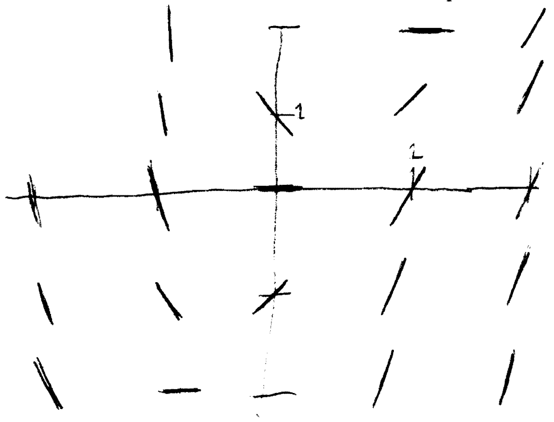
Math 116 - Quiz 3 **Name:** Key

Answer the following questions showing all work. 5 points each.

1. Let $\frac{dy}{dx} = 2x - y$. Sketch a slope field for this differential equation. Is $y = 2x$ a solution to the differential equation? Why or why not?



No, because if $y=2x$, then $\frac{dy}{dx}=2$; but $2x-y = 2x-(2x)=0$.

So $\frac{dy}{dx} \neq 2x-y$

2. Is $y = e^{e^x}$ a solution to $\frac{dy}{dx} = ln(y^y)$? [HINT: It may help to simplify first. Think about exponent rules.]

Suppose $y=e^{e^x}$. Then by chain rule, $\frac{dy}{dx} = e^x \cdot e^{e^x}$. Also, $\ln(y^y) = y\ln y$ $= e^{e^x}\cdot \ln(e^{e^x}) = e^{e^x} e^x$. Thus $\frac{dy}{dx} = \ln(y^y)$, so it is a soln

Alternatively, $\frac{dy}{dx} = \ln(y^y) = y\ln y$

$\int \frac{dy}{y\ln y} = \int dx$

$\ln|\ln y| = x+c$

w-sub w/ $w=\ln y$ & $dw = \frac{1}{y}dy$

$\rightarrow |\ln y| = e^{x+c}$ & $B = \pm e^c$

$\ln y = Be^x$

$y = e^{(Be^x)}$ ← This is all solutions to this diff eq; we have the one where $B=1$

3. Let $\frac{dq}{dt} = q^2 t$ and assume $q(0) = 1$. Using Euler's method with a Δt step of .5, estimate $q(1)$.

@ $t=0$ $q=1$ & $\frac{dq}{dt} = q^2 t = 1^2\cdot 0 = 0 = m$ So our tangent line is $q \approx 0(t-0)+1$, which gives us $q(.5) \approx 0(.5-0)+1 = 1$

@ $t=.5$ $q\approx 1$, so $\frac{dq}{dt} = (1)^2(.5) = .5 = m$. So our tangent line is $q \approx .5(t-.5)+1$, which gives $q(1) \approx .5(1-.5)+1 = 1.25$

So $q(1) \approx 1.25$

4. The most interesting man in the world wants a refreshing beer after a long day of wrestling tigers. Unfortunately, his liquid nitrogen chamber has no beer in it. So he puts a room temperature (70^oF) Dos Equis in. One second later, the temperature has already dropped to 69^oF. Set up and solve a differential equation to determine how long the beer needs to be before it is cold enough to drink (the most interesting man in the world prefers his beers to be 34^oF). You may assume Newton's Law of Cooling[1] and his liquid nitrogen chamber is kept at a chilly -350^oF.

$$\frac{dT}{dt} = k(T_{beer} - T_{air})$$

$$\frac{dT}{dt} = k(T+350)$$

$$\int \frac{dT}{T+350} = \int k\,dt$$

$$\ln|T+350| = kt + C$$

$$|T+350| = e^{kt+C} \quad B = \pm e^C$$

$$T+350 = Be^{kt}$$

$$T = Be^{kt} - 350$$

@ t=0 $\quad 70 = Be^{0k} - 350$

$$420 = B(1)$$

$$B = 420$$

@ t=1 $\quad 69 = Be^{1k} - 350$

$$419 = 420e^k$$

$$k = \ln\left(\frac{419}{420}\right) \approx -.00238$$

$$34 = 420e^{t(-.00238)} - 350$$

$$384 = 420e^{-.00238t}$$

$$-.00238t = \ln\left(\frac{384}{420}\right) \approx -.0896$$

$$t = \frac{-.0896}{-.00238} \approx 37.65 \text{ seconds!}$$

5. BONUS: Willy Wonka's 1000 gallon chocolate swimming pool starts with 100 gallons of water in it with a chocolicity (chocolate concentration) of 20 grams of chocolate per gallon. Fresh water is flowing into the tank at a rate of 10 gallons per minute, while 1 gallon of the mixed chocolate water leaks out every minute. What will the chocolicity of the water in the tank be when the tank is full?

Let $M(t)$: Mass of chocolate @ time t

Let $V(t)$ = Volume of H_2O @ time t

Let $C(t)$: Chocolicity @ time t

So $C(t)V(t) = M(t)$ $[CV=M]$

$$\frac{dM}{dt} = (\text{rate chocolate enter} - \text{rate chocolate leave})$$

$$\frac{dM}{dt} = (0 - (1)C) = -\frac{M}{V}$$

$$V(t) = 100 + (10-1)t$$

$$V(t) = 100 + 9t$$

$$\frac{dM}{dt} = \frac{-M}{100+9t}$$

$$\int \frac{-dM}{M} = \int \frac{dt}{100+9t}$$

$$-\ln|M| = \frac{1}{9}\ln|100+9t| + C$$

$$* \quad -\ln M = \frac{1}{9}\ln(100+9t) + C$$

$$e^{-\ln M} = e^{\frac{1}{9}\ln(100+9t) + C}$$

Let $B = e^C$

$$M^{-1} = B(100+9t)^{1/9}, \quad D = \frac{1}{B}$$

$$M = D(100+9t)^{-1/9}$$

@ $t=0$, we have $M = 20 \cdot 100 = 2000$

$$\rightarrow 2000 = D(100+0)^{-1/9} \rightarrow$$

$$D = 2000(100)^{1/9} \approx 1199$$

Tank full when $V = 1000 = 100 + 9t$

tank full when $t = 100$

$$M = 1199(1000)^{-1/9} \approx 557.67$$

$$C = \frac{M}{V} = \frac{557.67}{1000} \approx .557 \ \frac{g}{gal}$$

[1] The rate of change of the temperature of a body is directly proportional to the difference between the temperature of the body and the temperature of the surrounding air.

* b/c M & $100+9t$ both always positive