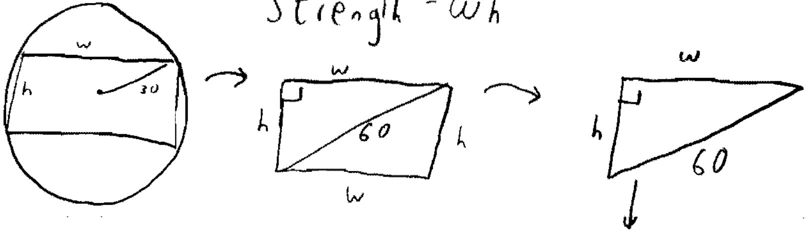
4.3 33)

Strength $= wh^2$



Constraint: $w^2 + h^2 = 60^2$

Solve $h^2 = (60^2 - w^2)$ with $0 \le w \le 60$

$Str = wh^2 = w(60^2 - w^2) = 60^2 w - w^3$

$\frac{dStr}{dw} = 60^2 - 3w^2 = 0 \rightarrow 3w^2 = 60^2$

$w^2 = \frac{60^2}{3}$

$w = \frac{\sqrt{60^2}}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 34.64\,cm$

critical pt

Compare: Str when $w = 34.64 \rightarrow (34.64)(60^2 - 34.64^2) = 83138$

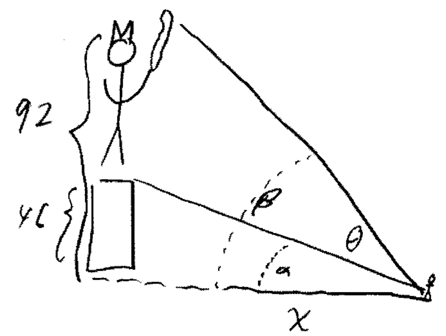
endpoints $\begin{cases} \text{Str when } w = 0 \rightarrow 0(60^2 - w^2) = 0 \\ \text{Str when } w = 60 \rightarrow 60(60^2 - 60^2) = 0 \end{cases}$

WINNER.

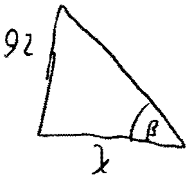
Max strength when

$\underline{w = 34.64\,cm}$ & $h = \sqrt{60^2 - w^2} = \sqrt{3600 - 1200} = \underline{48.98\,cm}$

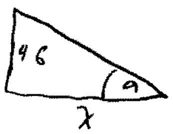
(48)



Maximize $\theta = \beta - \alpha$

β: 

$\tan\beta = \frac{92}{x} \rightarrow \beta = \tan^{-1}\left(\frac{92}{x}\right)$

α: 

$\tan\alpha = \frac{46}{x} \rightarrow \alpha = \arctan\left(\frac{46}{x}\right)$

x can be any real # >0

$\theta = \beta - \alpha$

$\theta = \arctan\left(\frac{92}{x}\right) - \arctan\left(\frac{46}{x}\right)$

$$\frac{d\theta}{dx} = \left(\frac{-92}{x^2}\right) \cdot \frac{1}{1+\left(\frac{92}{x}\right)^2} - \left(\frac{-46}{x^2}\right) \cdot \left(\frac{1}{1+\left(\frac{46}{x}\right)^2}\right)$$

$$= \left(\frac{46}{x^2}\right)\left[\frac{1}{1+\left(\frac{46}{x}\right)^2} - 2\frac{1}{1+\left(\frac{92}{x}\right)^2}\right]$$

Never 0

$$\frac{1}{1+\left(\frac{46}{x}\right)^2} - 2\frac{1}{1+\left(\frac{92}{x}\right)^2} = 0 \Longrightarrow \frac{2}{1+\left(\frac{92}{x}\right)^2} = \frac{1}{1+\left(\frac{46}{x}\right)^2}$$

cross multiply

$$2\left(1+\left(\frac{46}{x}\right)^2\right) = 1+\left(\frac{92}{x}\right)^2$$

times x^2

$$2x^2 + 2 \cdot 46^2 = x^2 + 92^2$$

$$x^2 = 4232$$

ignore $\rightarrow$ $x = \pm\sqrt{4232}$

$x = 65.0$ ~~ft~~ m

We note $\frac{d\theta}{dx}$ is negative for any $x > 65$ ~~ft~~ m & $\frac{d\theta}{dx} > 0$ for $0 < x < 65$ ~~ft~~ m so our critical point must be a global max.