Name: Key

Midterm 1 Quiz (61 points)
Time Limit[1]: 1 Hour.
You must show all of your work!

1. Bo and Samuel own Neverland Water Park, a highly sucessful theme park with aquatic rides. To better understand how well their business is doing, they hire Jesse to do some market research. Jesse determines that the number of people, N (in hundreds), who come to the park on a given day is a continuous function of the temperature, T (in ^oF) that day. Below is some of the date Jesse gathered. [13 points]

T	60	65	70	75	80	85	90	95
N	1	15	25	33	38	42	44	41

A) What are the units of $\frac{dN}{dT}$? [2 points]

$$\text{Unit of } \frac{dN}{dT} = \frac{\text{unit of } N}{\text{unit of } T} = \frac{\text{hundred of ppl}}{\text{degree F}}$$

B) Estimate $\frac{dN}{dT}$ when $T = 70$, $T = 75$, and $T = 90$. [3 points]

@ 70^o: $\frac{dN}{dt} \approx \frac{33-25}{75-70} = \frac{8}{5} = 1.6$

@ 75^o: $\frac{dN}{dt} \approx \frac{38-33}{80-75} = \frac{5}{5} = 1$

@ $T=90$: $\frac{dN}{dt} \approx \frac{41-44}{95-90} = \frac{-3}{5} = -.6$

(Answers may Vary!)

All have unit $\frac{\text{hundred ppl}}{^oF}$

C) Using your answer from A), how many people do you expect will come to the park on a day where the temperature is 77^oF? [3 points]

@ $T=75$, $N=33$ & $\frac{dN}{dt} = 1$

so @ $T=77$, $N \approx 33 + 2(1) = 35$

We expect about 3500 people to come on a 77^oF day.

D) Using your answer from A), estimate $\frac{d^2N}{dT^2}$ when $T = 70$. [2 points]

$$\frac{d^2N}{dT^2} \approx \frac{dN/dT\ (\text{when } T=75) - dN/dt\ (\text{when } T=70)}{75-70} = \frac{1-1.6}{5} = -.12 \left(\frac{\text{hundred ppl}}{(\text{deg F})^2}\right)$$

E) Is N an invertible function of T? Justify your answer. [3 points]

No, Since the fxn is continuous, there must be a temp between 90^o & 95^o where 4200 ppl. come. But 4200 ppl come if temp is 85^o. Can't be invertible if 2 Temps have the same # of ppl.

[1]This is how long I think you should spend on this to be 'on pace' to finish a full midterm in 90 minutes.

2. True or False (circle[2] **TRUE** if the statement is ALWAYS TRUE and circle **FALSE** otherwise). [10 points; 1 point each]

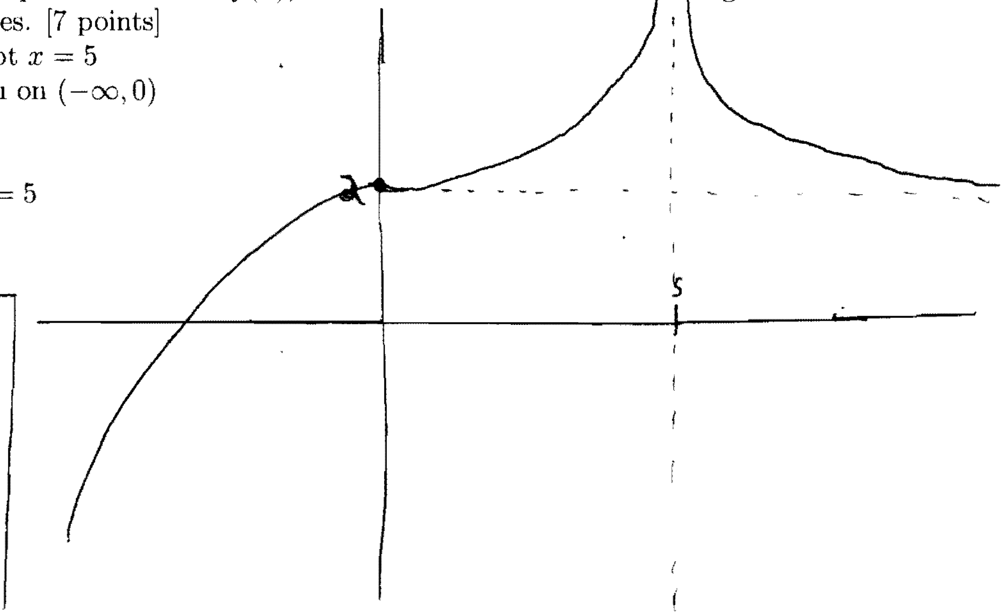
TRUE **FALSE** If $f(x)$ is a cubic polynomial, then $f(x)$ has at most 3 roots.
TRUE **FALSE** For any A and B greater than 0, $log(A \overset{\times}{+} B) = log(A) + log(B)$.
TRUE **FALSE** The function $g(y) = tan(y)$ is monotone. (on $-\pi/2 < y < \pi/2$)
TRUE **FALSE** Every invertible function is monotone. (monotone $\Rightarrow$ invertible)
TRUE **FALSE** If $a(2) = 2$ and $a(10) = -6$, then a has a root between 2 and 10. (True if a is cts)
TRUE **FALSE** If $p(q)$ is differentiable for $q = 7$, then $p(q)$ is continuous at $q = 7$.
TRUE **FALSE** The sum of two odd functions is an odd function. $\underbrace{f(-x)+g(-x)}_{(f+g)(-x)} = -f(x) + -g(x) = -(f+g)(x)$
TRUE **FALSE** If $l'(j)$ is concave down, then $l''(j) < 0$. [Change $l''(j)$ to $l'''(j)$ OR <0 to dec. OR $l'(j)$ to $l(j)$]
TRUE **FALSE** $3e^x$ can be obtained from e^x by a horizontal shift. Tricky! Shift by $ln(3)$.
TRUE **FALSE** $\lim_{x \to 0} \frac{|x|}{x} = 1$ (True if we take lim from right)

