Solution to quiz 1

1. A biologist is growing mold in a Petri dish for an experiment. At noon she starts the experiment by innoculating (putting mold into) the dish. At 4 pm, she notices that the mold has grown to cover an area of 5 $cm^2$. At 7pm, she returns and finds the mold now covers an area of 7 $cm^2$.

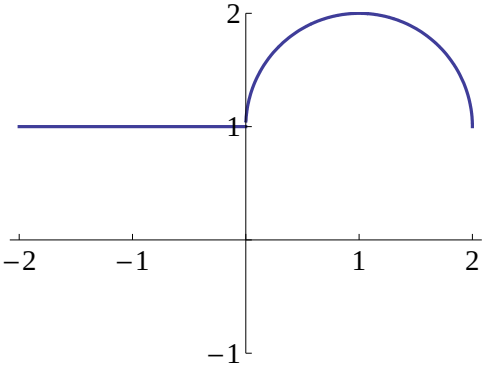
(a) Assuming the area of the mold grows linearly over time, find a formula $l(t)$ which gives the area the mold covers $t$ hours after noon. (3 points).
**Solution:** let $y = l(t) = mt + b$, then $l(4) = 5$ and $l(7) = 7$, so slope $m = \frac{7-5}{7-4} = \frac{2}{3}$. Use the point slope form, we have $y - 7 = \frac{2}{3}(x - 7)$, simplify, $y = l(t) = \frac{2}{3}t + \frac{7}{3}$.
(b) Assuming the area of the mold grows exponentially over time, find a formula $e(t)$ which gives the area the mold covers $t$ hours after noon. (3 points).
**Solution:** Let $y = e(t) = P_0 a^t$, then

$$e(4) = 5 \Rightarrow P_0 a^4 = 5$$

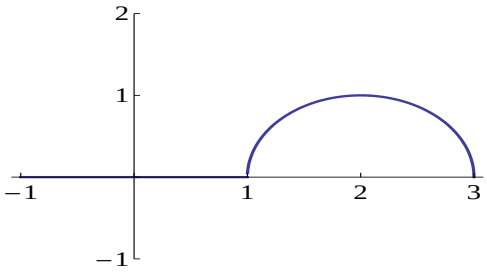
$$e(7) = 7 \Rightarrow P_0 a^7 = 7$$

Divide the second equation by the first and cancel out $P_0$, we have $1.4 = a^3$, so $a = 1.4^{\frac{1}{3}} = 1.1187$, plug this back into either equation, we have $P_0 = 3.1925$. So $e(t) = 3.1925 \cdot (1.1187)^t$.
(c) Using your formula $e(t)$ from part (b), how long does it take for the area of the mold to triple? (3 points).
**Solution:** $3P_0 = P_0(1.187)^t$, cancel out $P_0$, $3 = (1.1187)^t$, so $t = \ln(3)/\ln(1.1187) \approx 9.795$ years.
(d) Suppose the biologist returns at 9pm to find the mold has grown to 8.8 $cm^2$. Which of your formulas more accuately predicts this growth? Justify your reasoning. (3 points).
**Solution:** $l(9) = \frac{2}{3} \cdot 9 + \frac{7}{3} = 8.33$, $e(9) = 3.1925 \cdot (1.1187)^9 = 8.76$, so exponential is more accurate because it predicts 8.76 $cm^2$ of mold, which is closer than the 8.33 $cm^2$ of mold predicted by $l(t)$.

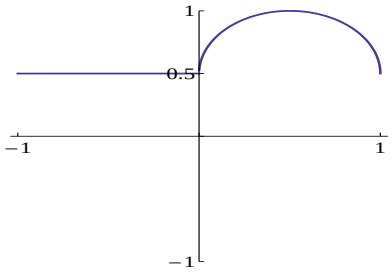
2. Given below is the graph of $y = f(x)$.



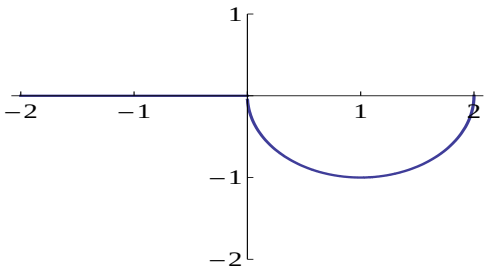

Find the function for the above graphs in terms of $f(x)$. (No partial credit, 8 points)



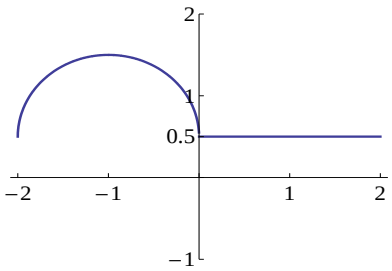

(a)




(b)




(c)




(d)

**Write your answer here**

(a) shift down by 1 and right by 1, so $f(x-1)-1$.
(b) vertical and horizontal scaling by $1/2$, so $\frac{1}{2}f(2x)$
(c) reflect over $x$-axis and shift up by 1, so $-f(x)+1$.
(d) reflect over $y$-axis and shift down by 0.5, so $f(-x)-0.5$.