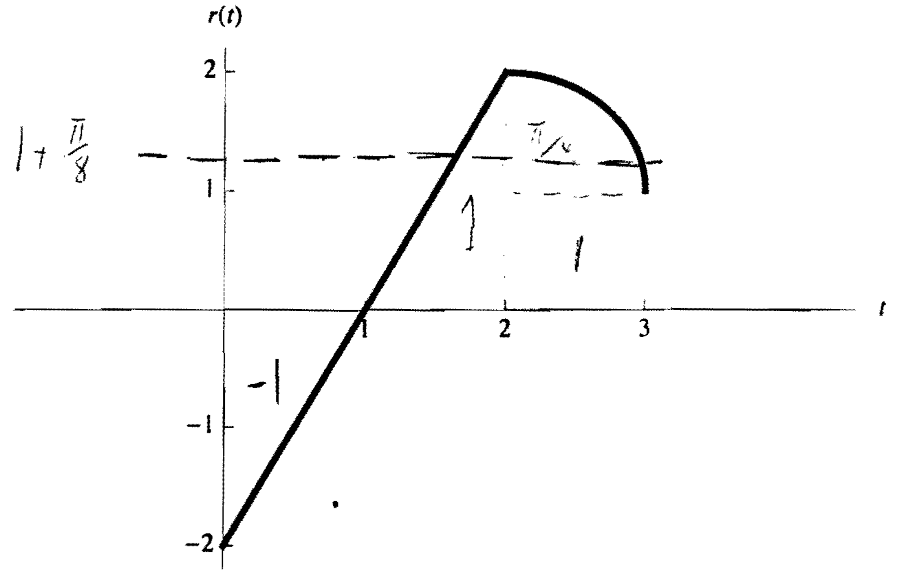
Key Quiz 1

3. [12 points] Shown below is a graph of a function $r(t)$. The graph consists of a straight line between $t = 0$ and $t = 2$ and a quarter circle between $t = 2$ and $t = 3$.



Calculate the following using the graph and the properties of integrals.

a. [4 points] $-3\int_0^3 (2 + r(t))dt$.

$$= -3\int_0^3 2\,dt + -3\int_0^3 r(t)\,dt = (-3)(3)(2) + (-3)\left[-1+1+1+\frac{\pi}{4}\right]$$

$$= -18 + (-3)\left[1+\frac{\pi}{4}\right] = -21 - 3\pi/4 \approx -23.35$$

b. [4 points] $\int_{1/2}^{3/2} r'(t)dt$.

FTOC

$$= r\left(\frac{3}{2}\right) - r\left(\frac{1}{2}\right) = 1 - (-1) = 2$$

c. [4 points] The average value of r on the interval $[1, 3]$. width = 2

$$\frac{1}{2}\int_1^3 r(t)\,dt = \frac{1}{2}\left[1 + \pi/4 + 1\right]$$

$$= \frac{1}{2}\left[2 + \frac{\pi}{4}\right]$$

$$= 1 + \frac{\pi}{8} \approx 1.3926$$ (see graph) (looks reasonable)

7. (4 points each) Table 1 below displays some values of an invertible, twice-differentiable function $f(x)$, while Figure 2 depicts the graph of the function $g(x)$.

Table 1

x	1	2	3	4	5
$f(x)$	-5	-2	2	4	7
$f'(x)$	5	6	2	3	3
$f''(x)$	1	-1	-3	-2	0

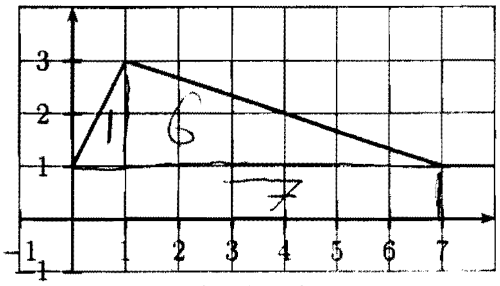


Figure 2: Graph of $g(x)$

Evaluate each of the following. Show your work.

(a) $\int_0^7 g(x)\,dx$ = Area under g from $x=0$ to $x=7$

$= 1+6+7 = 14$.

F.T.C

(b) $\int_1^3 f'(x)\,dx = f(3)-f(1) = 2-(-5)=7$

(c) $\int_1^5 \left(3f''(x)+4\right)dx = 3\int_1^5 f''(x)\,dx + \int_1^5 4\,dx$

FT.C $= 3(f(5)-f(1)) + 16$

$= 3(3-(5)) + 16$

$= 10$

(d) $\int_1^4 \underbrace{\left(f'(x)g(x)+f(x)g'(x)\right)}_{\frac{d}{dx}[f(x)g(x)]}dx = [f(4)g(4)] - [f(1)g(1)] =$

$[4 \cdot 2] - [3(-5)]$

Product Rule

$8 - (-15) = 23$