Quiz 4 Name: Key

1. Table 1 below displays some values of an invertible, differentiable function $f(x)$, while Figure 2 depicts the graph of the function $g(x)$. Set $h(x) = f(g(x))$ and $j(x) = \dfrac{f(x)}{g(x)}$.

Table 1

x	1	2	3	4	5
$f(x)$	-5	-2	2	4	7
$f'(x)$	5	6	2	3	3
$f''(x)$	1	-1	-3	-2	0

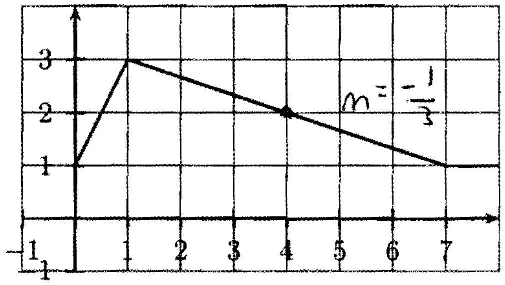

Figure 2: Graph of $g(x)$

Evaluate each of the following. **To receive partial credit you must show your work!**

(a) (2 points) $(f^{-1})'(2)$

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(3)} = \frac{1}{2}$$

(b) (2 points) $h'(4)$

$$h'(x) = f'(g(x))\, g'(x)$$

$$h'(4) = f'(g(4))\, g'(4) = f'(2)\cdot \frac{-1}{3} = 6\cdot \frac{-1}{3} = \boxed{-2}$$

0

(c) (2 points) $h''(4)$ *[Hint: you may want to use your work from part (b).]*

$$h''(x) = [f'(g(x))\cdot g''(x)] + [f''(g(x))\, g'(x)\cdot g'(x)]$$

$$h''(4) = [f'(g(4))\cdot g''(4)] + [f''(g(4))\, g'(4)\, g'(4)]$$

$$h''(4) = [f'(2)\cdot 0] + [f''(2)\cdot (\tfrac{-1}{3})^2] = 0 + (-1)(\tfrac{1}{9}) = \boxed{\frac{-1}{9}}$$

(d) (2 points) $j'(4)$

$$j'(x) = \frac{g(x)f'(x) - g'(x)f(x)}{(g(x))^2}$$

$$j'(4) = \frac{g(4)f'(4) - g'(4)f(4)}{(g(4))^2}$$

$$j'(4) = \frac{2\cdot 3 - (\frac{-1}{3})(4)}{2^2} = \frac{6 + \frac{4}{3}}{4} = \boxed{\frac{11}{6}}$$

2. [12 points] The equation below implicitly defines a hyperbola.

$$x^2 - y^2 = 2x + xy + y + 2.$$

a. [4 points] Find $\frac{dy}{dx}$.

$2x - 2yy' = 2 + y + xy' + y'$

$-2yy' - xy' - y' = -2x + 2 + y$

$y' = \dfrac{-2x + 2 + y}{-2y - x - 1}$

b. [4 points] Consider the two points $(4, 2)$ and $(2, -1)$. Show that one of these points lies on the hyperbola defined above, and one does not.

$4^2 - 2^2 = 2 \cdot 4 + 4 \cdot 2 + 2 + 2$

$16 - 4 = 8 + 8 + 4$

$12 \neq 20$

$4 - 1 = 4 - 2 - 1 + 2$

$3 = 3$

c. [4 points] For the point in part (b) which lies on the hyperbola, find the equation of the line which is tangent to the hyperbola at this point.

$y' = \dfrac{-2(2) + 2 - 1}{2 - 2 - 1}$

$= \dfrac{-3}{-1}$

$y + 1 = 3(x - 2)$

$y = 3x - 7$