3. Fun with graphs! [13 points]

A) On the axes below, sketch the graph of a function $f(x)$, that satisfies ALL of the following properties. Be sure to label your axes. [7 points]

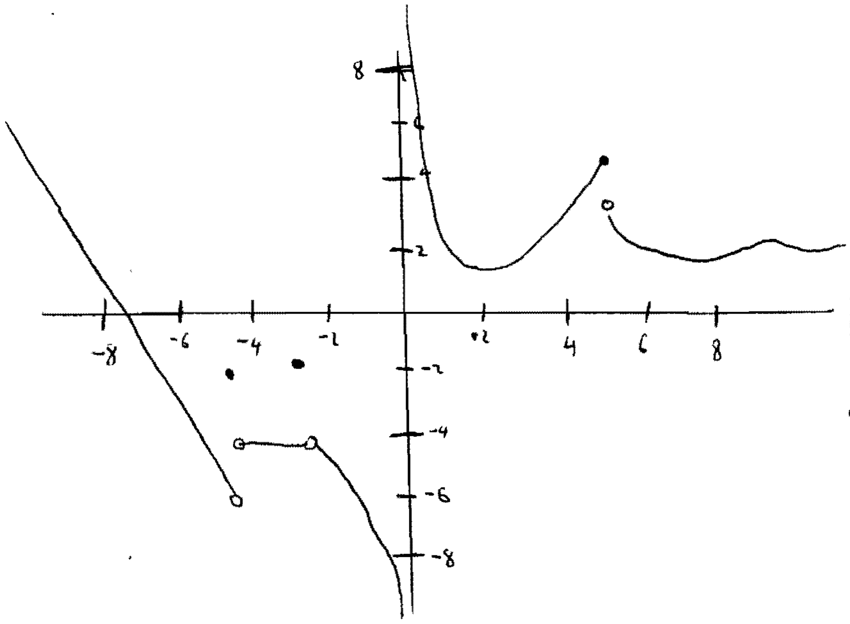
$f(x)$ is continuous everywhere except $x = 5$
$f(x)$ is increasing and concave down on $(-\infty, 0)$
$f(0) = 2$
$f''(x) > 0$ on $(0, 5)$
$f(x)$ has a vertical asymptote at $x = 5$
$\lim_{x \to \infty} f(x) = 2$
$f'(x) < 0$ for $x > 5$

NOTE: These Q's are common on exams. This one is a bit on the easy side. Try doing many of them. The key is using the $f(x)/f'(x)/f''(x)$ table to translate these statements into statements about f, then draw.



B) Using the graph of $g(x)$ below, compute the following limits. Write DNE if a limit does not exist. [6 points; 1 point each]

$\lim_{x \to -\infty} g(x) = +\infty$ (or DNE)
$\lim_{x \to -2} g(x) = -4$
$\lim_{x \to 0} g(x) =$ DNE
$\lim_{x \to 3} g(x) = 2$
$\lim_{x \to 5^-} g(x) = 3$
$\lim_{x \to \infty} g(x) = 2$



[2] For this just circle the correct answer. But for your own benefit, be able to justify your answers fully. For statements that are false, can you change the statement slightly to make it true?

THIS PROBLEM IS VERY Subtle

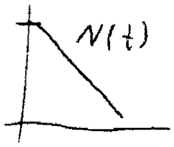
4. The Detroit Tigers are hosting the New York Yankees in a baseball game Tuesday[3] night in Comerica Park. The game will start at 8pm. Let $N(h)$ be the number of fans, in thousands, in Comerica Park h hours before the start of the game. Assume $N(h)$ is invertible. [13 points]

A) Interpret $N(3) = 5$. [2 pt]

At (8-3=) 5pm (3hrs before game starts), there will be 5000 fans in the stadium.

B) What is the sign of $N'(1)$? Justify your answer.[3 pts]

$N'(1)$ is negative because as our input (time before game) gets bigger, there are fewer fans in the stadium. [Think: Should $N(2)$ or $N(1)$ be bigger? $N(2) < N(1)$.]



