Math 116 - Quiz 2

Name: Key

Answer the following questions showing all work.

1. Let \( f(x) = \int_x^2 \cos^2(t) \, dt \).
   
a) Compute \( f(-1) \). [3 points]
   
   \[
   f(-1) = \int_{-1}^2 \frac{\cos^2(t)}{t} \, dt = \int_{-1}^1 \frac{\cos^2(t)}{t} \, dt \not= 0
   \]
   
   We integrate over a width of \((-1,1) = 0\), we get \( f(-1) \).

   b) Compute \( f'(3) \). [4 points]
   
   Let \( g \) be an antiderivative of \( \frac{\cos^2(t)}{t} \) (i.e. \( g'(t) = \frac{\cos^2(t)}{t} \)).
   
   Then \( f(x) = g(x^2) - g(1) \) by FTOC, so
   
   \[
   f'(x) = 2x \frac{\cos^2(x^2)}{x} \quad \text{so} \quad f'(3) = \frac{2 \cos^2(9)}{3} \approx 0.5534
   \]

2. Simplify \( \int \sin^6(5\theta) \cos(5\theta) \, d\theta \). [3 points]
   
   Let \( w = \sin^5 \theta \rightarrow \frac{dw}{d\theta} = 5 \cos \theta \rightarrow \frac{dw}{5} = \cos \theta \, d\theta \)
   
   \[
   \int \sin^6(5\theta) \cos(5\theta) \, d\theta = \int w^6 \frac{dw}{5} = \frac{1}{5} (\frac{1}{5} w^5) + C = \frac{1}{35} (\sin^5 \theta)^7 + C
   \]
3. Let $f(x)$ be a continuous function, with some of its values in the table below. Compute the following definite integrals. Show all work.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>2</td>
<td>1</td>
<td>-1</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>$f'(x)$</td>
<td>-2</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>12</td>
</tr>
</tbody>
</table>

a) $\int_0^3 4xf''(x)dx$ [5 points]

$$
\int_0^3 2 f'(w)dw = 2\int_0^3 f'(x)dx = 2f(2) - 2f(1)
$$

$$
2f(2) - 2f(1) = 2(2) - 2(3) = 1
$$

b) $\int_0^1 f(x)e^f(x)dx$ [5 points]

$$
\int_0^1 e^w dw = \int_0^x f(w) e^{f(w)}dw = [e^w]_0^x - e^0 = 1 - 1 = 0
$$

$$
[0 - 1 - 3] = 4, 6, 7, 1
$$

c) [Extra Credit] $\int_0^3 x f'''(x)dx$ [3 points]

$$
\int_0^3 x^2 f''(x)dx = \int_0^3 x f''(x) dx - \int_0^3 f''(x) dx
$$

$$
= 0 - [f''(1) + f''(3) - f''(3)] = 0 - [0 - 1] = 1
$$