Math 116 - Midterm

May 29, 2012

Name: ____

Instructor: ____

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1. **Do not open this exam until you are told to begin.**
2. This exam has 11 pages including this cover. There are 10 questions.
3. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you turn in the exam.
4. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
5. Show an appropriate amount of work for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
6. You may use your calculator. You are also allowed two sides of a 3 by 5 notecard.
7. If you use graphs or tables to obtain an answer, be certain to provide an explanation and sketch of the graph to show how you arrived at your solution.
8. Please turn off all cellphones and remove all headphones.

PROBLEM	POINTS	SCORE
1	12	
2	13	
3	9	
4	11	
5	9	
6	9	
7	10	
8	12	
9	6	
10	9	
TOTAL	100	

1. (3 pts each) Circle whether each statement is true or false. Give a brief explanation of why your answer is correct.

(a) If $f(x)$ is continuous and positive for $x > 1$, and if $\lim_{x\to\infty} f(x) = \infty$, then $\displaystyle\int_1^\infty \frac{1}{f(x)}dx$ converges.

TRUE **FALSE**

(b) For constants a, b, and c, $\int_{-5}^{5}(ax^2 + bx + c)dx = 2\int_0^5 (ax^2 + c)dx$.

TRUE **FALSE**

(c) If $F(x)$ is an antiderivative of $f(x)$, and $G(x)$ is an antiderivative of $g(x)$, then $F(x) \cdot G(x)$ is an antiderivative of $f(x) \cdot g(x)$.

TRUE **FALSE**

(d) $\int_0^2 (x - x^3)dx$ represents the area under the curve $y = x - x^3$ from $x = 0$ to $x = 2$.

TRUE **FALSE**

2. You and your fellow Math 116 students are running from a horde of zombies. Suppose that the positions of the zombie horde and of the group of students are given by parametric equations. The zombie horde has position

$$x_z = 4 + t^2, \qquad y_z = t^2 - t^3,$$

and the students have position

$$x_s = -t^2 + 6t + 4, \qquad y_s = -t^2 - 9.$$

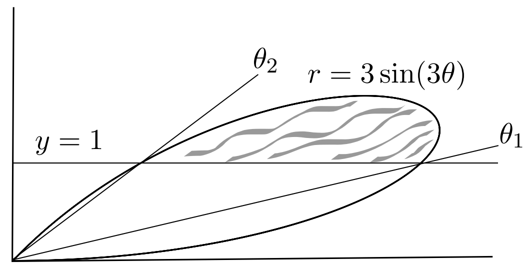
(a) Write parametric equations for the tangent line to the students' path at $t = 1$. (3 pts)

(b) Which group is moving faster at time $t = 2$? The zombies or the students? (3 pts)

(c) When plotted in the xy-plane, what is the concavity of the zombies' path at $t = 2$? (Justify your answer using calculus.) (3 pts)

(d) Do the zombies ever catch the students? Explain. (4 pts)

3. Consider the region in the first quadrant inside the curve $r = 3\sin(3\theta)$ and above the line $y = 1$.



(a) Find θ_1 and θ_2, the angles in the first quadrant at which the curve and line intersect. (Hint: You will need to use your calculator.) (3 pts)

(b) Use inequalities to describe the region using polar coordinates. (3 pts)

(c) Write out, but do not evaluate, an expression involving integral(s) giving the area of the region. (3 pts)

4. The error function, usually denoted by erf(x), is very important in statistics and probability. It is defined as

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt.$$

(a) Suppose $x > 0$. Is erf(x) increasing or decreasing? Is erf(x) concave up or concave down? Justify your answers using calculus. (4 pts)

(b) Use MID(4) to estimate erf(0.4). (4 pts)

(c) Is your answer to part b an overestimate or underestimate? Justify your answer using calculus. (Be careful.) (3 pts)

5. Suppose that $f(x), g(x)$, and $h(x)$ are continuous for $x > 0$, and that

$$0 \leq f(x) \leq g(x) \leq h(x) \text{ for } x \geq 1.$$

Suppose that $\int_1^\infty g(x)dx$ converges, while $\int_1^\infty h(x)dx$ diverges.

State whether the following improper integrals converge or diverge. Explain your reasoning. (3 pts each)

(a) $\displaystyle\int_{1000}^\infty h(x)dx$

(b) $\displaystyle\int_1^\infty f(3x+2)dx$

(c) $\displaystyle\int_1^\infty (h(x) - g(x))dx$ (Hint: Thinking about it graphically may help.)

