Name: Key

Quiz 3 (20 points)

You must show all of your work!

1. Let $f(x) = x^{\cos(x)}$. Write an expression for $f'(3)$. DO NOT EVALUATE. (3 pts)

$$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{(3+h)^{\cos(3+h)} - 3^{\cos(3)}}{h}$$

2. Let $g(x)$ be the piecewise function defined by

$$g(x) = \begin{cases} x^2 - s & : x < 4 \\ t & : x = 4 \\ 3x+1 & : x > 4 \end{cases}$$

Find s and t such that $g(x)$ is continuous. (7 pts)

Need $\lim_{x\to 4^+} g(x) = \lim_{x\to 4^-} g(x) = g(4)$

$= \lim_{x\to 4^+} (3x+1) = 13$

$= t$

$\lim_{x\to 4^-} (x^2 - s) = 16 - s$

So $13 = 16 - s = t$

$s = 3$

$t = 13$

3. Ron Jeremy and Ron Paul are runing a 20 meter three legged race. The distance they have run (in meters) is a function, $s(t)$ of the time t (in seconds) since the beginning of the race. Below is a table of certain values of $s(t)$. (13 pts)

t (seconds)	0	1	2	3	4	5	6
$s(t)$ (meters)	0	1.1	2.7	4.9	8.7	14.1	20

Avg rates of change: 1.1 (0→1), 1.6 (1→2), 2.2 (2→3), 3.8 (3→4), 5.4 (4→5), 5.9 (5→6)

a) Is $s(t)$ concave up or concave down? What does this mean in practical terms? (4 pts)

Concave up because the avg rates of change of $s(t)$ are getting larger. This means the Rons are speeding up as the race progresses

b) Estimate $s'(4)$. Include units. What does this mean in practical terms? (6 pts)

(1) $s'(4) \approx \frac{s(5)-s(4)}{5-4} = 5.4$ m/s

(2) $s'(4) \approx \frac{s(4)-s(3)}{4-3} = 3.8$ m/s

(3) $s'(4) \approx \frac{s(5)-s(3)}{5-3} = 4.6$ m/s

(4) $s'(4) \approx \frac{5.4+3.8}{2} = 4.6$ m/s

Any one works!

This is the speed (in meters per second) 4 seconds after the race starts