(1 Uses FTOC; 2 uses w-sub)

Math 116 - Quiz 2 Name: Key

Answer the following questions showing all work.

1. Let $f(x) = \int_1^{x^2} \frac{\cos^2(t)}{t} dt$.

   a) Compute $f(-1)$. [3 points]

$f(-1) = \int_1^{(-1)^2} \frac{\cos^2(t)}{t} dt = \int_1^1 \frac{\cos^2 t}{t} dt = 0$ ↑ b/c if we integrate a fxn over a width of $(1-1)=0$, we get 0.

   b) Compute $f'(3)$. [4 points]

Let g be an antiderivative of $\frac{\cos^2 t}{t}$ (ie $g'(t) = \frac{\cos^2 t}{t}$).

Then $f(x) = g(x^2) - g(1)$ by FTOC, so $f'(x) = 2x\, g'(x^2) - 0$ ...

$f'(x) = 2x\left(\frac{\cos^2(x^2)}{x^2}\right) = \frac{2\cos^2(x^2)}{x}$. So $f'(3) = \frac{2\cos^2(9)}{3} \approx .5534$

2. Simplify $\int \underbrace{\sin^6(5\theta)}_{w^6} \underbrace{\cos(5\theta)d\theta}_{dw}$. [3 points]

Let $w = \sin 5\theta \rightarrow \frac{dw}{d\theta} = 5\cos 5\theta \rightarrow \frac{dw}{5} = \cos 5\theta\, d\theta$

$\rightarrow \int \sin^6(5\theta)\cos(5\theta)\, d\theta = \int \frac{w^6 dw}{5} = \frac{1}{5}\cdot\left(\frac{1}{7}w^7\right) + C = \frac{1}{35}(\sin 5\theta)^7 + C$

a&b use w-sub   c uses IBP

3. Let $f(x)$ be a continuous function, with some of its values in the table below. Compute the following definite integrals. Show all work.s

x	0	1	2	3	4
f(x)	2	1	-1	3	8
f'(x)	-2	0	2	1	12

$w = x^2$

$dw = 2x\,dx$

a) $\int_1^2 4xf'(x^2)dx$ [5 points]

$$= \int_{1^2}^{2^2} 2f'(w)\,dw = 2f(w)\Big|_{w=1}^{w=4} = 2f(x^2)\Big|_{x=1}^{x=2} = 2f(2^2) - 2f(1^2)$$

$$= 2f(4) - 2f(1)$$

$$= 14$$

either way

$$2f(4) - 2f(1) = 14$$

b) $\int_0^1 f'(x)e^{f(x)}dx$ [5 points]

$w = f(x)$

$dw = f'(x)dx$

$$= \int_{f(0)}^{f(1)} e^w\,dw = e^w\Big|_{w=f(0)}^{w=f(1)} = e^{f(x)}\Big|_{x=0}^{x=1} = e^{f(1)} - e^{f(0)} = e^1 - e^2$$

either way

$$e^1 - e^2$$

$$\left[e^1 - e^2 \approx -4.671\right]$$

c) [Extra Credit] $\int_1^3 xf''(x)dx$ [3 points]

$u = x$ $v = f'(x)$

$du = dx$ $dv = f''(x)dx$

$$\int x f''(x)\,dx = xf'(x) - \int f'(x) = xf'(x) - f(x)$$

$$\int_1^3 xf''(x)\,dx = \Big[xf'(x) - f(x)\Big]_{x=1}^{x=3} = \big[3f'(3) - f(3)\big] - \big[1f'(1) - f(1)\big]$$

$$= [3\cdot 1 - 3] - [1(0) - 1]$$

$$= [0] - [-1]$$

$$= 1$$