Math 116-101 or 102 (circle one) (Spring 2012) **Name:** ______________________

Quiz 5: §9.3-9.5; 10.1-10.2

6/12/2012

Show all work and include units where appropriate. You have 30 minutes to complete this quiz. (25 pts)

1. Determine the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{5^n(x-2)^n}{n^2+1}$ (5 pts)

   Using the ratio test, $\displaystyle\lim_{n\to\infty} \left|\frac{5^{n+1}(x-2)^{n+1}}{(n+1)^2+1} \div \frac{5^n(x-2)^n}{n^2+1}\right| = \lim_{n\to\infty} \left|\frac{5^{n+1}}{5^n} \frac{n^2+1}{n^2+2n+2}(x-2)\right| = |5(x-2)|$. Thus our radius of convergence is $\frac{1}{5}$.

   Testing our end points, when $x = 2+\frac{1}{5}$, our power series evaluates as $\displaystyle\sum_{n=1}^{\infty} \frac{5^n((2+\frac{1}{5})-2)^n}{n^2+1} = \sum_{n=1}^{\infty} \frac{5^n(\frac{1}{5})^n}{n^2+1} = \sum_{n=1}^{\infty} \frac{1}{n^2+1}$, which converges by p-test[1] ($p = 2 > 1$).

   When $x = 2-\frac{1}{5}$, our power series evaluates as $\displaystyle\sum_{n=1}^{\infty} \frac{5^n((2-\frac{1}{5})-2)^n}{n^2+1} = \sum_{n=1}^{\infty} \frac{5^n(\frac{-1}{5})^n}{n^2+1} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$, which converges absolutely, as shown above. Therefore our interval of convergence is $[2-\frac{1}{5}, 2+\frac{1}{5}]$.

2. Does $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$ converge absolutely, converge conditionally, or diverge? Justify your answer. (5 pts)

   We note that $\displaystyle\lim_{n\to\infty} \frac{(-1)^n}{\ln(n)} = 0$, sucessive terms alternate signs, and $|\frac{(-1)^n}{\ln(n)}| > |\frac{(-1)^{n+1}}{\ln(n+1)}|$ for all n. Thus the series converges by the alternating series test. However, $\displaystyle\sum_{n=2}^{\infty} \left|\frac{(-1)^n}{\ln(n)}\right| = \sum_{n=2}^{\infty} \frac{1}{\ln(n)} > \sum_{n=2}^{\infty} \frac{1}{n}$, whih diverges by p-test. Therefore, $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$ converges conditionally.

[1] Or more accurately, by comparison with $\sum \frac{1}{n^2}$, which converges by p-test.

3. Determine whether each of the folllowing series converges or diverges. Justify your answer. (5 pts each)

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(n-3)}{5n+2}$

   $\displaystyle\lim_{n\to\infty} \frac{(-1)^n(n-3)}{5n+2}$ does not exist, therefore by the n^{th} term test, the series diverges.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(\frac{1}{n})}{n^2\cos(\frac{1}{n})}$

   As n increases from 1 to ∞, $\frac{1}{n}$ decreases from 1 to 0, thus causing $\sin(\frac{1}{n})$ to decrese and $\cos(\frac{1}{n})$ to increase. Therefore $f(n) = \frac{\sin(\frac{1}{n})}{n^2\cos(\frac{1}{n})}$ is a positive decreasing function for $n \geq 1$. Therefore we may apply the integral test, thus $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(\frac{1}{n})}{n^2\cos(\frac{1}{n})} \approx \int_1^{\infty} \frac{\sin(\frac{1}{n})}{n^2\cos(\frac{1}{n})} =^2 \int_{\cos(1)}^{0} \frac{1}{w}dw = \ln(w)|_{\cos(1)}^{0} = \ln(0) - \ln(\cos(1)) = -\ln(\cos(1)) \approx .6156$. Because the integral converges, the sum also converges.

4. Find the degree 3 taylor polynomial for $f(x) = \sqrt[3]{1+x}$ around $x = 0$. (5 pts)

$f(x) = \sqrt[3]{1+x} = (1+x)^{\frac{1}{3}}$, so $f'(x) = \frac{1}{3}(1+x)^{\frac{-2}{3}}$, $f''(x) = \frac{-2}{9}(1+x)^{\frac{-5}{3}}$ and $f'''(x) = \frac{10}{27}(1+x)^{\frac{-8}{3}}$. Thus $f(0) = 1$, $f'(0) = \frac{1}{3}$, $f''(0) = \frac{-2}{9}$, and $f'''(0) = \frac{10}{27}$. Thus our taylor polynomial is

$$f(x) \approx P_3(x) = 1 + \frac{1}{3}x + \frac{\frac{-2}{9}x^2}{2!} + \frac{\frac{-10}{27}x^3}{3!}$$

[2] By w-sub with $w = \cos\frac{1}{n}$, and $dw = \frac{\sin(\frac{1}{n})}{n^2}$. This causes our bounds to become $\cos(\frac{1}{1}) = \cos(1)$ and $\cos\frac{1}{\infty} = \cos(0) = 1$.