Quiz 4 (20 points in total)
Section 201/202 (circle one) Key
Name:

to check, compare graphs of $f(x)$ & $L(x)$ near $x=1$

1. [6 points] Suppose $f(x) = e^{x^2}$.

chain rule

(1) [2 points] Find the tangent line approximation of $f(x)$ at $x = 1$.

$f(1) = e^{1^2} = e^1 \approx 2.72$ $\quad f'(x) = (2x)e^{x^2}$ $\quad f'(1) = 2e^{1^2} = 2e \approx 5.44$ $\quad (y-e) = 2e(x-1)$ $\quad \boxed{L(x) = 2ex - e \approx 5.44x - 2.72}$

(2) [2 points] Use your answer in part (1) to estimate $e^{1.01^2}$.

$$e^{1.01^2} = f(1.01) \approx L(1.01) = 2e(1.01) - e = 1.02e \approx \boxed{2.77}$$

To check, compare to $e^{1.01^2} = e^{1.0201} \approx 2.77$

(3) [2 points] Is your estimation in part (2) an overestimate or underestimate? Use concavity to explain your answer.

$f''(x) = (2)e^{x^2} + (2x)(2x)e^{x^2}$ (from product/chain rules)

$= (2+4x^2)e^{x^2}$

always ≥ 2 so always +

always +

Since $f''(x) > 0$ always, f is concave up always. In particular, $f''(1) > 0$ so f is concave up near 1, so the line lies under the function. Thus our estimate is an underestimate.

2. [6 points]Suppose that x and y satisfy the relation given by the curve

$$x^2 + xy + y^2 = 3$$

(1) [3 points] Find $\frac{dy}{dx}$ (in other words, y').

$2x + (1)y + x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = 0$

$x\left(\frac{dy}{dx}\right) + 2y\left(\frac{dy}{dx}\right) = -2x - y$

$(x+2y)\left(\frac{dy}{dx}\right) = -2x - y$

$$\boxed{\frac{dy}{dx} = \frac{-2x-y}{x+2y}}$$

From before

Need $\frac{dy}{dx} = 0$

(2) [3 points] Find all points on the curve at which the tangent line is horizontal.

$\frac{dy}{dx} = \frac{-2x-y}{x+2y} = 0$

So $y = -2x$

Need $x^2 + xy + y^2 = 3$

$x^2 + x(-2x) + (-2x)^2 = 3$

$x^2 - 2x^2 + 4x^2 = 3$

$3x^2 = 3$

$x = \pm 1$

$x = 1 \rightarrow y = -2(1) = -2$

$x = -1 \rightarrow y = -2(-1) = 2$

So, $\boxed{(1,-2)\ \&\ (-1,2)}$

3. [8 points] Let $f(x) = x^2(x-1)^2$. Find all the critical points of $f(x)$ and use either the first derivative test or the second derivative test to classify these critical points as a local max, local min, or neither. Show all your steps!

$f'(x) = 2x(x-1)^2 + x^2(2)(x-1)$

$= 2x(x-1)[x-1+x]$

$= 2x(x-1)(2x-1)$ exists everywhere

Roots: $x = 0$ $\quad x = 1$ $\quad x = \frac{1}{2}$

f': $- -$ 0 $+ +$ 0 $- -$ 0 $+ +$

$0 \qquad \frac{1}{2} \qquad 1$

$f'(-1) = 2(-1)(-2)(-3) = -12 < 0$

$f'(\frac{1}{4}) = 2(\frac{1}{4})(\frac{-3}{4})(\frac{-1}{2}) = \frac{3}{16} > 0$

$f'(\frac{3}{4}) = 2(\frac{3}{4})(\frac{-1}{4})(\frac{1}{2}) = \frac{-3}{16} < 0$

$f'(2) = 2(2)(1)(3) = 12 > 0$

CP's	class
0	Local min
$\frac{1}{2}$	Local Max
1	Local min