6. Suppose $f(x)$ is a continuous, positive, twice-differentiable function with values given in the table below.

x	0	1	2	3	4
$f(x)$	12	10	6	3	2
$f'(x)$	-1	-3	-4	-1	1
$f''(x)$	-2	-1	2	1	3

(a) Compute $\displaystyle\int_2^3 \frac{f'(x)\ln(f(x))}{f(x)}dx$. (3 pts)

(b) Compute $\displaystyle\int_1^2 zf''(2z)dz$ (3 pts)

(c) Let $g(x) = \displaystyle\int_x^{x^2} f(t)dt$. Compute $g'(2)$. (3 pts)

7.

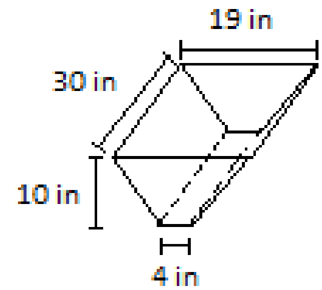
(a) Determine whether each of the following improper integrals converges or diverges. You do not necessarily need to compute the values of the integrals, but you must give a rigorous justification of your answer. You must show your work to receive full credit. (3 pts each)

(a) $\displaystyle\int_2^\infty \frac{x^2+1}{x^3-x^2-1}dx$

(b) $\displaystyle\int_2^\infty \frac{1}{x\ln^5(x)}dx$

(b) For which values of p does $\displaystyle\int_1^\infty \frac{\sqrt{1+x^p}}{x^p}dx$ converge? Justify your answer. (4 pts)

8. The pigs at the local farm are fed out of a trough that is in the shape of a trapezoidal prism. The height of the trough is 10 inches, the bottom has width 4 inches and the top has width 19 inches. The trough is 30 inches long. When it is time to feed the pigs, the farmer adds enough slop[1] to the trough to fill it to a height of 8 inches. Slop has a density of $.04\frac{lbs}{in^3}$.

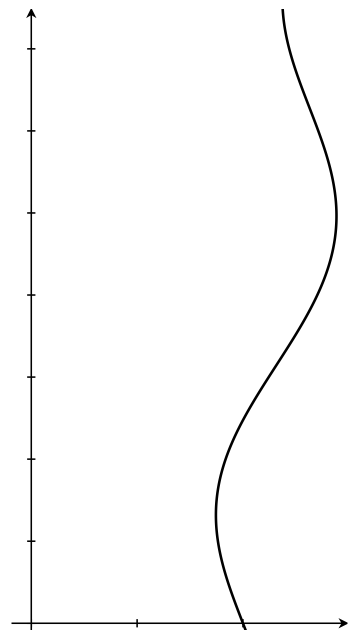


(a) Compute the work required to pump the slop out the top of the trough. Include units. (6 pts)

(b) Compute the force the slop exerts on one of the trapezoidal faces of the trough. Include units. (6 pts)

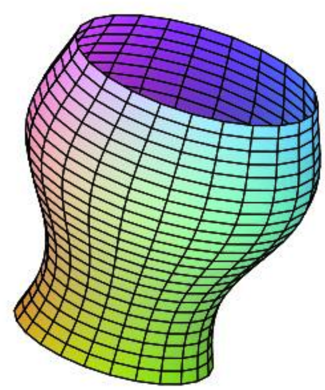
[1] Slop is a soup-like liquid fed to pigs as food.

9. A 7 inch tall beer glass can be modeled by the curve $x = 1 + .05y - 0.2\sin(y)$ rotated about the y-axis (x and y measured in inches).



(a) Set up, BUT DO NOT EVALUATE, an integral that gives the volume the glass can hold. (3 pts)

(b) Lucy the ladybug is sitting on top of the rim of the beer glass when she decides to walk down. Suppose she walks straight down along the outside of the glass to the bottom. Write an integral expression for how far she must walk to reach the bottom. You do not need to evaluate this integral. (3 pts)



10. Consider a metal sheet in the shape of the region bounded by the curves $y = 2x + 1$ and $y = x^2 - 4x + 1$. Suppose the density at any given point is directly proportional to the distance from that point to the y-axis with constant of proportionality k.

(a) Compute the area of the plate. (3 pts)

(b) Compute the mass of the plate.(3 pts)

(c) Let $(\bar{x}, \bar{y})$ be the coordinates of the center of mass of the plate. Compute $\bar{x}$. (3 pts)