Name: Key
Quiz 3 (20 points)

You must show all of your work!

1. Let \( f(x) = x \cos(x) \). Write an expression for \( f'(3) \). DO NOT EVALUATE. (3 pts)

\[
\begin{align*}
    f'(3) &= \lim_{h \to 0} \frac{f(3+h) - f(3)}{h} \\
    &= \lim_{h \to 0} \frac{\cos(3+h) - \cos(3)}{h} \\
    &= \lim_{h \to 0} \frac{\cos(3+h) - \cos(3)}{h} \\
    &= -3 \cos(3)
\end{align*}
\]

2. Let \( g(x) \) be the piecewise function defined by

\[
g(x) = \begin{cases} 
    x^2 - s & \text{if } x < 4 \\
    t & \text{if } x = 4 \\
    3x + 1 & \text{if } x > 4 
\end{cases}
\]

Find \( s \) and \( t \) such that \( g(x) \) is continuous. (7 pts)

\[
\begin{align*}
    \lim_{x \to 4^-} g(x) &= \lim_{x \to 4^-} (x^2 - s) = 16 - s \\
    \lim_{x \to 4^+} g(x) &= \lim_{x \to 4^+} (3x + 1) = 13 + 4 = 17 \\
    \lim_{x \to 4} g(x) &= g(4) = t
\end{align*}
\]

So, \( 13 = 16 - s = t \)

\[
\begin{align*}
    s &= 3 \\
    t &= 13
\end{align*}
\]
3. Ron Jeremy and Ron Paul are running a 20 meter three legged race. The distance they have run (in meters) is a function, \( s(t) \) of the time \( t \) (in seconds) since the beginning of the race. Below is a table of certain values of \( s(t) \). (13 pts)

<table>
<thead>
<tr>
<th>( t ) (seconds)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s(t) ) (meters)</td>
<td>0</td>
<td>1.1</td>
<td>2.7</td>
<td>4.9</td>
<td>8.7</td>
<td>14.1</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( t ) (seconds)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average change in ( s(t) )</td>
<td>( 1.1 )</td>
<td>( 1.6 )</td>
<td>( 2.2 )</td>
<td>( 3.8 )</td>
<td>( 5.4 )</td>
<td>( 5.9 )</td>
</tr>
</tbody>
</table>

a) Is \( s(t) \) concave up or concave down? What does this mean in practical terms? (4 pts)

Concave up because the avg rate of change of \( s(t) \) are getting larger. This means the Jons are speeding up as the race progresses.

b) Estimate \( s'(4) \). Include units. What does this mean in practical terms? (6 pts)

\[
\begin{align*}
1. & \quad s'(4) \approx \frac{s(5) - s(3)}{5 - 3} = 5.4 \text{ m/s} \\
2. & \quad s'(4) \approx \frac{s(5) - s(1)}{4 - 3} = 3.8 \text{ m/s} \\
3. & \quad s'(4) \approx \frac{s(5) - s(2)}{5 - 3} = 4.6 \text{ m/s} \\
4. & \quad s'(4) \approx \frac{5.4 + 3.8}{2} = 4.6 \text{ m/s} \\
\end{align*}
\]

Any one works!