Solution to quiz 2

1. [6 points] The average weight of a squirrel in Ann Arbor oscillates sinusoidally between a low of 5 pounds on January 1 and a high of 9 pounds on July 1, and a low of 5 pounds on January 1 next year again. Suppose that the function $P(t)$ gives the average weight in pounds of an Ann Arbor squirrel t months after January 1.

a). What is the amplitude of $P(t)$?
Solution: $\frac{9-5}{2} = 2$

b). What is the period of $P(t)$?
Solution: 12 (months)

c). Find a formula for $P(t)$.
Solution: Suppose $y = A\cos(Bt) + k$, then $A < 0$. Midline is $y = \frac{5+9}{2} = 7$, so $k = 7$; amplitude is 2, so $|A| = -A = 2$, so $A = -2$; period is $\frac{2\pi}{B} = 12$, so $B = \frac{\pi}{6}$. Therefore the formula is

$$P(t) = -2\cos(\frac{\pi}{6}t) + 7$$

2. [2 points]$f(x) = \sin(x)^x$, write down the limit definition of $f'(3)$. (You don't need to calculate it, just write the definition.)
Solution:

$$f'(3) = \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{\sin(3+h)^{3+h} - \sin(3)^3}{h}$$

3. [4 points]The cost, C (in dollars) to produce g gallons of ice cream can be expressed as $C = f(g)$. Assume f is invertible. Interpret

(1) $f'(100) = 2.5$
Solution: When 100 gallons of ice cream are produced, it costs approximately 2.5 dollars to produce 1 more gallon of ice cream.
Or, It costs approximately 2.5 more dollars to produce 101 gallons of ice cream than producing 100 gallons.

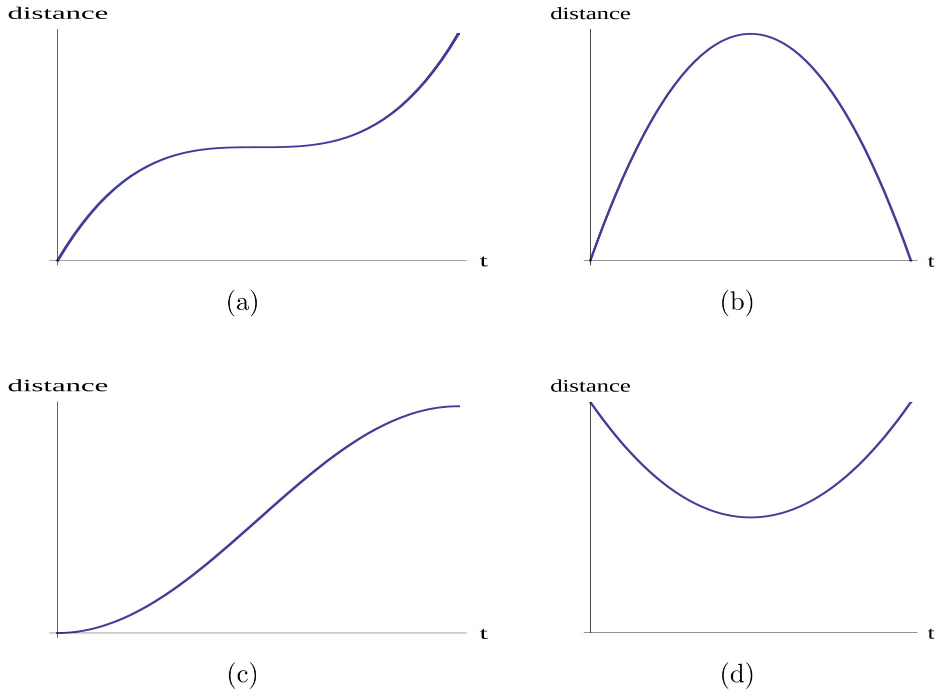
(2) $(f^{-1})'(100) = 2.5$
Solution: When the cost is 100 dollars, approximately 2.5 gallons more ice cream will be produced if 1 more dollar is spent.

4. [6 points]Consider a particle, whose position, s, is given by the table

t (seconds)	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6
S (feet)	0.5	1.4	3.8	6.5	9.6	9.2	8.7	6.1

(1) Estimate the velocity of the particle at $t = 0.2$. (Don't forget the unit)

Figure 1: Problem 5



Solution: $v(0.2) = S'(0.2) \approx \frac{S(0.4)-S(0.2)}{0.4-0.2} = \frac{1.4-0.5}{0.2} = 4.5$ ft/s

(2) Use (1) and the table to estimate the position of the particle at $t = 0.17$. (Don't forget the unit)
Solution: $S(0.17) \approx S(0.2) + S'(0.2)(0.17 - 0.2) = 0.5 + 4.5 \cdot (-0.03) = 0.365$ ft.

(3) For which t, does the velocity appear to be positive?
Solution: For $0.2 \leq t \leq 1.2$, $S(t)$ is increasing, so $v(t) = S'(t)$ appear to be positive for $0.2 \leq t \leq 1.2$.

5.[2 points] See figure 1, which graph represents the position of an object that is speeding up and then slowing down?
Answer: **(c)** Since speed is $v(t) = S'(t)$ = slope of tangent line at t, speed is increasing then decreasing, so slope of tangent lines is increasing then decreasing.