Name:

Show all work and include units where appropriate. (25 pts)

- 1. Circle "True" if the statement is always true. Otherwise, circle "False." You do not need to include an explanation. (2 pts each)
 - (a) On the interval $a \le x \le b$, the definite integral of a function f(x) is the total area between the graph and the x-axis between x = a and x = b.

True False The definite integral gives the area between the curve and the x-axis, but area below the x-axis counts negative, so if f(x) < 0 anywhere for $a \le x \le b$ the statement is false.

(b) For the function
$$f(x) = \int_0^x e^{t^3} dt, f'(x) = e^{x^3} \cdot 3x^2.$$

True False The second fundamental theorem asserts that $f'(x) = e^{x^3}$.

2. (a) State the Fundamental Theorem of Calculus. (2 pts) If f(x) is continuous for $a \le x \le b$, and F'(x) = f(x), then

$$\int_{a}^{b} f(x) \, dx = F(b) - F(a).$$

(b) Use the fundamental theorem of calculus to determine the positive value of b if the area under the graph of f(x) = 4x + 1 between x = 2 and x = b is equal to 11. (5 pts)

We have,

$$11 = \int_{2}^{b} (4x+1) \, dx = (2x^2+x) \Big|_{2}^{b} \tag{1}$$

$$= 2b^2 + b - (8+2).$$
(2)

We then solve for the roots of the quadratic equation $2b^2 + b - 21 = 0$, which are -3.5 and 3, and since we want b > 0 we have b = 3.

3. Find the derivatives of the following functions. (3 pts each)

(a) $f(x) = 3x \cos(\pi x)$

Using the product rule and chain rule, we obtain

$$f'(x) = 3\cos(\pi x) + 3x(-\sin(\pi x) \cdot \pi).$$

(b) $g(x) = 2x^2 e^{2x+3}$ Using the product rule and chain rule, we obtain

$$g'(x) = 4xe^{2x+3} + 2x^2e^{2x+3} \cdot 2.$$

(c) $h(x) = \frac{2x+7}{\sin(3x)}$ Using the quotient rule and chain rule, we obtain

$$\frac{\sin(3x)\cdot(2) - (2x+7)(3\cos(3x))}{\sin^2(3x)}$$

4. Consider the velocity function given by the table below.

(a) Approximate the distance traveled by the object by using a left sum with 4 subdivisions. (3 pts) For n = 4 subdivisions we have $\Delta t = 3$ and so

LEFT(4) = 3(6 + 8 + 9 + 11) = 102 m.

(b) Based on the information given, is your estimate an overestimate or underestimate? Why? (2 pts)
The function v(t) is increasing, and so the left hand sum is an underestimate.