Omega$ .