C) Interpret $N^{-1}(10) = 1$. [2 pts]

$N^{-1}(10) = 1 \longleftrightarrow N(1) = 10$

AT 7pm (1hr before game start),

10,000 fans are in the stadium.

D) In addition to the fans coming to the stadium, there will be 5,219 players, coaches, security officers, and media writers are also in the stadium. In terms of N, write a function $P(m)$ which gives the number of total number of people in the stadium m minutes after noon on Sunday. [4 pts] If we are m minutes after noon, we are $8-\frac{m}{60}$ hours before start (8pm). Since P wants total ppl (not thousands) need to vert. rescale (stretch) by $\times 1000$. Also v-shift by 5219.

$$P(m) = 1000N\left(8-\frac{m}{60}\right) + 5219.$$

E) Suppose $P'(411) = 253$, where P is as in part D. Interpret this in practical terms. [2 pts]

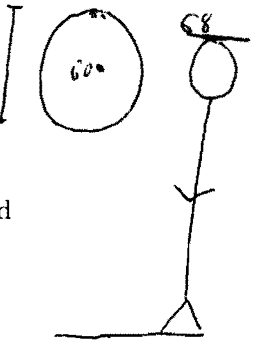
411 min before start = 1:09pm.

Between 1:09pm & 1:10pm, about 253 people entered the stadium.

[3]Yesterday? Two days ago? I wrote this question on Monday.

radius = 10 in   20 in

5) A circular dartboard of diameter 20 inches is on a wall, with the bull's-eye (center) exactly 5 feet above the floor. Lucy the Ladybug and an Albert the Ant are sitting at the very top of the dartboard having a fascinating conversation about pesticide . Suddenly, Paul the Person decides to spin the dartboard (clockwise) at a rate of 10 roations per minutes (RPM's). [12 points]



A) How fast is the dartboard spinning, in radians per second? How long, in seconds, is one period of rotation? [2 points]

$$\frac{10 \text{ rotations}}{1 \text{ minute}} = \frac{1 \text{ rotation}}{6 \text{ seconds}} = \frac{2\pi \text{ radians}}{6 \text{ seconds}} = \frac{\pi}{3} \text{ radians per second.}$$

The period is 6 seconds

B) Because ladybugs have suction-like grip on their feet, Lucy does not fall off the dartboard. Instead she spins around with the edge of the dartboard, going in circles. Find a formula for $L(t)$ which gives Lucy's height, in inches, off the ground t seconds after Paul starts spinning the dartboard. [4 pts]

max = 60 in + 10 in = 70 in

min = 60 in − 10 in = 50 in

start @ top

midline = 5 ft = 60 in

amplitude = 10 in = $\left(\frac{20 \text{ in}}{2}\right)$

$$L(t) = 10\cos\left(\tfrac{\pi}{3}t\right) + 60$$

C) Paul is exactly 5'8" tall. When is the first time after Paul spins the dartboard that Lucy will be exactly even with the top of Paul's head? [3 points]

68 in

$68 = 60 + 10\cos\left(\tfrac{\pi}{3}t\right)$ solve for t

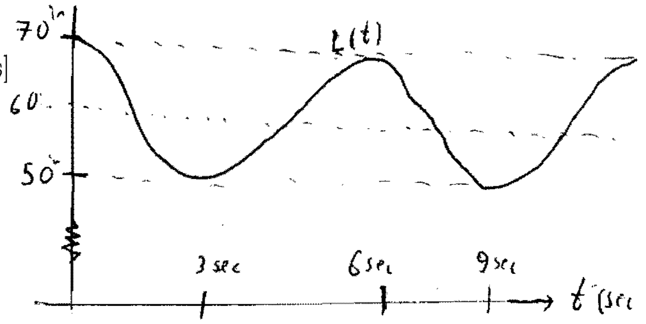
$8 = 10\cos\left(\tfrac{\pi}{3}t\right)$

$.8 = \cos\left(\tfrac{\pi}{3}t\right)$

$\tfrac{\pi}{3}t = \cos^{-1}(.8) \approx .6435$

$t = \tfrac{3}{\pi}\cos^{-1}(.8) \approx .6145$

D) Plot $L(t)$ on the axes below. [3 points]



BONUS) Albert also has very good grip in his feet. However, instead of sitting in one place and riding the edge of the dartboard, Albert starts walking toward the bull's-eye at a rate of 1 inch per second. Once he gets to the center, he sits there. Find a formula for $A(t)$ which gives Albert's height, in inches, off the ground t seconds after Paul starts spinning the dartboard. Draw a sketch of the graph of $A(t)$. [4 points]

$$A(t) = \begin{cases} (10-t)\cos\left(\tfrac{\pi}{3}t\right)+60 & \text{for } t \le 10 \\ 60 & \text{for } t \ge 10 \end{cases}$$

Technically $L(t) = \begin{cases} \sim & t \ge 0 \\ 70 & t < 0 \end{cases}$

& $A(t) = \begin{cases} \sim & t \le 10 \\ 60 & t \ge 10 \\ 0 & t \le 0 \end{cases}$